

# WAVELET DOMAIN CHANNEL ESTIMATION FOR MULTIBAND OFDM UWB COMMUNICATIONS

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## ABSTRACT

*This paper presents a receiver that combines semi-blind channel estimation with the decoding process for multiband OFDM UWB communications. We particularly focus on reducing the number of estimated channel coefficients by taking advantage of the sparsity of UWB channels in the wavelet domain. The EM algorithm is used to estimate the channel without any need to pilot symbols inside the data frame. Channel estimation performance is enhanced by integrating a thresholding/denoising scheme within the EM algorithm leading at the same time to a reduction of the estimator complexity. Simulation results using IEEE UWB channel models show 3 dB of SNR improvement at a BER of  $10^{-3}$  compared to training sequence based channel estimation.*

## 1. INTRODUCTION

Ultra-Wide-Band (UWB) technology is defined as any wireless transmission scheme that occupies a bandwidth of more than 25 % of its center frequency, or greater than 500 MHz [1]. In February 2002, the federal communication commission (FCC) agreed to allocate 7500 MHz of spectrum for unlicensed use of UWB transmission in the 3.1-10.6 GHz frequency band. This wide spectrum allocation has initiated a lot of research activity from both industry and academia.

In recent years, UWB system design has experienced a shift from the traditional “single-band” radio that occupies the whole allocated spectrum in favor of a “multiband” design approach [2]. “Multiband” consists in dividing the available UWB spectrum into several subbands, each one occupying approximately 500 MHz (minimum bandwidth for a UWB system according to FCC definition). This bandwidth reduction relaxes the requirement on sampling rates of ADCs, consequently enhancing digital processing capability. Multiband orthogonal frequency division multiplexing (OFDM) [2] is a scheme which enables high data rate UWB transmission inherits all the strength of OFDM that has already been proven for wireless communications (ADSL, DVB, 802.11a, 802.16.a, etc.). This approach uses a conventional OFDM system [3] combined with bit interleaved coded modulation (BICM) and frequency hopping over different subbands for improved diversity and multiple access.

In the multiband OFDM proposal [4], channel estimation is performed using known symbols (pilots) transmitted periodically inside the information frame assuming that the channel is static between two pilot sequences. Thus in time varying channels one must send more pilot patterns resulting in a significant loss in spectral efficiency.

We propose here an alternative semi-blind method based on the sparse wavelet domain representation of UWB channels addressed in [5]. The proposed algorithm requires only one pilot symbol per frame for initialization and can be used for estimating both static and time varying UWB channels. The Expectation Maximization (EM) [6] algorithm is used to approximate a maximum likelihood (ML) solution of the unknown channel. In the decoder part we used a “soft” output BCJR decoder [7] to provide the probability

of encoded bits which is exactly what the EM algorithm requires for channel estimation. Thus, in our technique the iterative process of the EM algorithm is naturally combined with the decoding operation of encoded data. Although joint channel estimation and soft detection of coded OFDM has already been studied in the literature [8] [9], the originality of our method lies in the parsimonious representation of UWB channels in the wavelet domain for reducing channel estimation complexity and improving its accuracy.

After presenting an overview of the multiband OFDM system in section 2, we introduce the wavelet domain problem formulation in section 3. Section 4 describes the proposed joint channel estimation and decoding algorithm and shows how the number of estimated parameters can be reduced through the iterations. Section 5 illustrates via simulations the performance of the proposed method in realistic UWB channel environments and section 6 concludes the paper.

Notational conventions are as follows :  $\mathbf{I}_M$  represents an  $M \times M$  identity matrix;  $(\cdot)^T$ ,  $(\cdot)^H$  and  $(\cdot)^*$  denote vector transpose, Hermitian transpose and conjugation, respectively.

## 2. MULTIBAND OFDM SYSTEM OVERVIEW

In a multiband OFDM system, the whole UWB spectrum is divided into 14 smaller non-overlapping subbands each one occupying 528 MHz of bandwidth [2]. The three lower bands are used for standard and mandatory operation whereas the rest of the bands are reserved for optional use or future expansions. Information is transmitted using OFDM modulation over one of the subbands in a particular time-slot. The transmitter architecture for the multiband OFDM system is very similar to that of a conventional wireless OFDM system. The main difference is that multiband OFDM system uses a time-frequency code (TFC) to select the center frequency of different subbands which is used not only to provide frequency diversity but also to distinguish between multiple users (see figure 1). As shown in figure 2, after channel coding, a block of bits is interleaved and mapped to QPSK symbols. Different puncturing patterns of a 1/3 convolutional mother code combined with time and/or frequency repetition, generate ten data rates from 55 Mbps to 480 Mbps. One OFDM symbol has a duration of 312.5 ns and a bandwidth of 528 MHz. A 128 point IFFT is used along with a cyclic prefix (CP) length of 60.6 ns to modulate 122 subcarriers among which 100 subcarriers are allocated to data, 12 subcarriers are used for frame synchronization and 10 subcarriers provide 9.5 ns of guard interval for switching between subbands. Here, we consider multiband OFDM in its mandatory mode *ie.* employing 3 first subbands of 528 MHz with center frequencies at 3.432, 3.960 and 4.448 GHz. More details about multiband OFDM system parameters and its advantages for UWB transmission can be found in [2] [4].

### 3. OBSERVATION MODEL AND WAVELET DOMAIN PROBLEM FORMULATION

#### 3.1 Traditional data modeling for OFDM channel estimation

Let's consider the multiband OFDM transmission of figure 2 using  $N$  subcarriers among which  $K$  are allocated to data. At the receiver, assuming a CP longer than the channel maximum delay spread and perfect synchronization, OFDM converts a frequency selective channel into  $K$  parallel flat fading subchannels [3]. Hence, the observation model corresponding to each subband can be written in frequency domain as:

$$\mathbf{y}_i = \text{diag}(s_i)\mathbf{h}_i + \mathbf{n}_i \quad i = 1, 2, 3 \quad (1)$$

where  $K \times 1$  vectors  $\mathbf{y}_i, s_i, \mathbf{h}_i$  and  $\mathbf{n}_i$  denotes respectively the observation, the unknown QPSK symbols, the channel frequency response and a zero mean white complex Gaussian noise with variance  $\sigma^2$  regarding to data subcarriers and  $i$  is the corresponding subband index. Major part of previous work in OFDM semi-blind channel estimation [8] [9] were based on a data model similar to (1) and proposed to estimate directly the channel frequency response ( $\mathbf{h}_i$ ) which requires the estimation of  $3 \times K$  different channel coefficients for the 3 subbands.

#### 3.2 Wavelet domain data modeling and motivations

Instead of working with the above observation model for estimating independently the channel for each subband, we propose to derive from (1) an equivalent observation model in which the channel corresponding to all 3 subband is involved (see figure 1):

$$\mathbf{Y} = \text{diag}(\mathbf{S})\mathbf{H} + \mathbf{n} \quad (2)$$

where  $\mathbf{Y}^T = [\mathbf{y}_1^T, \mathbf{y}_2^T, \mathbf{y}_3^T]$ ,  $\mathbf{S}^T = [s_1^T, s_2^T, s_3^T]$  and  $\mathbf{H}^T = [\mathbf{h}_1^T, \mathbf{h}_2^T, \mathbf{h}_3^T]$  are  $1 \times M$  ( $M = 3K$ ) vectors of observation, sent QPSK symbols and total channel frequency response over 3 subbands. In the above data model, the channel is assumed to be constant over 3 OFDM symbol period which is not a restrictive supposition due to the slow time variability of UWB channels.

Wavelet transform is known for its ability to provide *parsimonious* expansion for a large family of signals. In [5] we showed that orthogonal discrete wavelet transform (ODWT) provides a very sparse representation (a few large coefficients and many small ones) of UWB channel impulse response. This observation is the key to our approach since it suggests to express the UWB channel impulse response in terms of its wavelet coefficients. More clearly, in our model, we are estimating the wavelet coefficients of the channel impulse response, taken over the 1.584 GHz bandwidth, even if in the transmitter it is practically used by slices of 528 MHz bandwidth. Estimating the channel over a wider bandwidth is important to have a good sparsity in the wavelet domain.

Let  $\mathbf{F}_{M,L}$  be the truncated FFT matrix constructed from the  $3N \times 3N$  complete FFT matrix by keeping the  $M$  rows corresponding to data tones and the first  $L$  columns where  $L$  is the length of the total channel impulse response over 3 subbands. Let define  $\mathbf{W}$  as the  $L \times L$  inverse ODWT matrix. We can write  $\mathbf{H} = \mathbf{F}_{M,L}\mathbf{W}\boldsymbol{\theta}$  where  $\boldsymbol{\theta}$  is the vector of wavelet coefficients. Observation model (2) is rewritten as:

$$\begin{aligned} \mathbf{Y} &= \text{diag}(\mathbf{S})\mathbf{F}_{M,L}\mathbf{W}\boldsymbol{\theta} + \mathbf{n} \\ &= \text{diag}(\mathbf{S})\mathbf{T}\boldsymbol{\theta} + \mathbf{n} \end{aligned} \quad (3)$$

where  $\mathbf{T} = \mathbf{F}_{M,L}\mathbf{W}$ . Equation (3) is the starting point of our EM based channel estimation algorithm.

### 4. REDUCED COMPLEXITY EM ALGORITHM FOR JOINT CHANNEL ESTIMATION AND DECODING

This section presents the EM based algorithm for multiband OFDM channel estimation. We start with the reasons for using the EM algorithm then derive the iterative channel update formula. We show

how the number of estimated coefficients can be significantly reduced at each iteration of the EM algorithm thanks to the wavelet domain sparse representation of the UWB channel. Finally we describe the employed decoding method along with the reasons it is combined with the channel estimation process.

#### 4.1 EM principle and application to wavelet domain channel estimation

Our first step consists in decomposing the white Gaussian noise in (3) into the sum of two different Gaussian noises, as proposed in [10]:

$$\mathbf{n} = \alpha \text{diag}(\mathbf{S})\mathbf{n}_1 + \mathbf{n}_2 \quad (4)$$

where  $\alpha$  ( $\alpha^2 < \sigma^2$ ) is a positive parameter and  $\mathbf{n}_1$  and  $\mathbf{n}_2$  are independent noises such that  $\mathbf{n}_1 \sim \mathcal{N}(0, \mathbf{I}_M)$  and  $\mathbf{n}_2 \sim \mathcal{N}(0, \sigma^2 \mathbf{I}_M - \alpha^2 \text{diag}(\mathbf{S})\text{diag}(\mathbf{S})^H)$ . Since QPSK is a constant envelope modulation,  $\mathbf{n}_2 \sim \mathcal{N}(0, (\sigma^2 - \alpha^2)\mathbf{I}_M)$ . The idea behind the above noise decomposition is that it allows the introduction of the hidden vector  $\mathbf{z}$  (see (5)) which provides us a direct relation between the true and estimated wavelet coefficients corrupted by additive white Gaussian noise. Using  $\mathbf{n}_1$  and  $\mathbf{n}_2$ , we can rewrite the observation model (3) as:

$$\begin{cases} \mathbf{z} &= \mathbf{T}\boldsymbol{\theta} + \alpha\mathbf{n}_1 \\ \mathbf{Y} &= \text{diag}(\mathbf{S})\mathbf{z} + \mathbf{n}_2 \end{cases} \quad (5)$$

The general problem is to jointly estimate/update  $\boldsymbol{\theta}$  and detect the information symbols  $\mathbf{S}$  while taking advantage of any *a priori* on them. Since  $\mathbf{S}$  and  $\mathbf{z}$  are unknown, we have an observation model with missing data and hidden variables where the ML solution of  $\boldsymbol{\theta}$  has no close form. In such situations, the EM algorithm [6] is usually used to maximize the expectation of the likelihood function over all possible missing and hidden variables.

Let  $\mathbf{x} = \{\mathbf{Y}, \mathbf{S}, \mathbf{z}\}$  be the *complete data set* in the EM algorithm terminology. Note that the observation set  $\mathbf{Y}$  determines only a subset of the space  $\chi$  of which  $\mathbf{x}$  is an outcome. We search  $\boldsymbol{\theta}$  that maximizes the *complete log-likelihood*  $\log p(\mathbf{x}|\boldsymbol{\theta})$ . After initialization, the EM algorithm alternates between the following two steps (until some stopping criterion) to produce a sequence of estimates  $\{\boldsymbol{\theta}^{(t)}, t = 0, 1, \dots, t_{\max}\}$ .

- **Expectation step (E-step):** The conditional expectation of the complete log-likelihood given the observed vector and the current estimate  $\boldsymbol{\theta}^{(t)}$  is calculated. This quantity is called the *auxiliary* or *Q-function*:

$$Q(\boldsymbol{\theta}, \boldsymbol{\theta}^{(t)}) = \mathbb{E}_{\mathbf{S}, \mathbf{z}} \left[ \log p(\mathbf{Y}, \mathbf{S}, \mathbf{z} | \boldsymbol{\theta}) \middle| \mathbf{Y}, \boldsymbol{\theta}^{(t)} \right] \quad (6)$$

- **Maximization step (M-step):** The estimated parameter is updated by the maximization of the *Q-function*:

$$\boldsymbol{\theta}^{(t+1)} = \arg \max_{\boldsymbol{\theta}} \{Q(\boldsymbol{\theta}, \boldsymbol{\theta}^{(t)})\} \quad (7)$$

Next, we derive the channel update formula by applying the EM principle to our channel estimation problem using the data model (5).

##### 4.1.1 E-step: Computation of the Q-function

The complete likelihood is  $p(\mathbf{Y}, \mathbf{S}, \mathbf{z} | \boldsymbol{\theta}) = p(\mathbf{Y} | \mathbf{S}, \mathbf{z}, \boldsymbol{\theta}) p(\mathbf{S} | \mathbf{z}, \boldsymbol{\theta}) p(\mathbf{z} | \boldsymbol{\theta})$ . According to (5), conditioned on  $\mathbf{z}$ ,  $\mathbf{Y}$  is independent of  $\boldsymbol{\theta}$ . Furthermore,  $\mathbf{S}$  which results from coding and interleaving of bit sequence is independent of  $\mathbf{z}$  and  $\boldsymbol{\theta}$ . Since  $\alpha\mathbf{n}_1$  is a white complex Gaussian noise vector such that  $\alpha\mathbf{n}_1 \sim (\pi\alpha^2)^{-M} \exp(-\|\mathbf{n}_1\|^2/\alpha^2)$ , the complete log-likelihood

can be simplified to:

$$\begin{aligned}
 \log p(\mathbf{Y}, \mathbf{S}, \mathbf{z} | \theta) &= \log [p(\mathbf{Y} | \mathbf{S}, \mathbf{z}) p(\mathbf{S}) p(\mathbf{z} | \theta)] \\
 &= \log p(\mathbf{z} | \theta) + \text{cst.} \\
 &= -\frac{\|\mathbf{z} - \mathbf{T}\theta\|^2}{\alpha^2} + \text{cst.} \\
 &= -\frac{\theta^H \mathbf{T}^H \mathbf{T} \theta - 2\theta^H \mathbf{T}^H \mathbf{z}}{\alpha^2} + \text{cst.} \quad (8)
 \end{aligned}$$

where cst. stands for constant terms that do not depend on  $\theta$ . According to (6) we have:

$$\begin{aligned}
 Q(\theta, \theta^{(t)}) &= \mathbb{E}_{\mathbf{S}, \mathbf{z}} \left[ -\frac{\theta^H \mathbf{T}^H \mathbf{T} \theta - 2\theta^H \mathbf{T}^H \mathbf{z}}{\alpha^2} + \text{cst.} \mid \mathbf{Y}, \theta^{(t)} \right] \\
 &= -\frac{\theta^H \mathbf{T}^H \mathbf{T} \theta - 2\theta^H \mathbf{T}^H \mathbb{E}_{\mathbf{S}, \mathbf{z}} [\mathbf{z} | \mathbf{Y}, \theta^{(t)}]}{\alpha^2} + \text{cst.} \\
 &= -\frac{\|\tilde{\mathbf{z}}^{(t)} - \mathbf{T}\theta\|^2}{\alpha^2} + \text{cst.} \quad (9)
 \end{aligned}$$

where  $\tilde{\mathbf{z}}^{(t)} = \mathbb{E}_{\mathbf{S}, \mathbf{z}} [\mathbf{z} | \mathbf{Y}, \theta^{(t)}]$ . It is clear from (9) that the E-step involves only the computation of the following conditional expectation:

$$\begin{aligned}
 \tilde{\mathbf{z}}^{(t)} &= \mathbb{E}_{\mathbf{S}, \mathbf{z}} [\mathbf{z} | \mathbf{Y}, \theta^{(t)}] \\
 &= \sum_{\mathbf{S} \in \Psi} \int_{\mathbf{z} \in \zeta} \mathbf{z} p(\mathbf{z}, \mathbf{S} | \mathbf{Y}, \theta^{(t)}) d\mathbf{z} \\
 &= \sum_{\mathbf{S} \in \Psi} \underbrace{\left( \int_{\mathbf{z} \in \zeta} \mathbf{z} p(\mathbf{z} | \mathbf{Y}, \theta^{(t)}) d\mathbf{z} \right)}_{\tilde{\mathbf{z}}^{(t)}} p(\mathbf{S} | \mathbf{Y}, \theta^{(t)}) \quad (10)
 \end{aligned}$$

where the last equation results from the independence between  $\mathbf{S}$  and  $\mathbf{z}$  belonging respectively to the sets  $\Psi$  and  $\zeta$  which contain all of their possible values. Note that each entry of  $\mathbf{S}$  takes one (unknown) discrete value out of four complex points inside the QPSK constellation whereas components of  $\mathbf{z}$  are continuous variables. Since both  $p(\mathbf{Y} | \mathbf{z})$  and  $p(\mathbf{z} | \theta^{(t)})$  are Gaussian densities,  $p(\mathbf{z} | \mathbf{Y}, \theta^{(t)}) \propto p(\mathbf{Y} | \mathbf{z}) p(\mathbf{z} | \theta^{(t)})$  is also Gaussian. By standard manipulation of Gaussian densities we have:

$$\tilde{\mathbf{z}}^{(t)} = \mathbf{T}\theta^{(t)} + \frac{\alpha^2}{\sigma^2} \text{diag}(\mathbf{S}^*) (\mathbf{Y} - \text{diag}(\mathbf{S}) \mathbf{T}\theta^{(t)}) \quad (11)$$

where  $(\cdot)^*$  denotes the conjugation. By using (11) in (10) and after some simplifications we get:

$$\tilde{\mathbf{z}}^{(t)} = \left(1 - \frac{\alpha^2}{\sigma^2}\right) \mathbf{T}\theta^{(t)} + \frac{\alpha^2}{\sigma^2} \text{diag}(\mathbf{S}^*) \mathbf{Y} \quad (12)$$

where  $\overline{\text{diag}(\mathbf{S}^*)} = \sum_{\mathbf{S} \in \Psi} \text{diag}(\mathbf{S}^*) p(\mathbf{S} | \mathbf{Y}, \theta^{(t)})$ . The E-step is completed by inserting  $\tilde{\mathbf{z}}^{(t)}$  into  $Q(\theta, \theta^{(t)})$ , equation (9).

#### 4.1.2 M-step: Derivation of the Parameter Update Formula

In this step the estimate of parameter  $\theta$  is updated via the maximization of the auxiliary function. Using (9) in (7) leads to a least square (LS) problem the solution of which is [11]:

$$\theta^{(t+1)} = \arg \max_{\theta} \left\{ -\frac{\|\tilde{\mathbf{z}}^{(t)} - \mathbf{T}\theta\|^2}{\alpha^2} \right\} \quad (13)$$

$$= (\mathbf{T}^H \mathbf{T})^{-1} \mathbf{T}^H \tilde{\mathbf{z}}^{(t)} \quad (14)$$

Recall from section 3 that  $\mathbf{T} = \mathbf{F}_{M,L} \mathbf{W}$ . By assuming that all subcarriers of the multiband OFDM system are allocated to data

( $K=N$ ),  $\mathbf{T}$  becomes orthogonal ( $\mathbf{T}^H \mathbf{T} = \mathbf{I}_L$ ) and we get the following update formula for  $\theta$ :

$$\theta^{(t+1)} = \mathbf{T}^H \tilde{\mathbf{z}}^{(t)} \quad (15)$$

$$= \left(1 - \frac{\alpha^2}{\sigma^2}\right) \theta^{(t)} + \frac{\alpha^2}{\sigma^2} \mathbf{T}^H \overline{\text{diag}(\mathbf{S}^*)} \mathbf{Y} \quad (16)$$

where  $\tilde{\mathbf{z}}^{(t)}$  is replaced from (12).

#### 4.1.3 Reducing Channel Estimation Complexity via Wavelet Sparseness Property

Equation (13) shows that  $\theta^{(t+1)}$  coincides with the ML solution of  $\tilde{\mathbf{z}}^{(t)} = \mathbf{T}\theta + \alpha \mathbf{n}'$ . Left multiplying both sides by  $\mathbf{T}^H$  and recalling (15) get  $\theta^{(t+1)} = \theta + \alpha \mathbf{n}'$ . Due to the orthogonality of  $\mathbf{T}$ ,  $\alpha \mathbf{n}'$  is also an array of independent zero mean white Normal noise with variance  $\alpha^2$ . Thus,  $\theta^{(t+1)}$  can be interpreted as a *virtual observation* vector which is equal to the unknown parameter  $\theta$  corrupted by "white" Gaussian noise. The noise decorrelation is a crucial requirement since it opens the issue to process wavelet coefficients independently of each other. As mentioned before, wavelet transform allows parsimonious representation for UWB channels [5] so it is reasonable to assume that only a few "large" components of  $\theta^{(t+1)}$  really contain information about the unknown parameter  $\theta$  and should be kept, while the "small" coefficients are probably attributed to the noise and should be shrunk or replaced by zero. The extraction of those "significant" coefficients can be naturally done by thresholding each ODWT coefficient [12]. Let  $\theta_j$  denote an arbitrary element of the complex vector  $\theta^{(t+1)}$ . The hard thresholding function is defined as:

$$\delta_{\lambda}(\theta_j) = \begin{cases} \theta_j & |\theta_j| > \lambda \\ 0 & |\theta_j| \leq \lambda \end{cases} \quad (17)$$

where  $\lambda$  is the threshold level. Among several approaches for the choice of a particular threshold, we adopt the simple *Minimax* procedure [12] which suggest to set  $\lambda = \alpha$ . In order to reduce the channel estimation computational load, we propose to discard at each iteration, the elements of  $\theta^{(t+1)}$  that fall below  $\alpha$ . The above procedure can be modeled as:

$$\tilde{\theta}^{(t+1)} = \Omega \theta^{(t+1)}, \quad \tilde{\mathbf{T}} = \mathbf{T} \Omega^T \quad (18)$$

where the selection matrix  $\Omega$  gathers in  $\tilde{\theta}^{(t+1)}$  the components of  $\theta^{(t+1)}$  that must be kept and  $\tilde{\mathbf{T}}$  is constructed from  $\mathbf{T}$  by keeping the rows corresponding to kept indexes. At the beginning ( $t=0$ ),  $\Omega$  is initialized with an identity matrix and the EM algorithm estimates all coefficients. After each M-step, the number of unknown parameters to be estimated in the next iteration is reduced by replacing  $\theta^{(t+1)}$  and  $\mathbf{T}$  by  $\tilde{\theta}^{(t+1)}$  and  $\tilde{\mathbf{T}}$  in the update formula (16).

## 4.2 Decoding Method and Implementation Issues

In this section we describe how the proposed channel estimation algorithm is related to the decoding process via the a posteriori probability  $p(\mathbf{S} | \mathbf{Y}, \theta^{(t)})$  involved in the channel update formula. According to the Bayes law, we have  $p(\mathbf{S} | \mathbf{Y}, \theta^{(t)}) \propto p(\mathbf{Y} | \mathbf{S}, \theta^{(t)}) p(\mathbf{S})$ . Since we deal with a BICM OFDM system with convolutional code,  $p(\mathbf{S})$  can be derived from a soft output decoder as the product of its corresponding bit probability assuming the interleaver adequately breaks the correlation between encoded bits (see figure 2). Furthermore, soft output decoders require as input an estimate of the channel. Hence, the iterative channel estimation can be naturally combined with the decoding process as shown in figure 3. Here we use a soft demapping [13] followed by a BCJR decoder [7] to provide directly from the observations, the encoded bits probability needed in the EM algorithm. This decoding strategy is motivated

by the fact that the BCJR [7] decoder minimizes the BER for QPSK modulations [13]. Among several possible ways to practically implement a joint channel estimation and decoding receiver, we adopt the following global procedure (see figure 3):

- Iteration  $t = 0$  :
  - Initialize all probabilities of coded bits with 0.5 and derive  $p(\mathbf{S})$ .
  - Initialize the EM algorithm by pilot symbols and estimate the channel wavelet coefficients according to (16).
- Iteration  $t = 1, \dots, t_{\max}$  :
  - Update  $\theta^{(t)}$  according to (16) by using the previous estimate  $\theta^{(t-1)}$ .
  - Use the new estimate  $\theta^{(t)}$  to update the probability of coded bits and derive  $p(\mathbf{S})$ .
  - Discard from the estimation process the wavelet coefficients that are replaced by zero in (17).

Note that at the convergence ( $t_{\max}$ ), updating coded bits probability is not necessary since we are interested to the decoded (data) bits probability provided by the decoder. Decoded bits are obtained by thresholding these probabilities.

## 5. SIMULATION RESULTS

The performance of the proposed method is evaluated according to the parameters described in section 2. For each transmitted frame, a different realization of the time invariant UWB channel model CM1 specified by the IEEE802.15.3a channel modeling subcommittee report [14] has been drawn. The entire channel bandwidth is 1584 MHz and consists of 3 equal subbands. The information data rate is 480 Mbps which is generated by a punctured convolutional code with rate  $R = 1/2$ . Monte Carlo simulations are run and averaged over the transmission of at least 10000 frame. Each frame has a payload of 1 kB along with 3 pilot symbols for initializing the channel on each subband and estimating the noise variance. Among different wavelet families, “symmetric” wavelets providing the sparser representation have been considered. The joint estimation-decoding process is repeated until the square error between successive estimates of  $\theta$  becomes lower than  $10^{-5}$ . From the second iteration, the thresholding rule (17) with  $\alpha^2 = 0.5 \sigma^2$  is applied to the estimated coefficients.

Figure 4 depicts the mean square error (MSE) as a function of  $E_b/N_0$  between the true and estimated channel. Comparison is done between a pilot based estimation and the proposed method. It is observed on figure 5 that the number of estimated wavelet coefficients is reduced across the iterations. For example at  $E_b/N_0 = 2\text{dB}$  the number of estimated parameters is reduced from 96 to 50 parameters at the last iteration. Note that applying the EM algorithm to traditional data models, would require the estimation of 384 coefficients at each iteration. Figures 4 and 5 show that in addition to reducing the estimation complexity, the proposed method leads to a more precise estimate of the channel in terms of MSE thanks to the inherent denoising of the thresholding rule. The BER performance is illustrated in figure 6 and compared with a non iterative pilot assisted channel estimation. As can be seen, for a BER of  $10^{-4}$ , our iterative algorithm outperforms the pilot based estimator with about 0.4 dB degradation from the performance obtained with perfect channel state information.

## 6. CONCLUSION

In this paper we proposed a channel estimation algorithm that integrates the advantages of wavelet based estimation. We derived an equivalent data model for the multiband OFDM system involving the channel over all 3 subbands expressed in the wavelet domain. By combining a thresholding/denoising rule with the iterative channel estimation, the number of estimated coefficients can be significantly reduced while the precision is improved at the same time. The investigated method naturally combines the EM iterations with the decoding process. With only few iterations, this approach

outperforms pilot based design. Further work will focus on using Bayesian wavelet thresholding rules by choosing a prior distribution for the channel wavelet coefficients.

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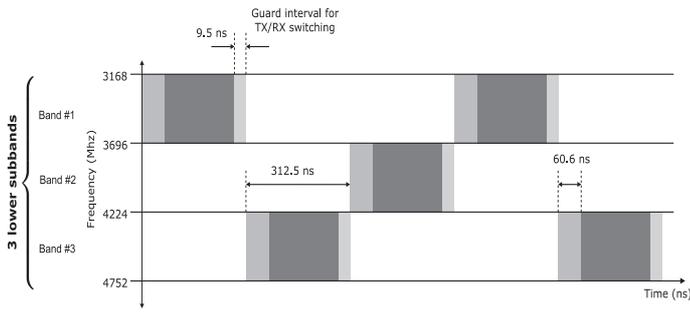


Figure 1: Example of time-frequency coding for the multiband OFDM system.

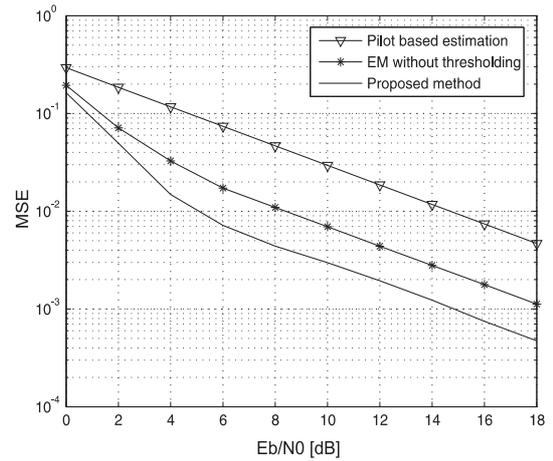


Figure 4: Mean square error between the true and estimated coefficients of CM1 channel.

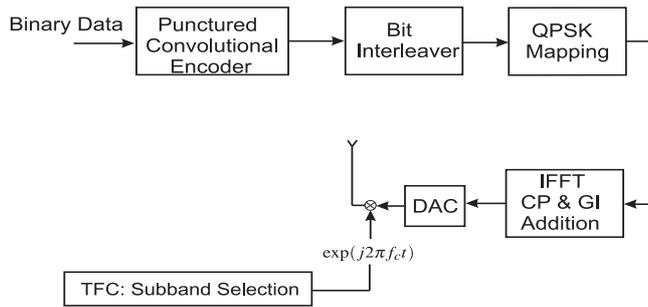


Figure 2: TX architecture of the multiband OFDM system.

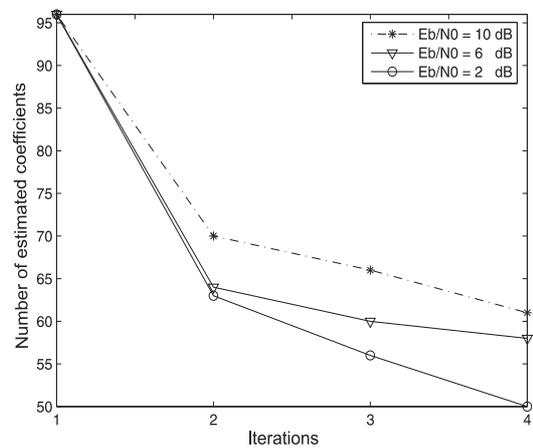


Figure 5: Reduction of the number of estimated parameters through iterations.

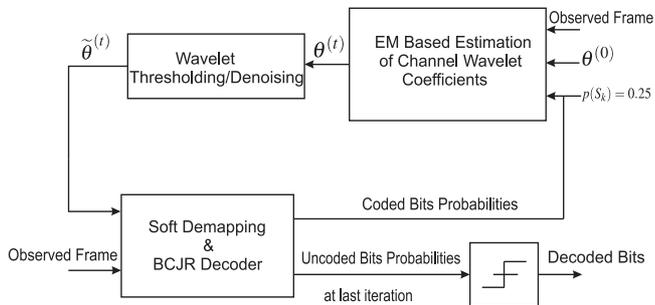


Figure 3: Joint channel estimation and decoding.

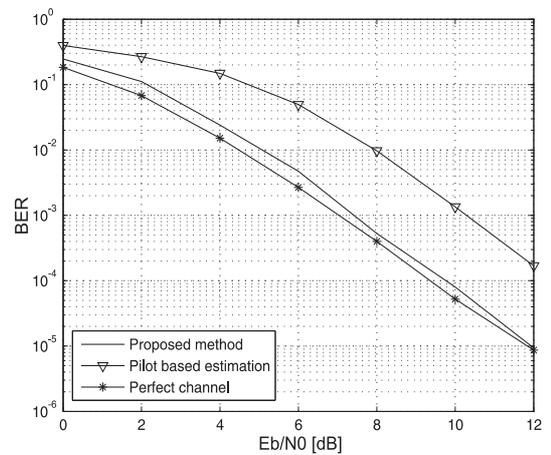


Figure 6: BER performance of the proposed receiver over the UWB channel CM1.