A TURBO RECEIVER FOR WIRELESS MC-CDMA COMMUNICATIONS USING PILOT-AIDED KALMAN FILTER-BASED CHANNEL TRACKING AND MAP-EM DEMODULATOR WITH LDPC CODES

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ABSTRACT

This paper presents a turbo receiver for wireless multi-carrier code division multiple access (MC-CDMA) communications using pilot-aided Kalman filter-based channel tracking and maximum a posteriori expectation-maximization (MAP-EM) demodulator with a soft low-density parity-check (LDPC) decoder. The pilot-aided Kalman filter considerably simplifies the tracking of the time-varying channel, which helps to improve the performance of the MAP-EM demodulator and LDPC decoder in fast fading channels. Simulation results show that the proposed system gives a better bit-error-rate (BER) performance than the conventional turbo receiver, at the expense of slightly lower data rate due to use of pilot symbols. It therefore provides a useful alternative to conventional approaches with a different tradeoff between BER performance, implementation complexity, and transmission bandwidth.

1. INTRODUCTION

MC-CDMA [1] was recently proposed as an efficient multicarrier transmission scheme for supporting multiple access communications, which combines code-division multiple access (CDMA) and orthogonal frequency division multiplexing (OFDM) techniques. It recently receives considerable attention because of its advantages in frequency diversity and multipath fading resilience [2]. Reliable channel estimation and detection of the information symbols in fast fading channels are crucial to the practical implementation of MC-CDMA systems.

Kalman filtering-based methods have been proposed for the estimation of fast fading channels in mobile OFDM systems [3] and MIMO-OFDM systems [4]. In [5], the Kalman-based channel estimator was investigated for MC-CDMA systems. However, in the fast fading channel, the performance of decision-directed receiver will degrade significantly due to feedback of wrongly detected symbols. In this paper, pilot-aided Kalman filter-based channel estimator is used to improve the performance in tracking fast fading channels.

To further improve the bit error rate (BER) performance, LDPC is employed. LDPC codes have been proposed for space-time coded (STC) OFDM system [6] and CDMA system [7]. Both of these approaches use the expectation-maximization (EM) algorithm and LDPC decoding to form a turbo receiver structure, which can significantly reduce the error floor in fast fading channels. In this paper, we propose a maximum a posteriori expectation-maximization (MAP-EM) demodulator and a soft LDPC decoder with pilot-aided channel estimator based on the Kalman filter, where the pilots are inserted in the training subcarriers at each MC-CDMA symbol to simplify the tracking of fast fading channels. Simulation results show that the proposed system has much better BER performance than conventional turbo receiver. Therefore, the proposed system offers another tradeoff between BER performance and transmission bandwidth through the use of pilot-assisted Kalman-based channel estimation.

The rest of this paper is organized as follows: Section 2 describes the MC-CDMA system model. Section 3 presents the Kalman filter based channel tracking scheme. In section 4, MAP-EM and LDPC decoding are described. Section 5 and 6 give the simulation results and conclusion respectively.

2. SYSTEM MODEL

Consider a $K$-user MC-CDMA system for downlink transmission. Fig. 1 shows the MC-CDMA transmitter system. The original serial data stream of the $k$th user is first converted into $P$ parallel data sequences $b_{k}(r) = [b_{k,0}(r), \ldots, b_{k,p-1}(r)]$ at the $n$th time. Each S/P converted output spreads with the user’s spreading sequence $c_{k} = [c_{k,0}, \ldots, c_{k,M-1}]^{T}$. The data chips after spreading are S/P converted into $M$ parallel subcarriers. In order to achieve the maximum frequency diversity, the data bit $b_{k,p}$ are transmitted on subcarriers with frequencies of $f_{1} + (p + mP) \cdot \Delta f$, $m = 0, \ldots, M - 1$. The resulting chips, $N = PM$, in total are expressed as $u_{k}(r) = [b_{k,0}(r)c_{k,0}, \ldots,$
\[ h_k(r) = ah_k(r-1) + w(r) \]  
\[(1)\]

where \( h[r] = [h_0[r], \ldots, h_L[r]]^T \) is a complex Gaussian random process with zero mean and variance \( \sigma_h^2 \), the parameter \( a \) is the fading correlation coefficient that characterizes the degree of time variation. The value of \( a \) is related to the 3-dB frequency \( f_d \) of the corresponding Doppler power spectrum as \( a = \exp(-w_d T_s) \) \( (w_d = 2\pi f_d) \) \[9\], and \( w[r] \) is the driving noise with zero mean and variance \( \sigma_w^2 = (1-a^2)\sigma_h^2 \).

To combat inter-symbol interference (ISI) caused by multipath fading, a cyclic prefix of \( N_g \) samples is added to an MC-CDMA symbol. When \( N_g \geq L-1 \), the effect of ISI can be eliminated. At the receiver, the signal is sampled at a rate \( (N + N_g)/T_s \). The samples corresponding to the cyclic prefix are then discarded. Finally, a fast Fourier transform (FFT) of size \( N \) is performed at the receiver. The discrete-time MC-CDMA signal in the frequency-domain can then be obtained in matrix notation as:

\[ y(r) = \sum_{k=0}^{K-1} \sqrt{P} U_k(r) g(r) + n(r) \]  
\[(2)\]

where \( P \) is the chip energy, \( U_k(r) = \text{diag}\{u_k(r)\} \) is a diagonal matrix, the diagonal elements of which are the transmitted data block of the \( k \)th user. \( g(r) = [g_0(r), \ldots, g_{N-1}(r)]^T \) represents the channel frequency response, given as

\[ g(r) = \text{F}_{N,L} h(r) \]  
\[(3)\]

where \( \text{F}_{N,L} \in \mathbb{C}^{N \times L} \) is the FFT matrix, and the elements of \( g(r) \) is denoted as \( g_n(r) = \sum_{l=0}^{L-1} h_l(r) \exp(-j2\pi nl/N), n = 0, \ldots, N-1 \). Finally, \( n(r) \) is the complex additive white Gaussian noise after the FFT, with zero mean and variance \( \sigma_n^2 \).

Given the state dynamic in (1), the Kalman filter gives the optimal linear estimate of the channel if the symbol is known and the state and measurement noise are known as Gaussian processes. We propose to insert pilots in MC-CDMA blocks in order to simplify the tracking of the time-varying channel. In \[10\], it is shown that in the presence of noise, equally-spaced placement of the pilot symbols is optimal. Therefore, \( N_p \) \( (N_p \geq L+1) \) out of \( N \) subcarriers are chosen as the training subcarriers from the set \{1,\ldots\,1+[N/N_p],\ldots,1+(N_p-1)\,[N/N_p]\}. The pilot symbols, denoted by \( d_p(r) = [d_p^0(r), \ldots, d_p^{N_p-1}(r)]^T \), are sent over the \( N_p \) training subcarriers in each MC-CDMA block.

Considering the channel model given in (2) and (3), the time-varying channel and the received data vector for the \( N_p \) training subcarriers satisfy the following state-space model:

\[ g_p(r) = \text{F}_{N_p,L} h(r), \]
\[ y_p(r) = \text{D}_p(r) g_p(r) + n_p(r), \]  
\[(4)\]

where \( \text{D}_p(r) = \text{diag}\{d_p(r)\} \) is the pilot matrix, the rows of the matrix \( \text{F}_{N_p,L} \in \mathbb{C}^{N_p \times L} \) are from the FFT matrix \( \text{F}_{N,L} \) corresponding to the training subcarriers, and \( n_p(r) \) is white Gaussian noise with zero mean and variance \( \sigma_n^2 \).

The state-space model of (4) allows the use of the Kalman filter to track the channel in the time domain. The algorithm \[11\] and \[15\] is given below.

1. Initialize the Kalman filter with \( h(0) = 0_{L \times 1} \) and \( \text{M}(0) = \delta I_L \), where \( \delta \) is a small positive constant.
2. Perform the Kalman filter update as the following
\[ \tilde{D}_p (\tau) = D_p (\tau) F_{N_p, L} \]

\[ M(\tau) = a M(\tau - 1) a^* + \sigma_n^2 I_L \]

\[ K(\tau) = M(\tau) \tilde{D}_p (\tau) \cdot (\tilde{D}_p (\tau) M(\tau) \tilde{D}_p (\tau) + \sigma_n^2 I_{N_p})^{-1} \]

\[ h(\tau) = a h(\tau - 1) + K(\tau) (y_p (\tau) - \tilde{D}_p (\tau) a h(\tau - 1)) \]

\[ M(\tau) = (I_L - K(\tau) \tilde{D}_p (\tau)) \cdot M(\tau). \]

The channel gains at each subcarrier can then be obtained as \( g(\tau) = F_{N_p} h(\tau). \)

Once the channel state information has been estimated, the minimum mean squared-error (MMSE) detector can be employed to recover the data symbols. Referred to the matrix form of the system model in (2), the received data vector corresponding to the \( p \)th data stream can be written as follows, \( (p = 1, \ldots, P) \):

\[ y^p (\tau) = \sqrt{P} G^p (\tau) b^p (\tau) + n^p (\tau), \] \( (5) \)

where \( y^p (\tau) \) is an \( M \times 1 \) vector denoting the received data vector, \( G^p (\tau) = \text{diag} \{g^p(\tau)\} \) with \( g^p (\tau) \) denoting the channel gains, \( b^p (\tau) = [b^p_0 (\tau), \ldots, b^p_{P-1} (\tau)]^T \) represents the data vector of the users corresponding to the transmitted \( p \)th data stream, respectively, and \( C = [c_0, \ldots, c_{K-1}]_{M \times K} \) is the code matrix of the users which satisfies \( CC^T = I_M \).

Considering the \( p \)th data stream, the optimization criterion is to find a matrix such that

\[ V^p (\tau) = \arg \min V_{\text{V} (\tau)} E \{|| b^p (\tau) - V(\tau) y^p (\tau) ||^2 \}. \] \( (6) \)

The data vector estimate is then obtained as the following, \( \hat{y}^p (\tau) = V^p_0 (\tau) y^p (\tau) = [\hat{y}^p_0 (\tau), \ldots, \hat{y}^p_{P-1} (\tau)]^T \), and the hard decision of \( \hat{y}^p (\tau) \) is \( \hat{b}^p (\tau) = \text{sgn}[\Re(V^p_0 (\tau) y^p (\tau))] \), which will be used for MAP-EM initialization to be described in next session.

The Wiener solution of (6) is given by

\[ V^p_0 (\tau) = R_{by}^p (\tau) (R_{yy}^p (\tau))^{-1}, \] \( (7) \)

where \( R_{by}^p (\tau) = E \{b(\tau)y^{H} (\tau)\} = \sqrt{P} C^T G^p (\tau) \)

\[ R_{yy}^p (\tau) = E \{y^p (\tau)y^{H} (\tau)\} = PG^p (\tau) C C^T G^p (\tau) + \sigma_n^2 I \]

Therefore the updated symbol using Kalman filter tracking for the \( k \)th user is \( \hat{y}_k (\tau) = [\hat{y}^p_0 (\tau), \hat{y}^p_1 (\tau), \ldots, \hat{y}^p_{P-1} (\tau)]^T \).

4. MAP-EM DEMODULATION

The turbo receiver consists of two stages, the soft demodulator and the soft LDPC decoder which exchange the information iteratively to improve the receiver performance. Let the DFT coefficients vector be \( f_L (p) = [e^{j0}, \ldots, e^{-j2\pi(P-1)/P}]^H \) and \( x_k (\tau, p) = b_k (\tau, p) \). The filtered signal of \( k \)th user during one data burst can be rewritten as

\[ \hat{y}_k (\tau) = X_k (\tau) F_{P,L} h(\tau) + n_k (\tau), \] \( (8) \)

with \( X_k (\tau) = \text{diag} \{x_k (\tau,0), x_k (\tau,1), \ldots, x_k (\tau, P-1)\} \), \( F_{P,L} = [f_L (0), f_L (1), \ldots, f_L (P-1)]_{P \times L} \), where \( \hat{y}_k (\tau) \) and \( n_k (\tau) \) are \( P \)- sized vectors which contain respectively the received signals and the ambient Gaussian noise at all \( P \) subcarriers and at the \( \tau \)th time; the diagonal elements of \( X_k (\tau) \) are the \( P \) symbols transmitted from the transmitter antenna and at the \( \tau \)th time.

Without CSI, the maximum a posteriori (MAP) detection problem is written as

\[ \hat{X}_k (\tau) = \arg \max_{X_k (\tau)} \log p(X_k (\tau) | \hat{y}_k (\tau)) \] \( (9) \)

To solve (9), so we will use the expectation-maximization (EM) algorithm [12]. The MAP-EM algorithm can be summarized in the following steps according to [6].

E-step:
Compute \( Q(X_k | X_k^{(i)}) = E\{\log p(\hat{y}_k | X_k, h) | \hat{y}_k, X_k^{(i)}\} \) \( (10) \)

M-step:
Solve \( X_k^{(i+1)} = \arg \max_{X_k} Q(X_k | X_k^{(i)}) + \log P(X_k) \), \( (11) \)

where \( X_k^{(i)} \) denotes hard decisions of the data symbols at the \( i \)th EM iteration and \( P(X_k) \) represents the a priori probability of \( X_k \), which is fed back by the LDPC decoder from the previous turbo iteration.

In the E-step, the expectation can be rewritten as

\[ Q(X_k | X_k^{(i)}) = -E_{b(\hat{y}_k, \hat{X}_k^{(i)})} \{|| \hat{y}_k - X_k F_{P,L} h(\tau) ||^2 \} + \text{const.} \]

\[ = -\sum_{p=0}^{P-1} [\hat{x}_k (p) - x_k^{(H)} (p) f_L (p)^{H} \hat{h}]^2 + [x_k^{(H)} (p) \hat{S}_h (p) x_k (p)] \} + \text{const.} \]

\[ \hat{h} = (F_{P,L}^{H} X_k^{(i)} X_k^{(i)} F_{P,L} + \hat{S}_h)^{-1} F_{P,L}^{H} X_k^{(i)} \hat{y}_k, \]

\[ \hat{S}_h = \sum_h - (F_{P,L}^{H} X_k^{(i)} X_k^{(i)} F_{P,L} + \hat{S}_h)^{-1} F_{P,L}^{H} X_k^{(i)}, \]

\[ \times F_{P,L}^{H} X_k^{(i)} X_k^{(i)} + F_{P,L} \sum_h. \] \( (12) \)

\[ \hat{S}_h = \sum_h (F_{P,L}^{H} X_k^{(i)} X_k^{(i)} F_{P,L} + \hat{S}_h)^{-1} F_{P,L}^{H} X_k^{(i)} \]

and \( \hat{S}_h (p) = F_{P,L} \hat{S}_h F_{P,L}^{H} \). \( \hat{S}_h \) is defined as the pseudo inverse of \( \sum_h \), the covariance matrix of the channel response \( h \). The details of the derivation of (12) and (13) can be found in [6]. Next, based on (12), the M-step shows as follows:
In (15) the M-step can be decoupled into \( P \) independent minimization problems. Within each turbo iteration, the above E-step and M-step are iterated \( J \) times. At the end of the \( Ith \) EM iteration, the extrinsic a posteriori Log-Likelihood ratio (LLRs) of the LDPC code bits are computed and then send to the soft LDPC decoder. In LDPC decoder, it iteratively computes the extrinsic LLRs and then feeds them back to the MAP-EM demodulator and thus completes one turbo iteration. At the end of the last turbo iteration, hard decisions of the information bits are output by the LDPC decoder. The details of the definition of LLRs in MAP-EM can be found in [6]. For details of the soft LDPC decoder, see [13] [14]. The initialization of the MAP-EM demodulator is very important because the performance and convergence speed of the demodulator depends on the quality of the initial value of \( \mathbf{X}_k^{(0)}(\tau) \). Except the first turbo iteration, \( \mathbf{X}_k^{(0)}(\tau) \) is taken from previous turbo iteration to initialize the MAP-EM demodulator in each turbo iteration. In the proposed demodulator, \( \mathbf{X}_k^{(0)}(\tau) \) at the first turbo iteration is taken from MMSE detection after the Kalman filter tracking of the time-varying channel mentioned in equations (6) and (7).

### 5. NUMERICAL RESULTS

In this section, numerical results are presented to illustrate the performance of the systems. The signal-to-noise ratio (SNR) is defined to be proposed and conventional average received bit energy to noise ratio \( P/\sigma_w^2 \). A frequency-selective channel is considered with \( L = 6 \) paths. The multipath intensity profile decays exponentially, and the total channel power is normalized to 1. We set \( M = 8 \) and \( P = 16 \). Therefore, the total number of subcarriers is \( N = 128 \). Consequently, the original data sequence is first serial/parallel converted into 16 parallel data streams, then each data symbol after S/P spreads with a Walsh code of length \( M = 8 \). The subcarrier spacing is \( \Delta f = 10 \text{ kHz} \), and the time duration at the subcarrier is \( T_s = 100 \mu \text{s} \). An additional guard tones \( N_g = 8 \) is added to prevent ISI due to channel frequency selectivity. There are 8 simultaneous users in the channel. The pilot symbols of length \( N_p = 8 \) are placed at the training subcarriers for each MC-CDMA symbol. The fading rate under consideration is in the range of \( 1 \times 10^{-3} \leq w_d T_s \leq 5 \times 10^{-2} \). In the simulation, the turbo iteration and EM iteration are 3 respectively. Unless otherwise specified, all the simulations are assumed to use eight users and three turbo iterations. And the number of iteration in the LDPC code is 30 times with a code rate of 1/2. In the following figures, the suffix ‘kf (1e-3)’ denotes the results of using Kalman filter at a fading rate of \( 1 \times 10^{-3} \). The suffix ‘turboEM N (1e-3)’, \( N=60, 108, \) and \( 200 \), is used to denote the results of using the proposed turbo receiver with a code length \( N \) at a fading rate of \( 1 \times 10^{-3} \).

![Fig. 2. BER comparison of the various systems at a fading rate of 1e-3.](image1)

![Fig. 3. BER comparison of the various systems at a fading rate of 5e-2.](image2)

First of all, we evaluate the performance of the various systems assuming that the fading correlation coefficient \( a \) is known. Figs. 2 and 3 show the overall BER performance of the systems using pilot-aided Kalman filter, MAP-EM demodulator and LDPC decoder at fading rates \( 1 \times 10^{-3} \) and \( 5 \times 10^{-2} \), respectively. It can be seen that the performance of the proposed turbo receiver with pilot-aided Kalman channel tracking and MAP-EM demodulator with LDPC code is considerably better than the receiver using pilot-aided Kalman channel tracking alone, and the performance improves with increasing code length of the LDPC used. Also, the system performance is still good even at relatively short or medium code length. Fig. 4 shows the BER performance of proposed systems at a fading rate of \( 5 \times 10^{-2} \) with estimated fading factor. The true value of the fading factor \( a \) is 0.998. It can be seen that the Kalman filter-based channel tracker [15] offers certain robustness to unknown
fading factors and the performance is slightly degraded. Fig. 5 shows the BER performances of the proposed system and the systems using either pilot-aided Kalman filter or turbo receiver in single user mode. Suffix ‘iter#N’ denotes that N turbo iterations are employed. It can be seen that the proposed system has a much better BER performance than the conventional turbo receiver even though the conventional turbo receiver performs 7 turbo iterations. Also, we can see that the BER performance of the conventional turbo receiver becomes worse when the SNR is low, while the performance of the proposed system is significantly better. Moreover, it can be seen that the proposed system can significantly reduce the number of turbo iteration in order to attain a given BER and this translates into lower implementation complexity. The improvement achieved by the proposed system is mainly attributed to the use of pilot symbols (which slightly reduce the data rate of the system) and the good performance of the Kalman filter-based channel tracker. It therefore offers a useful alternative to conventional approaches with a different tradeoff between BER performance, implementation complexity and transmission bandwidth.

![Fig. 4: BER comparison of the various systems with estimated fading factor.](image1)

![Fig. 5: BER comparison for the proposed system, systems using Kalman filter channel estimation only and turbo receiver only.](image2)

6. CONCLUSION

A turbo receiver for wireless MC-CDMA communications using pilot-aided Kalman filter-based channel tracking and MAP-EM demodulator with a soft LDPC decoder is presented. The pilot-aided Kalman filter considerably simplifies the tracking of the time-varying channel, which helps to improve the performance of the MAP-EM demodulator and LDPC decoder in fast fading channels. Simulation results show that the proposed system gives a better BER performance than the conventional turbo receiver, at the expense of slightly lower data rate due to use of pilot symbols. It therefore offers a useful alternative to conventional approaches with a different tradeoff between BER performance, implementation complexity, and transmission bandwidth.

REFERENCE


