14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

MODELLING ELASTIC WAVE PROPAGATION IN THIN PLATES

D. Rovetta, A. Sarti, G. De Sanctis, M. Fabiani

Electronics and Information Department of Politecnico di Milano

Viale delle Rimembranze di Lambrate, 14, 20133, Milano, Italy
phone: (+39) 02-2399 9640, fax: (+39) 02-2399 9611, email: rovetta@elet.polimi.it
web: www.elet.polimi.it

ABSTRACT

In this work we propose an in-depth study of elastic wave propagation in thin plates. We show that at the frequency range of interest and for modest plate thicknesses, the only waves that can be excited and propagate in the structure are guided waves (also called Lamb waves). This propagation modeling approach is based on the theory of Viktorov. The elastic properties of the panel and the finger touch signature are usually unknown, therefore we propose a method for estimating them through a simple experimental procedure. The obtained estimates are then used to simulate the propagation in the boards. Our approach is to implement the general solution of the elastic wave equation for infinite plates, and introduce the boundary conditions afterwards using a real-time beam tracer. We finally prove the effectiveness of the approach by comparing the predicted response of a finger touch with the measured one on materials such as MDF (Medium Density Fiberboard) and PLX (Plexiglass).

1. INTRODUCTION

We refer to a tactile interaction on a thin panel where the vibrational signals transmitted by a finger touch (*the source*) and propagated like elastic waves in the board, are acquired by some receivers.

In this paper we show how to calculate the received signals, knowing the elastic properties of the board and the exact position of the touch (*modelling problem*). To achieve this purpose the study of the elastic wave propagation in plates is required: at the frequencies of our interest and for small thicknesses of the panels, the only waves that can be excited and propagate in the medium are guided waves (also called *Lamb waves*). The propagation follows the theory of Viktorov [1]. Moreover, in our experiments, the elastic properties of the panel and the finger touch signature are unknown and so we have to face the problem to estimate them from the data.

Finally the estimates of the plate elastic properties and of the transmitted signature are used to simulate the propagation in the boards. Our approach is to implement the general solution of the elastic wave equation for infinite plates and to introduce the boundary conditions afterwards using a real-time beam tracer [2-3].

The experimental equipment (Fig. 1) is made up of a panel and an acquiring system, composed by some receivers (*sensors*), a semi-professional audio card and a PC.

We assume that the boards used for the tests are characterized by homogeneity, isotropy and relatively thin, layered, plate-like geometry. We found two different common materials with these requirements: plexiglass (PLX) and Medium Density Fiberboard (MDF). This latter is a composite wood product similar to particleboard, made up of wood waste fibers glued together with resin, heat, and pressure. They exhibit large attenuation coefficient in the high-frequency range.

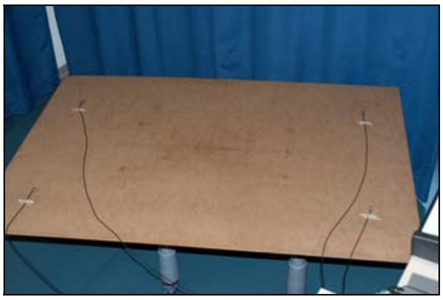

Figure 1 - The experimental equipment of the tests.

In this paper we show results for a MDF panel, whose dimensions are $l=152$ cm, $w=106$ cm, $t=0.5$ cm and whose elastic properties are unknown.

The receivers are used to acquire the signals transmitted by the finger touch and propagated in the board. Several kinds of sensors have been tested, principally piezoelectric devices and the best results have been achieved with the BU-1771 (Fig. 2).

Figure 2 - Piezo-sensor BU 1771 applied to the MDF board.

They are based on a piezoelectric transducer followed by a FET, which ensures low output impedance and high output level. Moreover, their small dimensions make them suitable for this application.

These receivers are sensitive only to displacements along the *z*-axis (normal to the plate) and have a large bandwidth (*10 kHz*). Their frequency response is shown in Fig. 3 where it is evident that the acquired signals have to be low-pass filtered at a frequency ranging from *5 kHz* to *8 kHz* to compensate the undesired effects of the non-linearity.

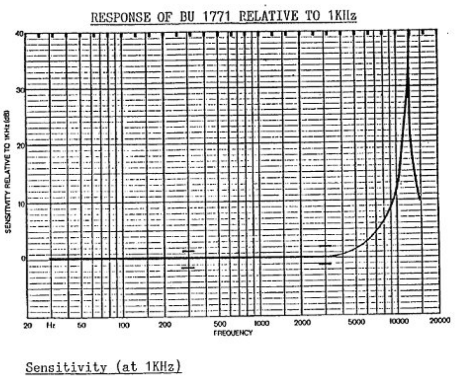

Figure 3 - Frequency response of the BU 1771 receivers.

The sensors are coupled to the plate surface using double sided adhesive tape and are connected to the audio card using the three wires method (signal-power supply-ground).

All the tests and measuring campaign are taken using the standard semi-professional *M-Audio Delta 44* audio card with an external box for analog connections (all audio connectors are 1/4" jacks). It is modified to have the power supply for the microphones on the input sockets. In fact the balanced inputs are transformed in unbalanced, using the ring contact to carry the *5V* power supply.

The PC used for the signal processing is a Pentium IV (*3GHz*, *1GB* RAM, Win2003 Server Operating System).

2. THE GENERAL SOLUTION OF ELASTIC WAVE EQUATION FOR INFINITE PLATES

Nicholson at al. [4] analyzed the elastic wave propagation in panels considering two different types of geometry: a semi-infinite solid half-space with the transducers on the free surface, and infinite solid plates, with thickness of *10* and *5 mm*, with the transducers on the upper free surface.

In the semi-infinite solid half-space the situation is clear: we can recognize the longitudinal (*P-wave*) and shear (*S-wave*) wavefronts, the Rayleigh wave localized close to the surface and the lateral or head wave, shown as the component of the longitudinal wave propagating at, and parallel to, the surface [5].

In the infinite solid plate with thickness of *10 mm* the situation is much more complex, due to the presence of guided waves generated by the interaction of the different wavefronts with the surfaces of the plate.

Finally for infinite solid plates with smaller thicknesses, only guided wave arrivals are visible.

Tucker [6] showed that the previous behaviour can be found in other wood-based composite panels, like MDF. The boards are “perceived” by the wave as homogeneous (through the thickness), orthotropic plates as long as the wavelength remains much larger than the panel thickness. The “perception” of the material greatly reduces the complexity of the equations needed to describe the wave propagation [1].

Plate wave propagation occurs when the wavelength, λ, is much greater than the thickness, t, of the plate ($\lambda >> t$). Some authors recommend the wavelength be ten times greater than the thickness ($\lambda > 10\ t$), while others propose less stringent wavelength requirements [6]. The remaining dimensions (length, l and width, w) of the plate must be much greater than the wavelength.

Wavelength is calculated from the phase velocity, v, and the frequency, f as:

$$\lambda = \frac{v}{f} \qquad (1)$$

In our experiments the requirements $\lambda >> t$, $\lambda << l$ and $\lambda << w$ are always satisfied and thus a plate wave propagation occurs.

The theory of Viktorov, governing the wave propagation in thin plates (also commonly termed *Lamb* or *guided wave*), is documented in [1] and in this section we only consider the main concepts.

An ideal bulk wave is a spherical disturbance that originates from a point source and propagates through an infinite medium. A plate wave may be thought as a two-dimensional representation of a bulk wave bounded by an upper and a lower surface.

There are two distinct types of plate waves: extensional (*symmetric*, *s*) and flexural (*antisymmetric*, *a*), each of which have an infinite number of modes (s_0, s_1, s_2, …, s_n and a_0, a_1, a_2, …, a_n) at higher frequencies. Plate waves are dispersive by nature, meaning that different frequencies travel at different speeds (*phase velocities*). The phase velocity v is the fundamental characteristic of the Lamb wave and once it is known we can determinate the wave number and calculate the stresses and displacements at any point of the plate. v can be found by numerically solving the following characteristic equations [1].

If $t = 2d$ is the thickness of the plate, k_β is the S-wave number, v_α and v_β are the *P-wave* and *S-wave* velocities and $v_{s,a}$ is the phase velocity of the Lamb waves, then the characteristic equation for the symmetrical modes is:

$$\frac{tg\left(\bar{d}\sqrt{1-\zeta_s^2}\right)}{tg\left(\bar{d}\sqrt{\xi^2-\zeta_s^2}\right)} + \frac{4\zeta_s^2\sqrt{1-\zeta_s^2}\sqrt{\xi^2-\zeta_s^2}}{\left(2\zeta_s^2-1\right)^2} = 0 \qquad (2)$$

and for the antisymmetrical modes:

$$\frac{tg\left(\bar{d}\sqrt{1-\zeta_a^2}\right)}{tg\left(\bar{d}\sqrt{\xi^2-\zeta_a^2}\right)} + \frac{\left(2\zeta_a^2-1\right)^2}{4\zeta_a^2\sqrt{1-\zeta_a^2}\sqrt{\xi^2-\zeta_a^2}} = 0 \qquad (3)$$

where $\bar{d} = k_\beta d$, $\zeta_{s,a}^2 = \dfrac{v_\beta^2}{v_{s,a}^2}$ and $\xi^2 = \dfrac{v_\beta^2}{v_\alpha^2}$.

Many authors have performed calculations of the phase velocities and their dependence on the plate thickness and frequency (*dispersion curves*). To achieve this purpose the elastic properties of the medium (in our tests the velocities v_α and v_β of the *P-wave* and *S-wave* in the board) are necessary. In the next section we show how to estimate these properties and then how to calculate the dispersion curves for the s_0 and a_0 modes.

Extensional and flexural modes should be distinguished in the same received signal but it can be proved that the extensional mode does not propagate below certain frequencies. Rose and Tucker [6] offer an explanation: as the frequency-thickness product increases, the structure of a Lamb wave changes. For extensional waves, the out-of-plane displacement increases with the frequency, for a given thickness. Since normal contact transducers are only sensitive to out-of-plane motion, the phenomenon observed is explained and the flexural mode may then be easily isolated at low frequencies.

3. ESTIMATION OF THE PANEL ELASTIC PROPERTIES

The panels here analyzed are characterized by large attenuation coefficient in the high-frequency range. This attenuation forces the method described in this section into the low-frequency, long-wavelength region, where the prominent wave propagation modes are plate waves.

A transducer *T* converts electrical energy into mechanical energy, propagated through a thin panel in form of elastic waves. Two sensors Rx_1 and Rx_2, placed a known distance apart, can be used to receive the signals associated to these waves, to calculate their phase difference and thus to measure their phase velocity (Fig. 4). Then different phase velocity observations can be used to estimate the elastic properties (v_α and v_β) of the panel.

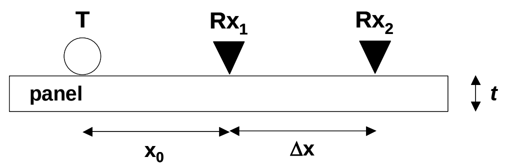

Figure 4 - Technique to calculate the dispersion curves of a thin panel.

The transducer *T* generates sinusoidal bursts ranging from *1500 Hz* to *5000 Hz* for the MDF, with a step of *250 Hz*. At these frequencies only a_0 Lamb mode can be excited: extensional plate waves are negligible, due to their small out-of-plane motion at lower frequencies.

A short number of sinusoidal bursts (*4* to *6* cycles) aids in reducing the duration of each wave mode.

Moreover the observed phase velocities are obtained by averaging several realizations (also varying Δx and x_0) to reduce the effect of noise.

For each frequency, the difference between the theoretical $v_{a,cal}$ and the experimental $v_{a,obs}$ phase velocities is computed and optimal values for *P-wave* and *S-wave* velocities, v_α and v_β, can be obtained by minimizing the data residual R_d:

$$R_d(v_\alpha, v_\beta) = \left\| v_{a,obs} - v_{a,cal}(v_\alpha, v_\beta) \right\|^2 \qquad (4)$$

The 2D objective function R_d is shown for different values of v_α and v_β in Fig. 5.

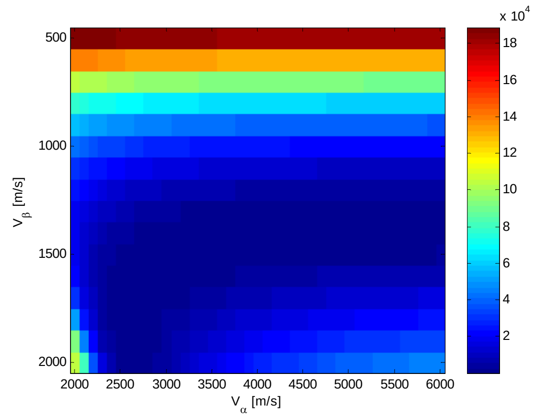

Figure 5 - Data residual as a function of the *P-wave* and *S-wave* velocities for the MDF.

The minimization of R_d can be achieved by using a grid search technique but it is difficult due to its flatness: an additional constraint is necessary to reach a solution with a good accuracy. As suggested by Tucker and Viktorov [6] we can constraint the solution of the minimization problem by fixing the value of the Poisson ratio ν of the board.

A confidence interval for ν in the MDF is obtained by using the commercial software CES selector 4.5: $\nu = 0.2 \div 0.3$. With a fix value of the Poisson ratio, the data residual becomes a 1D function. It is shown in Fig. 6 versus the *P-wave* velocity for three meaningful values of ν.

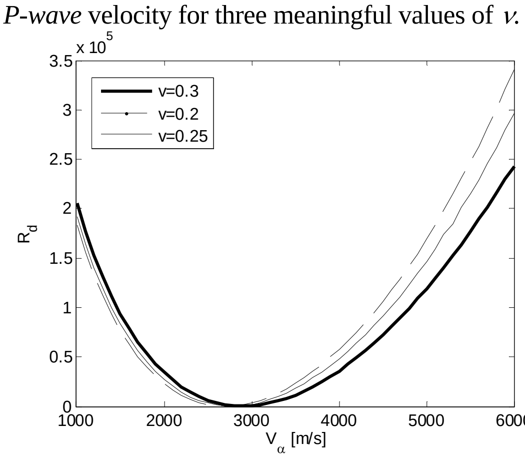

Figure 6 - Data residual as a function of the *P-wave* velocity for the MDF.

The minimization problem is now simple and choosing $\nu = 0.25$ the estimated elastic properties of the MDF board are $v_\alpha = 2900\ m/s$ and $v_\beta = 1600\ m/s$.

Using the estimated elastic properties of the boards and the plate wave propagation theory, the phase velocity at different frequencies can be computed (*calculated data*) for different materials. Observed data and calculated data for MDF are shown in the range of frequencies of the measurements in Fig. 7.

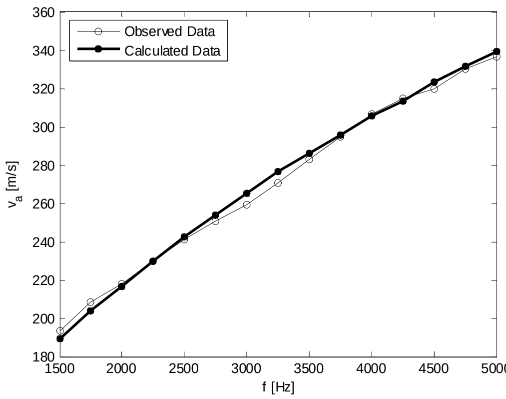

Figure 7 - Phase velocities of the a_0 mode in the MDF board: measured and calculated data.

The good agreement between the observed and the computed curves confirms the accuracy of the solution of the optimization problem.

4. ESTIMATION OF THE EXCITATION SIGNAL

In this section we propose a simple scheme to estimate the finger touch signature. Let us consider five receivers (Rx_1, …, Rx_5): four of them (Rx_1, …, Rx_4) are located at the corners and the fifth one (Rx_5) at the centre of the board (Fig. 8). The receivers Rx_1, …, Rx_5 acquire respectively the signals s_1, …, s_5.

The signal s_5, acquired by the sensor Rx_5, is not affected by problems of overlap between the direct arrival and the signals reflected from the borders of the panel (*edge reflections*).

If the position of the touch (x_T, y_T) is known then the transmitted signature can be estimated by *inverse propagating* s_5 of the exact distance between source and receiver. The inverse propagation is obtained by filtering and the filter is designed in the frequency domain by using the knowledge of the plate wave theory and of the estimated elastic properties of the panel (§ 3).

In a MDF experiment the estimated signature of the finger touch, after inverse propagating the signal s_5, is shown in Fig. 9. It is impulsive with a time duration of about *6 ms*.

The knowledge of the transmitted signature and of the propagation model allows the calculation of the direct arrivals acquired by all the receivers (*simulated or recalculated data*): the transmitted signature is forward propagated of the exact distance between source and receivers.

Since we want to simulate the complete elastic wave propagation in the plate, in order to compare the observed response with the calculated one, we have to take into account the edge reflections. A fast beam tracer [2, 3] can be used to achieve this purpose. We can therefore compute the complete board response as the result of the sum of the signals due to the direct arrival and to the most energetic reflected rays.

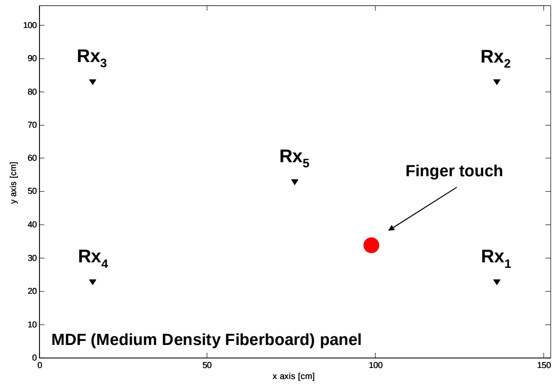

Figure 8 - Scheme used to estimate both the position and the signature of a finger touch.

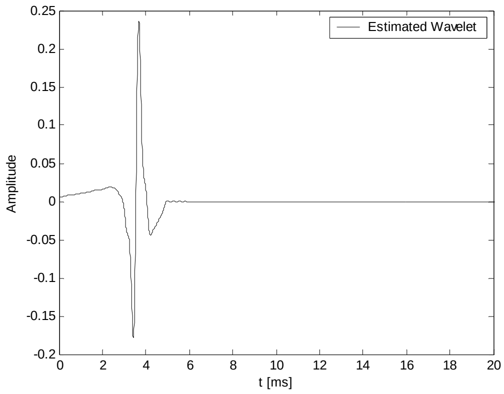

Figure 9 – Estimated signature of the finger touch in MDF.

The reflection coefficient of the borders of the plate can be also computed from the observations. It can be proved that the reflected signal is an attenuated and delayed copy of the incident one.

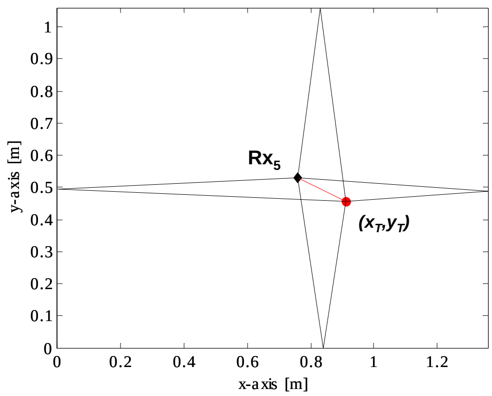

Figure 10 - Direct arrival and the first four reflected rays.

5. RESULTS

In the previous sections we estimated the MDF elastic properties (§ 3) and the transmitted signature (§ 4): it is therefore possible to simulate the propagation in the board. Let us consider an experiment, whose configuration is shown

14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

in Fig. 10. We calculate the direct arrival, corresponding to the ray directly linking the source with the receiver (red line) and the first four delayed arrivals, corresponding to the path of the rays reflected only once by the borders of the plate and linking the source with the receiver (black lines).

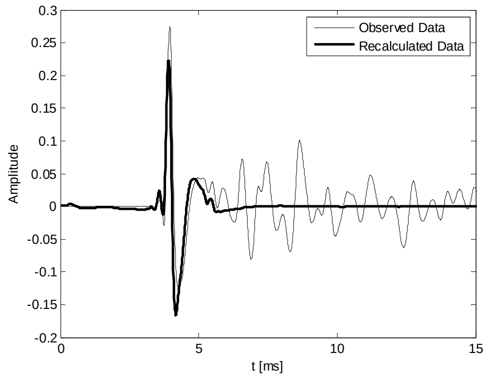

Figure 11 - Comparison of the observations with the simulated data only considering the direct arrival.

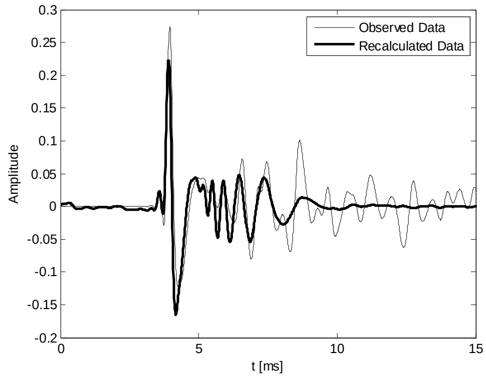

Figure 12 - Comparison of the observations with the simulated data considering the direct arrival and the first reflected ray.

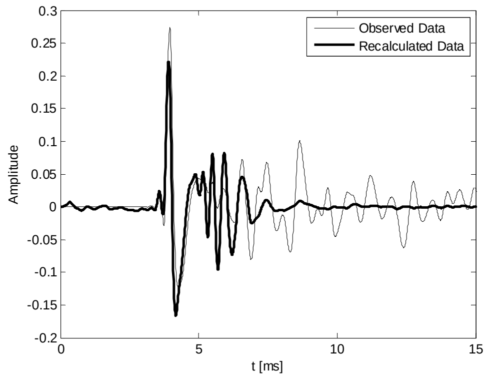

Figure 13 - Comparison of the observations with the simulated data considering the direct arrival and the first two reflected rays.

First we compare the observations with the simulated data only considering the direct arrival (Fig. 11). There is a good agreement before the arrival of the reflected waves (about *6 ms*). The more reflected rays we consider in the computation of the simulated signal response (Figs. 12-15), the more the agreement between observations and calculated data is good.

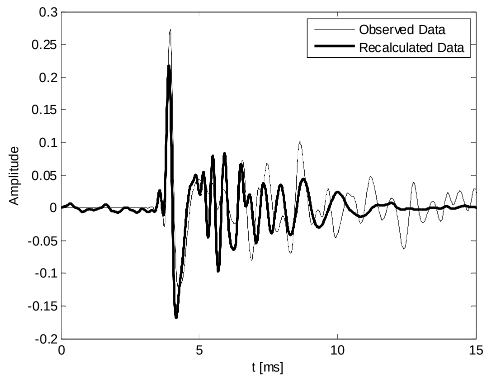

Figure 14 - Comparison of the observations with the simulated data considering the direct arrival and the first three reflected rays.

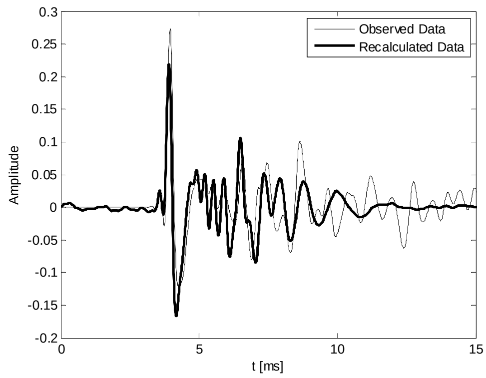

Figure 15 - Comparison of the observations with the simulated data considering the direct arrival and the first four reflected rays.

6. CONCLUSIONS

A study of the elastic wave propagation in thin plates has been conducted, following the formulation of Viktorov.

Estimates of the panel elastic properties and of the signature transmitted by a finger touch allow to simulate the propagation in the panels. The reflections from the borders of the panel can be also considered in the modelling by using a beam tracer.

REFERENCES

[1] I. A. Viktorov, *Rayleigh and Lamb Waves*, Plenum Press, New York, 1967.

[2] F. Antonacci, M. Foco, A. Sarti, S. Tubaro, “Fast Modeling of Acustic Reflections and Diffraction in Complex Environments Using Visibility Diagrams”, *Proc. DAFX-02, Hamburg, Germany, September 26-28, 2002*.

[3] M. Foco, P. Polotti, A. Sarti, S. Tubaro, “Sound Spatialization Based on Fast Beam Tracing in the Dual Space”, *Proc. DAFX-02, Hamburg, Germany, Sept. 26-28, 2002*.

[4] Nicholson et al., “Guided ultrasonic waves in long bones: modelling, experiment and *in vivo* application”, *IOP, 2002*.

[5] K. Aki, P. G. Richards, *Quantitative Seismology*, W. H. Freeman and Company, San Francisco, 1980.

[6] B. J. Tucker, *Ultrasonic Plate Waves in Wood-Based Composite Panels*, Washington State University, Department of Civil and Environmental Engineering, 2001.