14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

A FAST WINDOWING TECHNIQUE FOR DESIGNING DISCRETE WAVELET MULTITONE TRANSCEIVERS EXPLOITING SPLINE FUNCTIONS

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ABSTRACT

A very fast technique for designing discrete wavelet multitone (DWMT) transceivers without using time-consuming nonlinear optimization is introduced. In this method, the filters in both the transmitting and receiving filter banks are generated based on the use of a single linear-phase finite-impulse response prototype filter and a cosine-modulation scheme and the prototype filter is optimized by using the windowing technique. The novelty of the proposed technique lies in exploiting spline functions in the transition band of the ideal filter, instead of using the conventional brick-wall filter. In this approach, a simple line search is used for finding the passband edge of the ideal filter for minimizing a predetermined cost function. The resulting DWMT transceivers closely satisfy the perfect reconstruction property, as is illustrated by means of examples

1. INTRODUCTION

Cosine-Modulated Filter Banks (CMFBs) with applications to communications are generally referred to as discrete wavelet multitone (DWMT) or discrete subband multicarrier (DSBM) transceivers [1]-[3]. The use of CMFBs is a very attractive way of generating these transceivers for the following reasons. First, high-selectivity and high-discrimination systems can be easily designed. Second, the resulting transmitting and receiving filters can be generated based on the use of a single linear-phase finite-impulse response (FIR) prototype filter. Third, in particular cases, fast algorithms can be applied to efficiently implementing the sub-carrier modulators utilizing parallel processing structures [4, 9].

This work describes a very efficient technique for designing prototype filters for DWMT systems as shown in Fig. 1. This design scheme is an improved version of that described in [5, 6], where the design of prototype filters for nearly perfect-reconstruction (NPR) CMFBs is based on the use of the windowing technique in such a manner that the cut-off frequency of the ideal filter the 3-dB point of the magnitude response of the prototype filter is approximately located at $\omega_p = \pi/(2M)$. As illustrated in [5, 6], this technique results in improved analysis-synthesis (or receiving-transmitting) filter banks when compared with those achieved using other existing similar design methods [7, 8].

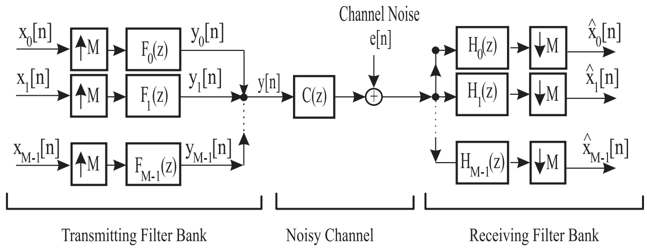

Figure 1 - Critically-sampled *M*-channel DWMT transceiver over a noisy channel.

In order to achieve even better overall filter bank performances, this work incorporates a spline transition function in the ideal low-pass filter definition as suggested in [10] for designing least-squared-error linear-phase FIR filters. Several examples are included illustrating the superiority of the proposed approach compared to those using brick-wall filters.

This work is organized as follows: Section 2 briefly reviews how to construct cosine-modulated DWMT transceivers. Section 3 describes the proposed technique for designing the prototype filter. Section 4 considers proper quality measures for evaluating the performance of the DWMT systems resulting when using the proposed technique compared with the perfect-reconstruction (PR) systems. Due to the close relations between the synthesis-analysis and analysis-synthesis filter banks, similar measures are also introduced for the latter banks. In Section 5, examples are included for both illustrating the benefits of the proposed design scheme and on how to properly select the window function and the spline function for the transition band shaping of the ideal filter. Finally, concluding remarks are drawn in Section 6.

2. COSINE-MODULATED DWMT TRANSCEIVERS

This section briefly reviews how to construct critically-sampled cosine-modulated DWMT systems as shown in Fig. 1, where the signal is transmitted over a noisy channel.

For such a system, based on an Nth-order linear-phase FIR prototype transfer function given by

$$H_p(z)=\sum_{n=0}^{N} h_p[n]z^{-n}, \quad h_p[n]=h_p[N-n] \text{ for } n=0,1,\ldots,N, \tag{1}$$

the impulse response coefficients of all the transmitting and receiving sub-channel filters, denoted by $f_k[n]$ and $h_k[n]$ for $k=0, 1, \ldots, M-1$, respectively, are generated as follows:

$$f_k[n]=2Mh_p[n]\cos\left[(2k+1)\frac{\pi}{2M}\left(n-\frac{N}{2}\right)-(-1)^k\frac{\pi}{4}\right], \tag{2a}$$

and

$$h_k[n]=2h_p[n]\cos\left[(2k+1)\frac{\pi}{2M}\left(n-\frac{N}{2}\right)+(-1)^k\frac{\pi}{4}\right], \tag{2b}$$

for $n=0, 1, \ldots, N$. Compared to the conventional construction of the $f_k[n]$'s (see, e.g., [4]), an additional constant M is included in equation (2a). This is because of the following reason. For the prototype filter as well as for the transmitting and receiving sub-channel filters resulting when applying the proposed technique, the maximum amplitude value in the passband is approximately equal to unity. Therefore, this constant is needed in order to preserve the signal energy when interpolating by a factor of M before using the transmitting sub-channel filters.

Depending on how to synthesize the prototype filter, the above cosine-modulation scheme results in either PR or NPR DWMT transceivers. This work concentrates on generating NPR systems.

3. PROPOSED DESIGN SCHEME FOR THE PROTOTYPE FILTER

This section describes the proposed design scheme for designing prototype filters based on applying the windowing technique to ideal filters, where the transition band is shaped by means of a spline function.

3.1 FIR filter design based on the use of transition-band spline functions

When generating an Nth-order linear-phase FIR prototype filter using the proposed technique, first a proper window function $w[n]$ is selected in such a way that it is nonzero only for $0 \le n \le N$ and satisfies $w[N-n]=w[n]$ for $n=0, 1, \ldots, N$. Second, the ideal filter with the passband and stopband edges being located at $\omega=\omega_p$ and $\omega=\omega_s$, respectively, is generated as follows:

$$h_{\text{ideal}}[n]=\frac{\sin\{[n-N/2](\omega_p+\omega_s)/2\}}{\pi[n-N/2]}\phi[n], \tag{3a}$$

where

$$\phi[n]=\left[\frac{\sin\{[n-N/2](\omega_p-\omega_s)/(2\psi)\}}{\pi[n-N/2](\omega_p-\omega_s)/(2\psi)}\right]^{\psi} \tag{3b}$$

is the ψth-degree spline function for shaping the transition band.

The frequency response of the resulting infinite-duration ideal filter is expressible as

$$H_{\text{ideal}}(e^{j\omega})=e^{-jN\omega/2}A_{\text{ideal}}(\omega), \tag{4}$$

where $A_{\text{ideal}}(\omega)\equiv 1$ in the passband region $[0, \omega_p]$ and $A_{\text{ideal}}(\omega)\equiv 0$ in the stopband region $[\omega_s, \pi]$, respectively, whereas in the transition band (ω_p, ω_s) the performance of $A_{\text{ideal}}(\omega)$ depends on ψ, the order of the spline function selected for shaping the transition band (for more details, see [10]).

After knowing both the window function and the impulse-response coefficients of the ideal filter, the impulse-response coefficients of the resulting prototype filter are given by

$$h_p[n]=w[n]\cdot h_{\text{ideal}}[n] \tag{5}$$

for $n=0, 1, \ldots, N$, as in the case of using a conventional brick-wall filter with $\omega_p \equiv \omega_s$.

3.2 Main benefits of using the spline function for the transition band shaping

The proposed method basically follows the same technique as described in [5, 6], where the impulse response of the prototype filter, as given by equation (5), is determined in the case of brick-wall ideal filters. The use of the ψth-order spline function in the transition band in the transition band (ω_p, ω_s) results in the following benefits. First, the stopband edge of the prototype filter can be fixed in terms of the roll-off factor, denoted by ρ, as

$$\omega_s=(1+\rho)\pi/(2M). \tag{6}$$

Second, by properly selecting the degree of the spline function for the transition band shaping gives an extra freedom in arriving at a prototype filter giving rise to a better overall transceiver performance compared to the case of using the brick-wall ideal filter.

3.3 Proposed Design Technique

Given N, the order of the prototype filter, M, the number of channels, ρ specifying the stopband edge according to equation (6), the window function $w[n]$ being nonzero for $0 \le n \le N$, and ψ, the degree of the spline function, the proposed design scheme carried out in the following steps:

Step 1: Fix $\omega_s=(1+\rho)\pi/(2M)$. Select the initial value for ω_p to be $\omega_p=(1-\rho)\pi/(2M)$.

Step 2: Generate the impulse response of the ideal filter $h_{\text{ideal}}[n]$ according to equations (3a) and (3b).

Step 3: Generate the impulse response $h_p[n]$ of the prototype filter according to equation (5).

Step 4: Determine the value of ω_p so that the magnitude response of the prototype achieves the value of

$1/\sqrt{2}$ at $\omega = \pi/(2M)$, that is, $\left|H_p\left(e^{j\pi/(2M)}\right)\right| = 1/\sqrt{2}$.

In this method, the optimized value of ω_p can be found by using a simple line search algorithm, thereby making the overall synthesis extremely fast. What is left is to select the window function and the degree of the spline function in such a manner that the resulting NPR DWMT transceiver closely approximates the PR one in the case, where the channel in Fig. 1 is ideal and there is no noise. Various alternatives for generating such a system will be considered in Section 5 by means of illustrative examples.

4. PERFORMANCE EVALUATION

This section considers proper quantities for measuring the distortions for evaluating the performance of NPR DWMT systems resulting when using the proposed technique compared to the PR ones. In addition, because of the close connections between the analysis-synthesis and synthesis-analysis filter banks, proper distortion measures are introduced for the corresponding NPR cosine-modulated filter bank (CMFB) as shown in Fig. 2.

4.1 Quality Measures for CMFB Systems

When omitting the effect of the processing unit and using definitions of the impulse-responses of the F_k(z)'s and H_k(z)'s, as given by equations (2a) and (2b), respectively, the relation between the output signal $\hat{x}[n]$ and the input signal $x[n]$ is expressible in the z-domain as

$$\hat{X}(z) = T_0(z)X(z) + \sum_{l=1}^{M-1} T_l(z)\, X\left(ze^{-j2\pi l/M}\right) \tag{7a}$$

where

$$T_0(z) = \sum_{k=0}^{M-1} F_k(z)H_k(z) \tag{7b}$$

is the distortion transfer function determining the distortion caused by the overall system for the un-aliased component $X(z)$ and

$$T_l(z) = \sum_{k=0}^{M-1} F_k(z)H_k\left(ze^{-j2\pi l/M}\right) \tag{7c}$$

for $l = 1, 2, \ldots, M$-1 are called the alias transfer functions and determine how well the aliased components $X\left(ze^{-j2\pi l/M}\right)$ of the input signal are attenuated.

For the PR condition, it is required that $T_0(z) \equiv z^{-N}$ and $T_l(z) \equiv 0$ for $l = 1, 2, \ldots, M$-1. Based on this, the quality measurements should concentrate on both the distortion on the un-aliased component and the aliasing distortion.

A good measure for the un-aliased distortion is the following quantity:

$$\delta_{pp} = \max_{\omega\in[0,\pi]}\left\{\left|T_0\left(e^{j\omega}\right)\right|\right\} - \min_{\omega\in[0,\pi]}\left\{\left|T_0\left(e^{j\omega}\right)\right|\right\}, \tag{8}$$

whereas a good measure for the overall aliasing distortion is the peak aliasing error as given by [4]

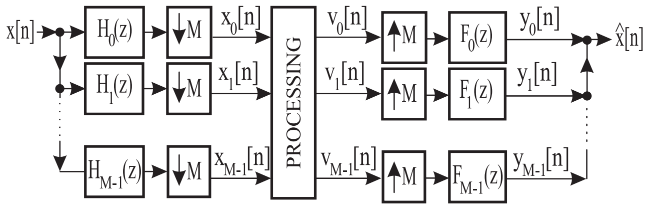

Fig. 2. Critically-sampled M-channel CMFB system.

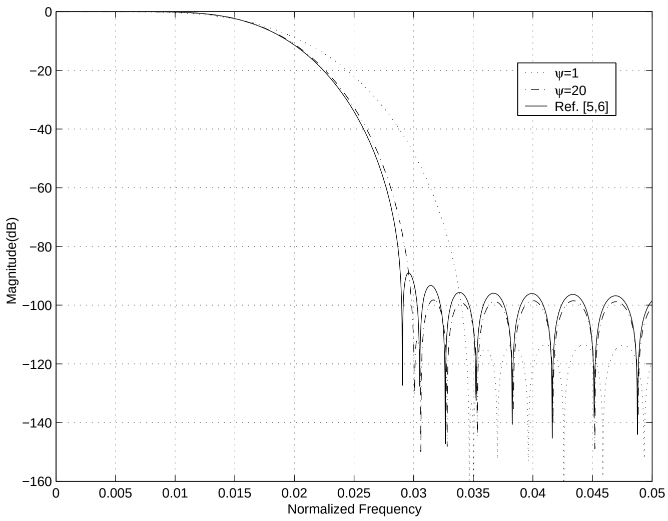

Fig. 3. Prototype filter magnitude responses $|H_p(e^{j\omega})|$ designed using the several techniques with the aid of the Kaiser window in the $\beta = 9$ case.

$$E_{alias} = \max_{\omega\in[0,\pi]}\left\{\sqrt{\sum_{l=1}^{M-1}\left|\sum_{k=0}^{M-1} F_k\left(e^{j\omega}\right)H_k\left(e^{j(\omega-2\pi l/M)}\right)\right|^2}\right\}. \tag{9}$$

4.2 Quality Measures for DWMT Transceivers

The performance of the DWMT transceivers as shown in Fig. 1 can be evaluated using the following two main signal-to-interference ratios (SIRs) in the case where there exist no noise. The first SIR, denoted by SIR_{ICI} for later use, is the ratio between the power of the received signal in the lth sub-channel due to the lth input signal and that due to the other input signals, called the inter-channel interference (ICI). This SIR_{ICI} is given by

$$SIR_{ICI}(l) = \frac{\frac{1}{\pi}\int_0^{\pi}\left(\left|T_{ll}\left(e^{j\omega}\right)\right|^2\right)\cdot d\omega}{\frac{1}{\pi}\int_0^{\pi}\left(\sum_{k=0,\,k\neq l}^{M-1}\left|T_{lk}\left(e^{j\omega}\right)\right|^2\right)\cdot d\omega}, \tag{10a}$$

where

$$T_{lk}(e^{j\omega}) = \sum_{m=0}^{M-1} H_l\left(e^{j(\omega-2\pi m)/M}\right)\cdot C\left(e^{j(\omega-2\pi m)/M}\right)\cdot F_k\left(e^{j(\omega-2\pi m)/M}\right) \tag{10b}$$

is the frequency response between the lth output and the kth input in the noisy-free transceiver of Fig. 1.

TABLE I
COMPARISON BETWEEN VARIOUS TECHNIQUES FOR DESIGNING 32-CHANNEL CMFBS AND DWMT TRANSCEIVERS WITH THE SUB-CHANNEL FILTERS OF LENGTH 512 ($N = 511$).

Prototype Filter Design Technique	δ_{pp}	E_{alias} (dB)	SIR_{ICI} (dB)	SIR_{ISI} (dB)
Blackman window (ref. [5, 6])	$\mathbf{1.4149 \cdot 10^{-3}}$	–93.885	99.1404	**67.7298**
Blackman window (proposed with $\psi = 1$)	$7.7261 \cdot 10^{-3}$	–98.604	105.1199	52.0328
Blackman window (proposed with $\psi = 20$)	$2.3331 \cdot 10^{-3}$	–93.289	99.2244	62.4770
Hamming window (ref. [5, 6])	$12.845 \cdot 10^{-3}$	–67.207	71.7194	47.5223
Hamming window (proposed with $\psi = 1$)	$15.344 \cdot 10^{-3}$	–78.561	83.0964	45.5444
Hamming window (proposed with $\psi = 20$)	$9.7849 \cdot 10^{-3}$	–69.133	74.5846	48.9237
Kaiser window (ref. [5, 6])	$2.5505 \cdot 10^{-3}$	–87.354	92.5326	62.8532
Kaiser window (proposed with $\psi = 1$, $\beta = 9$)	$1.7526 \cdot 10^{-3}$	**–103.26**	**108.1508**	67.1791
Kaiser window (proposed with $\psi = 20$, $\beta = 9$)	$6.4182 \cdot 10^{-3}$	–89.176	94.2620	54.7580
Ref. [7]	$2.2677 \cdot 10^{-3}$	–94.961	99.7242	65.3814
Ref. [8] with $\beta = 9$	$3.1786 \cdot 10^{-3}$	–90.888	95.8082	62.6819

The second SIR, denoted by SIR_{ISI}, is due to the inter-symbol interference (ISI). This SIR is defined as the ratio between the power of the received signal in the lth sub-channel due to the lth input signal and the reduction in this power due to ISI. This SIR is given by

$$SIR_{ISI}(l) = \frac{\frac{1}{\pi}\int_0^{\pi} \left|T_{ll}\left(e^{j\omega}\right)\right|^2 \cdot d\omega}{\frac{1}{\pi}\int_0^{\pi} \left|T_{ll}\left(e^{j\omega}\right) - 1\right|^2 \cdot d\omega}. \quad (11)$$

5. EXAMPLE DESIGNS

In order to illustrate the benefits of the proposed design scheme, several 32-channel CMFB and DWMT systems ($M = 32$) with 512-length prototype filters ($N = 511$) have been designed for $\rho = 1$, that is, $\omega_s = \pi/M$. The comparisons are summarized in Table I, considering an ideal environment in which there is an ideal channel $(C(z) = 1)$. The best results with respect to the above-defined quality measures, that is, δ_{pp}, E_{alias}, SIR_{ICI} and SIR_{ISI}, are indicated by the boldface letters. As seen from this table, the use of the Kaiser window with $\beta = 9$ and $\psi = 1$ results in significant global improvements and is a good choice for the $M = 32$ and $N = 511$ case.

The effect of the parameter ψ on the resulting prototype filter has also been studied. As an example, the resulting prototype filter responses designed using the Kaiser window in the $\beta = 9$ for $\psi = 1$ and $\psi = 20$ as well as that synthesized using the earlier technique described in [5, 6] are shown in Fig. 3. It is seen from this figure, the integer value of ψ has an influence on the transition band behavior of the prototype filter and thus also on the overlap factor between the adjacent channels.

The performance of transceivers with the prototype filter designed using various windowing techniques has also been tested inside a communication system. For comparison purposes, several bit-error rate (BER) curves based on Monte-Carlo simulations are shown in Figs. 4 and 5. For these figures, the proposed transceivers has been designed using the Kaiser window with $\beta = 9$ and $\psi = 1$. It is assumed that the input signals $x_k[n]$ for the system of Fig. 1 are independently chosen binary white data sequences. In addition, it is assumed that the transmission channel is ideal distorted with both additive white Gaussian noise and a narrow-band interference shown in Fig. 6. The detection is based on simple threshold detectors. As can be seen, all the simulated DWMT transceivers perform in almost the same manner including the low signal-to-noise ratios (SNRs).

According to the measurement results (shown in Figs. 4 and 5), the DWMT transceiver designed with the proposed technique (for $\psi = 1$) shows a good performance, jointly with that designed through technique proposed in [5, 6].

6. CONCLUSIONS

An improved windowing technique for designing prototype filter for cosine-modulated synthesis-analysis and analysis-synthesis multirate filter banks has been proposed. The key idea in this technique is to include a spline function for the transition band shaping of the ideal filter, instead of using a conventional brick-wall filter. Simulations results have shown the benefits of this approach.

When designing such multirate filter banks for practical applications, the main benefit of the proposed technique is that it enables one to design very quickly, without time-consuming non-linear optimization algorithms, systems with very many channels and very long prototype filters. Furthermore, by properly selecting the window function and the spline function for the transition band shaping of the ideal filter, the performance of the resulting NPR multirate systems approach the PR ones. Future work is devoted to further developing this technique by both properly selecting the degree of the spline function for the transition band shaping of the ideal filter and finding or generating new window functions.

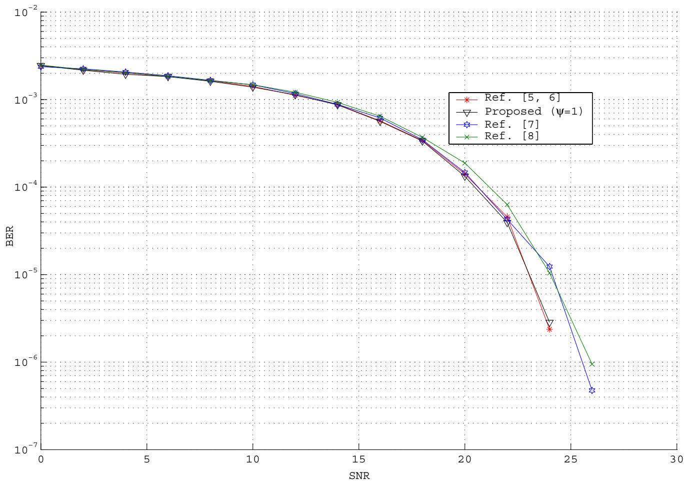

Fig. 4. BER results for various 32-channel DWMT transceivers.

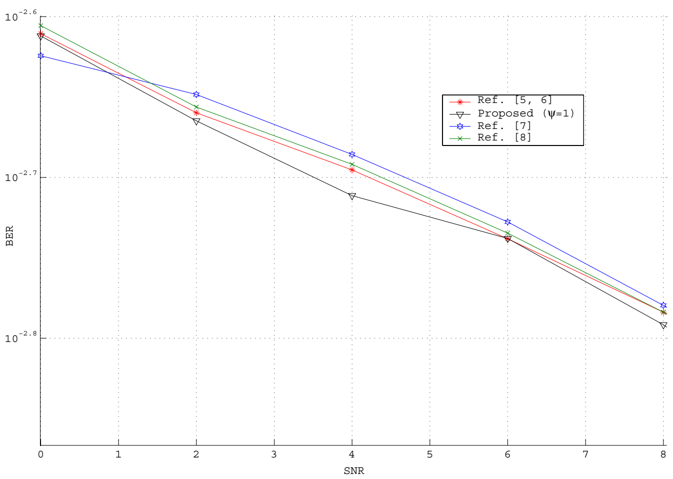

Fig. 5. Details of the BER results for various 32-channel DWMT transceivers.

ACKNOLEDGMENT

This work was supported in part by Comunidad Autónoma de Madrid and Universidad de Alcalá through project CAM-UAH2005/021.

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