TRANSMIT CORRELATION-AIDED OPPORTUNISTIC BEAMFORMING AND SCHEDULING

Marios Kountouris 1, David Gesbert 2, Lars Pittman 3

1France Telecom R&D, Issy-les-Moulineaux, France, marios.kountouris@francetelecom.com
2Eurecom Institute, Sophia-Antipolis, France, gesbert@eurecom.fr
3Norwegian University of Science & Technology, Trondheim, Norway, pittman@stud.ntnu.no

ABSTRACT
The problem of multiuser scheduling and beamforming with partial channel state information at the transmitter (CSIT) is considered here in the particular context of opportunistic beamforming. We consider a random multiuser beamforming scheme and show how long-term statistical information of users’ channels can be efficiently combined with short-term SINR feedback to increase system performance substantially over a conventional opportunistic beamforming scheme. We propose a channel estimation method to be used by the transmitter which exploits the second-order channel statistics, the fading statistics and the information contained in the instantaneous SINR feedback of random opportunistic beamforming. This coarse (low feedback-based) channel estimate is shown to be particularly valuable for the purpose of user selection, as well as for the precoding matrix design.

1. INTRODUCTION
Multiple-input multiple-output (MIMO) systems can significantly increase the spectral efficiency by exploiting the spatial degrees of freedom created by multiple antennas. In point-to-point MIMO systems, the capacity increases linearly with the minimum of the number of transmit/receive antennas, irrespective of the availability of channel state information (CSI). In the MIMO broadcast channel, it has recently been proven [1] that the sum capacity is achieved by dirty paper coding (DPC) [2]. However, the applicability of DPC is limited due to its computational complexity and the need for full channel state information at the transmitter (CSIT) across all active users. The latter may lead to prohibitive feedback requirements in frequency-division duplex (FDD) systems or lack of robustness to CSIT errors in time-division duplex (TDD) setups with mobility. The capacity gain of multiuser MIMO systems is highly dependent on the available CSIT. While having full CSI at the receiver can be assumed, this assumption is not usually reasonable at the transmitter side.

In [3], it was shown that for the no-feedback or error-prone feedback case, space-time coding combined with time-division multiple access (TDMA)-like scheduling algorithms exploiting multiuser diversity [4] seems a reasonable option. However, for a system encompassing even limited and reasonably accurate feedback of CSIT, it is beneficial to exploit the spatial multiplexing capability of transmit antennas to several users at once rather than trying to maximize the reliability/diversity of a single user link. To circumvent the problem of scheduling with partial CSIT, schemes linking random beamforming together with opportunistic scheduling were proposed [5], [6]. Such schemes are interesting as they yield optimal capacity scaling for large number of users. However, for moderate number of users their performance degrades significantly, since the feedback of signal-to-interference-plus-noise ratio (SINR) over the random beams is unable to characterize satisfactorily the spatial signature of the users and the random beams often miss their target [7], or equivalently a suboptimal user set is selected by the scheduler.

Recently, we identified that useful information relevant to the scheduler lies untapped in the correlation matrix, which can be easily acquired either by uplink/downlink reciprocity of the second-order statistics or via very low rate feedback channel. We have proposed to enhance multiuser communications by exploiting the channel correlation information at the transmitter in the design of the scheduler and combining it with the feedback of the channel norm [8]. We also proposed the idea that the scheduler (not the beamforming design) is the step that requires by far the most channel feedback since it involves processing of all users’ CSIT. Thus, we focus our attention on reducing the feedback requirements for that operation. Another scheme for combining the channel norm and the correlation knowledge was also reported in [9], however it has not been exploited in multiuser beamforming context. The disadvantage of the approaches above is that robustness is limited with respect to the situation where transmit correlation is negligible, leaving the scheduler with only channel gain information and no spatial information.

In this paper, we solve this problem by exploiting the transmit correlation information in the context of random beamforming where partial (scalar) feedback is obtained in the form of SINR information experienced by the users over the launched beams. We propose a scheme where the scheduling stage for multiuser MIMO is aided by forming a channel vector estimate based on the combination of transmit correlation matrix and instantaneous beam SINR feedback. We show the gain of this type of approach over standard opportunistic schemes in various settings. In particular the performance of the proposed schemes is the same as the optimal joint scheduling/beamforming with full CSIT, when the transmit correlation matrix is close to rank one (small angular spread at the base station). Furthermore,
our method exhibits robustness to the case of wide-angle spread as well, as it will decrease, in the worst case, down to a performance level similar to that of the conventional random beamforming of [6].

2. SYSTEM MODEL

We consider a multiple antenna broadcast channel consisting of $M$ transmit antennas and $K$ single-antenna receivers. The signal received by the $k$-th user at time slot $t$ is mathematically described as

$$y_k(t) = h_k^T x(t) + n_k(t), \quad k = 1, \ldots, K$$

(1)

where $x(t) \in \mathbb{C}^{M \times 1}$ is the transmitted signal at time slot $t$, $h_k \in \mathbb{C}^{M \times 1}$ is the channel vector, and $n_k(t)$ is additive white circularly symmetric complex Gaussian noise, which is assumed to be independent and identically distributed (i.i.d.) with zero mean and unit variance. We assume that the channel vector $h_k$ is perfectly known at the receiver. The transmitted signal is subject to transmit power constraint $P$, i.e., $\mathbb{E}(|x|^2) \leq P$. Due to the noise variance normalization, $P$ takes on the meaning of transmit signal-to-noise ratio (SNR). Let $H \in \mathbb{C}^{K \times M}$ refer to the concatenation of all channels, $H = [h_1, \ldots, h_K]^T$, where the $k$-th row is the channel of the $k$-th receiver ($h_k^T$).

We consider opportunistic beamforming using $M$ mutually orthogonal random beams, as proposed in [6]. The transmitted signal is given by

$$x(t) = \sum_{m=1}^{M} q_m(t) s_m(t)$$

(2)

where $s_m(t)$ is the transmit symbol associated to the $m$-th beam, and $q_m \in \mathbb{C}^{M \times 1}$ is the beamforming vector for the $m$-th beam in slot $t$. The beamforming vectors form a random orthonormal basis and correspond to the columns of an isotropically distributed $M \times M$ unitary matrix $Q$. The SINR of user $k$ on beam $m$ is

$$SINR_{k,m} = \frac{P |h_k^T q_m|^2}{\sum_{j \neq m} P |h_k^T q_j|^2 + 1}$$

(3)

The channel is considered invariant during each coded block, but is allowed to vary from block to block. We focus on the ergodic sum rate, which means that the capacity is averaged over the fading distribution, and thus the block length does not affect our results.

3. JOINT MMSE BEAMFORMING AND SCHEDULING WITH PERFECT CSIT

We focus here on the case of linear beamforming schemes with perfect CSIT, where exactly $M$ spatially separated users access the channel simultaneously. In this case the joint scheduling and beamforming problem can be stated as follows.

Let $w_k$ and $s_k$ be the (normalized) beamforming vector and data symbol of the $k$-th user, respectively. Let $Q$ be the set of all possible subsets of cardinality $M$ of disjoint indices among the complete set of user indices $\{1, \ldots, K\}$. Let $S \in Q$, be one such group of $M$ users selected for transmission at a given time slot. Then $H(S)$, $W(S)$, $s(S)$, $y(S)$ are the concatenated channel vectors, beamforming vectors, uncorrelated data symbols and received signals respectively for the set of scheduled users. The signal model is given by

$$y(S) = H(S)W(S)s(S) + n$$

(4)

For a group of users belonging to $S$, the objective is to find the linear precoding matrix $W_{\text{mmse}}(S)$ that minimizes the mean-square error (MSE) between the received and symbol vectors. Note that in contrast to opportunistic beamforming the design of the precoding matrix relies on full CSIT. Mathematically the problem can be expressed as

$$W_{\text{mmse}}(S) = \arg \min_{W : \|W\|^2 \leq P} \mathbb{E} \{ ||S(S) - y(S)||^2 \}$$

(5)

The optimal precoding matrix is given by

$$W_{\text{mmse}}(S) = (H(S)^H H(S) + \beta I)^{-1} H(S)^H$$

(6)

where the use of the regularization constant $\beta = M/P$ is motivated by the results in [10]. Note that the downlink MMSE filters do not in fact minimize the MSE, as the precoder effects all received signals before noise is introduced. As a user selection metric, the scheduler selects the set $S^*$ of users that minimizes the MSE, i.e.

$$S^* = \arg \max_{S \in Q} \mathbb{E} \left\{ \left\{ (\Psi(S) + \beta I)^{-1} (\Psi(S))^H \right\} \right\}$$

(7)

$$S^* = \arg \min_{S \in Q} \mathbb{E} \left\{ \left\{ (H(S)^H H(S) + \beta I)^{-1} H(S)^H \right\} \right\}$$

(8)

The sum rate is given by

$$R_{\text{mmse}} = \mathbb{E} \left\{ \sum_{k \in S^*} \log(1 + SINR_k) \right\}$$

(9)

However, the above framework has two major limitations. The optimum scheduler requires full CSIT for all $K$ users which cannot be assumed in practice. On the top of that, finding the optimal user set requires an exhaustive search over all possible subsets, which becomes prohibitively large even for moderate values of $K$.

4. PROPOSED CHANNEL ESTIMATION METHODS WITH PARTIAL CSIT

We present here a simple framework to exploit both long-term and partial short-term CSIT in correlated
multiple antenna channels. We consider transmission in a downlink where insufficient scattering around the transmitter makes the MIMO channel spatially correlated. This models an environment where antennas are placed for example at an elevated high-point base station. The receiver is located in a rich-scattering surrounding, and thus we have correlation only at the transmitter side.

The channel vector is complex Gaussian distributed with zero mean and full-rank covariance matrix $\mathbf{R}_k = \mathbb{E}\{\mathbf{h}_k \mathbf{h}_k^H\}$. We assume that $\mathbf{R}_k$ is perfectly known at both ends of the link, which can be obtained from up-link measurements (exploiting channel reciprocity) or using a low-rate feedback channel. The eigenvalue decomposition of the transmit correlation matrix is $\mathbf{R}_k = \mathbf{V}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$, where $\mathbf{\Sigma}_k$ is a diagonal matrix with the eigenvalues of $\mathbf{R}_k$ in descending order and $\mathbf{V}_k$ is a unitary matrix with the eigenvectors of $\mathbf{R}_k$.

The correlation matrix provides information about the spatial channel characteristics, especially if it is ill-conditioned, however it does not reveal any information about the quality of the current realization. In order to exploit multiuser diversity, the scheduler requires properly designed instantaneous feedback $\gamma_k$, which can be a measure of the quality of the current channel. Note that if the channel norm is chosen as instantaneous feedback, i.e. $\gamma_k = ||\mathbf{h}_k||^2$, no additional spatial information is provided and sign ambiguity in the channel direction cannot be eliminated.

In an opportunistic beamforming setting, for each user $k$, we combine the correlation information with a scalar $\gamma_k$ as $q_{\max,k} = \arg \max_{m=1,\ldots,M} |\mathbf{h}_k^T \mathbf{q}_{m,k}|^2 \quad (10)$

Evidently this scalar value provides a joint measure of the quality of the current channel realization and its direction. Although the amount of spatial information encapsulated into this metric cannot be decomposed from the channel gain information, it is particularly useful for users with strong channels, i.e., users that are very likely to be scheduled.

It can be also shown that the choice of $q_{\max,k}$ is equivalent to selecting the beam that provides the better SINR$_k$ in [6]. Consider a given user $k$, its best beam $i$ out of the $j \in \{1,\ldots,M\}$ is defined as:

$$i = \arg \max_j \frac{x_j}{c - x_j} \quad (11)$$

where $x_j = |\mathbf{h}_k^T \mathbf{q}_j|^2$ with $0 < x_j < c$, and $c = \sum_{m=1}^M |\mathbf{h}_k^T \mathbf{q}_m|^2 + M/P$ is a positive constant. Defining the function $f(x) = \frac{x}{c-x}$, we have that $\lim_{x \to 0} f(x) = 0$ and $\lim_{x \to \infty} f(x) = \infty$. Since $f(x)$ is always monotonous positive for $x \in (0,c)$, we have that

$$i = \arg \max_j f(x_j) = \max_j x_j \quad (12)$$

or, equivalently, $\arg \max_j \text{SIN}R_{k,i} = \arg \max_j |\mathbf{h}_k^T \mathbf{q}_j|^2$.

### 4.1 Constrained ML Channel Estimation

We propose a maximum likelihood (ML)-like channel estimate based on the limited long-term and instantaneous CSIT available. The intuition behind the method is to pick users whose channels span spatially separated cones of multipath and have good channel gains. The channel is modeled as Rayleigh flat-fading, so that $\mathbf{h}_k \sim \mathcal{CN}(0, \mathbf{R}_k)$ denotes a multivariate circularly symmetric zero-mean complex Gaussian vector with probability density function (pdf)

$$p_n(\mathbf{h}_k) = \frac{1}{\pi^M \det(\mathbf{R}_k)} \exp\left\{-\mathbf{h}_k^H \mathbf{R}_k^{-1} \mathbf{h}_k\right\} \quad (13)$$

The proposed Constrained Maximum Likelihood (CML) estimator is the one that maximizes the log-likelihood function of the pdf under the following scalar constraint

$$\gamma_k = |\mathbf{h}_k^T q_{\max,k}|^2 \quad (14)$$

Denoting $\Phi_k = q_{\max,k}^* \mathbf{R}_k^{-1} \mathbf{q}_{\max,k}$, it can be easily shown that this is equivalent to solving the generalized eigenvalue problem (GEV): $\mathbf{R}_k \mathbf{h}_k = \lambda \Phi_k \mathbf{h}_k$. The solution of (14), in the view of Rayleigh-Ritz quotient, is given by

$$\hat{\mathbf{h}}_k = \arg \max_{\mathbf{h}_k} \frac{\mathbf{h}_k^T \mathbf{R}_k \mathbf{h}_k}{\mathbf{h}_k^T \mathbf{\Phi}_k \mathbf{h}_k} \quad (15)$$

which corresponds to the (dominant) generalized eigenvector $\tilde{\mathbf{u}}_k$ associated with the largest positive generalized eigenvalue of the Hermitian matrix pair $(\mathbf{R}_k, \mathbf{\Phi}_k)$.

Therefore, the estimated channel vector is given by

$$\hat{\mathbf{h}}_k = \rho \tilde{\mathbf{u}}_k \quad (16)$$

where $\rho = \frac{\sqrt{\pi}}{|q_{\max,k} \tilde{u}_k|}$ to satisfy the constraint.

Once the above coarse channel estimate is derived for all users, we find the set of users to be scheduled using the scheduling metric in (8). At a second step, full channel feedback is demanded from the selected $M$ users and their optimal MMSE precoders are computed based on full CSIT. Additionally, an one-step, low feedback approach is to calculate the MMSE precoders based on this channel estimate and adapt only the selected users’ SINRs to the actual channel conditions.

### 4.2 Orthogonal Basis Expansion Channel Estimation

Since the CML approach requires the computation of the principal generalized eigenvector at each time slot, it exhibits remarkable computational complexity. Therefore, we propose an equivalent channel estimation method in which the channel of the $k$-th user is expressed as a linear combination of orthogonal vectors.
A natural choice of orthonormal basis is the random channel vector is expressed as
\[ q_k = \sum_{m=1}^{M} \alpha_m q_m^H \]
where \( q_m^H \) are the orthonormal vectors obtained from the matrix \( Q \) and \( \alpha_m \) are the (complex) weights of the orthogonal expansion.

Assume, without loss of generality, that \( q_1 \) corresponds to the best beam chosen by user \( k \). Substituting (17) into (14), and solving the optimization problem (P1) using Lagrange multipliers, we obtain that the optimal weights \( b_{\text{opt}} = [\alpha_2, \ldots, \alpha_M]^T \) equal to
\[ b_{\text{opt}} = -\alpha_1 A^{-1} c \]
where
\[ c = [q_2^T R_k^{-1} q_1^*, \ldots, q_M^T R_k^{-1} q_1^*]^T \]
and \( \alpha_1 = \sqrt{\gamma_k} \) so that the instantaneous feedback constraint is satisfied.

Observing the similarity in the structure of matrix \( A \) with that of \( Q^T R_k^{-1} Q^* \), the computational complexity of the matrix inversion of \( A \) can be further reduced through use of block matrix decomposition.

Denote \( F = Q^T R_k^{-1} Q^* \), then
\[ F = \begin{bmatrix} q_1^T R_k^{-1} q_1 & q_1^T R_k^{-1} q_2^* & \cdots & q_1^T R_k^{-1} q_M^* \\ q_2^T R_k^{-1} q_1^* & q_2^T R_k^{-1} q_2 & \cdots & q_2^T R_k^{-1} q_M \\ \vdots & \vdots & \ddots & \vdots \\ q_M^T R_k^{-1} q_1 & q_M^T R_k^{-1} q_2^* & \cdots & q_M^T R_k^{-1} q_M^* \end{bmatrix} \]
and the inverse \( A^{-1} \) is obtained from the equality
\[ S_A^{-1} \begin{bmatrix} 1 \\ -A^{-1} c \\ -c^H A^{-1} \end{bmatrix} = F^{-1} \]
where \( S_A = q_1^T R_k^{-1} q_1^* - c^H A^{-1} c \) is the Schur complement of \( A \) and \( F^{-1} = Q^T R_k Q^* \) as \( Q \) is unitary.

5. NUMERICAL RESULTS

At the transmitter side, we consider a uniform linear antenna array (ULA) and the channel evolves according to a specular, canonical model where the channel impulse response is a superposition of \( U \) spatially separated paths. Each of these paths have an angle of incidence with respect to the transmitter broadside of \( \theta_{k,u} \). The angles follow a Gaussian distribution with 2\( \pi \)-periodic continuation and mean \( \bar{\theta}_k \). The angle spread around its mean is given by \( \sigma_\theta_k = \sqrt{\text{E}[(\theta_{k,u} - \bar{\theta}_k)^2]} \).

The channel can be described as
\[ h_k = \frac{1}{\sqrt{U}} \sum_{u=1}^{U} \phi_{k,u} a(\theta_{k,u}) \]
where \( \phi_{k,u} \) is the gain of the \( u \)-th path seen at the receiver and is assumed to be zero-mean complex Gaussian distributed. Assuming that all paths have unit variance, the steering vectors \( a(\theta_{k,u}) \) are defined as
\[ a(\theta_{k,u}) = \left[ 1, e^{-jkd \cos(\theta_{k,u})}, \ldots, \right. \]
\[ \left. e^{-jk(M-1)d \cos(\theta_{k,u})} \right]^T \]

We compare the sum rate of the proposed channel estimation methods with that of optimal MMSE beamforming with full CSIT and with a random beamforming-based scheduling approach [6].

Figures 1 and 2 show the performance comparison as a function of the angle spread and the number of users, respectively. Once the scheduling user set is obtained based on each user’s channel estimate, the transmitter obtains full CSIT for the selected users and designs the corresponding MMSE beamformers. We use 5000 channel realizations and the correlation matrix is averaged over 60 slots. The SNR is set to 10 dB. It is observed that the scheduler, based on our coarse channel estimate, is able to identify better users than random opportunistic beamforming for all angle spreads. When the path angle spread is close to zero, our schemes close the throughput gap to the MMSE precoder with full CSIT. Note also that the two proposed estimation schemes exhibit exactly the same performance as they are solution variants of the same optimization problem, and they only differ in terms of computational complexity.

In Figure 3, we evaluate the performance of the channel estimation methods when both user selection and beamforming design are done based on the coarse channel estimate. Evidently, the MMSE precoders derived from the estimated channel are valid for highly correlated channels. Both our proposed methods show - without additional feedback - a significant gain over random beamforming for angle spread less than 35 degrees, making them a practical approach for cellular outdoor systems.

6. CONCLUSIONS

We show how statistical channel knowledge can be efficiently combined with instantaneous scalar feedback for the purpose of scheduling/beamforming in multiuser MIMO systems. We proposed an efficient channel estimation method based on maximum likelihood criterion, which offers performance close to the full CSIT scheme when the multipath angular spread per user at the base
station is small enough, making this approach suitable to wireless systems with elevated base stations such as outdoor cellular networks.

REFERENCES


Figure 1: Sum rate versus angle spread performance of proposed estimation methods for $M = 2$, and $K = 50$ users. Full CSIT is obtained for the selected users.

Figure 2: Sum rate versus the number of users performance of various channel estimation methods for $M = 2$, and $\sigma_\theta = 0.2\pi$. User selection is based on the channel estimate - full CSIT for the selected users is obtained for beamforming.

Figure 3: Sum rate versus angle spread performance of different estimation methods for $M = 2$, and $K = 50$ users. User selection and MMSE precoding design are derived based on the coarse channel estimate.