## A LINEAR CHIP-LEVEL MODEL FOR MULTI-ANTENNA SPACE-TIME CODED WIDEBAND CDMA RECONFIGURABLE RECEIVERS

Giovanni Garbo, Stefano Mangione, Vincenzo Maniscalco

Dipartimento Ingegneria Elettrica Elettronica Telecomunicazioni, Università di Palermo Viale delle scienze ed. 9, 90128, Palermo, Italy phone: + (39)0916615247, fax: + (39)091488452 e-mail: {giovanni.garbo,stefano.mangione,vincenzo.maniscalco}@tti.unipa.it web: http://www.tti.unipa.it

## ABSTRACT

Transmitter side reconfigurability is currently mostly implemented via adaptive modulation and coding. In the scenario of multiple antenna transmission which is foreseeable for fourth generation wireless mobile communications, a new level of reconfigurability might employ transmission scheme adaptation, i.e. dynamically switching between spatialmultiplex and space-time coded transmission.

This paper presents a generalized linear model for the received signal in a multi-antenna CDMA signaling scheme on frequency-selective fading MIMO channels. The proposed model is unprecedented in that it supports general complex space-time codes as well as spatial-multiplex multiantenna transmission and it may be implemented with minor modifications to the front-end of a matched filter receiver.

## 1. INTRODUCTION

The upcoming fourth generation of wireless mobile communications is expected to provide the user with the best possible experience in heterogeneous environments. From the medium access technology point of view, multiple antenna transmission is a key to achieve high spectral efficiencies and its use is being devised in many recent standards.

Anyhow, due to the heterogeneity of wireless mobile scenarios, neither a uniform transmitter scheme nor a single transmit antenna fall-back may provide a fitting solution. For example, in spatially-correlated fading scenarios, a relatively low-rate space-time coding scheme would be more suitable than a spatial-multiplex to approach channel capacity [1].

Nonetheless, it is widely accepted that transmitter-side reconfigurability poses an implementative issue on the receiver side, since a specialized (ASIC) approach will result in wasted chip area, while a fully software-defined radio approach is not yet viable for multi-megabit per second receiver front-ends [2].

Motivated by these issues, this work presents a novel generalized linear model for the received signal of a spacetime coded or/and spatial-multiplexed Wideband CDMA scheme on MIMO frequency selective channels. The novelty of the proposed model is that the traditional representation of a Space-Time Block Coded transmission has a non-linear dependency on the data. In fact, all complex-valued STBC's are a function of data symbols and their complex conjugates, which is non-linear in nature. The proposed model is based on the equivalence between complex numbers and real vectors. As it will be shown, our model has neither computational nor storage requirements when compared to a more traditional approach based on complex numbers.

The paper is structured as follows: section 2 reports the derivation of a traditional model and the novel linear model for the received signal, section 3 presents an example of application with some simulative results based on a reference receiver structure, and finally section 4 draws conclusions and directions for further work.

#### 2. SYSTEM MODEL

#### 2.1 MIMO frequency selective channel

Without loss of generality, we assume a basebandequivalent quasi-static time-varying multipath MIMO channel with channel impulse response given by

$$h_{n,m}(j,i) = \sum_{l=0}^{L} \rho_{n,m,l}(iT_c) \gamma_p(jT_c - \tau_l)$$

where  $0 \le n < N$  and  $0 \le m < M$  are respectively the receive and transmit antenna indexes,  $0 \le l \le L$  is the resolvable path index,  $T_c$  is the sampling period (at least equal to the chip signaling period),  $iT_c$  is the input time sample,  $jT_c$  is the output time sample,  $\rho_{n,m,l}(iT_c)$  is the complex-valued *l*-th path gain,  $\tau_l$  is the *l*-th path delay and  $\gamma_p(\tau)$  is the modulation shaping pulse self-correlation function.

The complex baseband sampled output from the n-th receive antenna chip matched filter is given by

$$r_{n}(j) = \sum_{i} \sum_{m=0}^{M-1} h_{n,m}(j-i,i) c_{m}(i) + v_{n}(j)$$
(1)

where  $c_m(i)$  is input to the *m*-th transmit antenna pulse shaping filter and  $v_n(j)$  is the sampled sequence of filtered additive noise and interference.

#### 2.2 W-CDMA orthogonal signaling

The channel access scheme which is assumed in the course of the derivation is the W-CDMA 3GPP standard [4]. However, the proposed model may in principle be extended to any linear modulation scheme. The assumption of a multi-

user access scheme allows for a straightforward generalization to other widely used access schemes such as Orthogonal Frequency Division Multiplex and Pulse Position Modulation.

With W-CDMA signaling,  $c_m(i)$  is given by

$$c_{m}(i) = \sum_{\xi} \sum_{k=0}^{K-1} s_{k}^{(\xi)}(i) x_{m,k}(\xi)$$
(2)

where  $0 \le k < K$  is the spreading code index,  $\xi$  is the CDMA codeword index,  $x_{m,k}(\xi)$  is the complex symbol associated to code k on antenna m,  $s_k^{(\xi)}(i) \ne 0$  for  $\xi G_k \le i < (\xi+1)G_k$  is the k-th code spreading sequence, which is specified in W-CDMA as a complex time-varying sequence given by a Walsh-Hadamard OVSF code of length  $G_k$  multiplied by a segment of complex valued long Gold Code used as scrambling sequence.

Joining equations (1) and (2) the following expression for the received signal is obtained:

$$r_{n}(j) = \sum_{\xi,m,k} \beta_{n,m,k}^{(\xi)}(j) x_{m,k}(\xi) + \nu_{n}(j)$$
(3)

where  $\beta_{n,m,k}^{(\xi)}(j) = \sum_{i} h_{n,m}(j-i,i) s_k^{(\xi)}(i)$  is the *m*-th to

*n*-th channel response to  $s_k^{(\xi)}$  and multiple summations have been joined to simplify the notation.

## 2.3 Space-Time Block Codes

A generic Space-Time Block Code is a map from  $\mathbb{C}^{P}$  to  $\mathbb{C}^{QM}$ , where *P* is the number of input symbols in a code block, *Q* is the number of time samples in a block and *M* in the number of transmit antennas, defined before. The rate of an STBC code is defined as P/Q.

Without loss of generality, we assume that the  $\phi$ -th STBC codeword which is associated to a block of complex constellation points  $\mathbf{d}(\phi) = \begin{bmatrix} d(\phi P) & \dots & d(\phi P + P - 1) \end{bmatrix}^T$  is given by  $\mathbf{F}(\mathbf{d}(\phi)) \in \mathbb{C}^{QM}$ . In this paper, boldface fonts denote column vectors and matrices.

For the so-called *linear* space-time block codes [3], **F** has the following form

$$\mathbf{F}(\mathbf{d}(\phi)) = \mathbf{A}\mathbf{d}(\phi) + \mathbf{B}\mathbf{d}^{*}(\phi)$$
(4)

where **A** and **B** are suitably defined matrices on  $\mathbb{R}^{\mathcal{Q}M\times P}$ . For example, the well-known Alamouti code [5] has Q = M = P = 2 and the following representation

$$\mathbf{F}(\mathbf{d}(\phi)) = \begin{bmatrix} d(2\phi) \\ d(2\phi+1) \\ -d^*(2\phi+1) \\ d^*(2\phi) \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Note that this model, although called linear, employs complex conjugation so it non-linear on the complex field.

#### 2.4 MIMO spatial multiplex

Spatial multiplex is a high spectral efficiency wireless communication scheme which has been introduced by Foschini in his pioneering work [6]. In the current context, spatial multiplexing may be seen as an STBC with M = P and Q = 1; the representation of a spatial multiplex scheme is  $\mathbf{A} = \mathbf{I}_{M}$  and  $\mathbf{B} = \mathbf{0}$ .

#### 2.5 Space-time Block Coded W-CDMA model

To derive a model for the received signal of space-time block coded W-CDMA it suffices to link  $x_{m,k}(\xi)$  and  $\mathbf{F}(\mathbf{d}(\phi))$ . Denoting with  $\mathbf{d}_k(\phi)$  the *k*-th user  $\phi$ -th block of modulation symbols,  $x_{m,k}(\xi) = \mathbf{F}(\mathbf{d}_k(\phi))_{qM+m}$ , where  $\phi = \lfloor \xi/Q \rfloor$ ,  $q = \xi - \phi Q$ .

To develop a compact expression for vector or matrix index selection, let us define the canonical base vector as

$$\mathbf{e}_{\alpha} = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^{t},$$

so that  $x_{m,k}(\xi) = \mathbf{e}_{qM+m}^T \mathbf{F}(\mathbf{d}_k(\phi))$ , and the following model for the received signal is finally obtained:

$$r_{n}(j) = \sum_{\phi,q,m,k} \beta_{n,m,k}^{(\phi Q+q)}(j) \mathbf{e}_{qM+m}^{T} \mathbf{F}(\mathbf{d}_{k}(\phi)) + \nu_{n}(j).$$
(5)

It must be noted that this model is not linear in  $d_k(\phi P + p)$ , so an *ad-hoc* receiving structure must be devised for the detection.

# 2.6 Representation of complex numbers via real vectors

A complex number  $\zeta$  may be represented by a column real-valued vector  $\tilde{\zeta} = [\Re(\zeta) \ \Im(\zeta)]^T$  and conversely a real two-element column vector  $\tilde{\zeta}$  may represent the complex number  $\zeta = [1 \ l]\tilde{\zeta}$ . Addition between two complex numbers results in vector addition, while multiplication between two complex numbers a+tb and c+td is accomplished by a matrix-vector multiplication:

$$\begin{bmatrix} ac-bd\\ ad+bc \end{bmatrix} = \begin{bmatrix} a & -b\\ b & a \end{bmatrix} \begin{bmatrix} c\\ d \end{bmatrix} = \begin{bmatrix} c & -d\\ d & c \end{bmatrix} \begin{bmatrix} a\\ b \end{bmatrix}.$$

Complex conjugation is also accomplished by a matrixvector multiplication:

$$\tilde{\zeta}^* = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tilde{\zeta}$$

but the matrix involved does not represent any complex number (since it is not anti-symmetric).

Thus the non-linear complex conjugation operation may be represented by a linear operation on real-valued vectors.

A complex valued column vector  $\mathbf{x} \in \mathbb{C}^N$  is equivalent to a real-valued vector  $\tilde{\mathbf{x}} \in \mathbb{R}^{2N}$  defined as follows:

$$\tilde{\mathbf{x}} = \Re(\mathbf{x}) \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \Im(\mathbf{x}) \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

A complex valued matrix  $\mathbf{C} \in \mathbb{C}^{N \times M}$  is equivalent to a real-valued matrix  $\tilde{\mathbf{C}} \in \mathbb{R}^{2N \times 2M}$  defined as follows:

$$\tilde{\mathbf{C}} = \Re(\mathbf{C}) \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \Im(\mathbf{C}) \otimes \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

where  $\otimes$  denotes the Kronecker (or external) product. Note that the storage and computational requirements for the real-valued vector associated to a complex-valued vector are the same, while the representation of a complex-valued matrix has twice the storage and same computational requirements. So the proposed equivalence trades-off functionality (ability to linearize the complex conjugation) with storage requirements for matrices involved in complex multiplications.

#### 2.7 Linear model

Using the results presented in the previous subsection, we derive an equivalent linear model for the received signal in terms of the symbols input to each of the K STBC encoders.

Observing that

$$\tilde{\mathbf{d}}^{*}(\phi) = \mathbf{I}_{P} \otimes \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tilde{\mathbf{d}}(\phi) ,$$

the model (4) for STBC encoding of a data block  $\mathbf{d}(\phi)$  may be represented by

$$\tilde{\mathbf{F}}(\mathbf{d}(\phi)) = \mathbf{G}\tilde{\mathbf{d}}(\phi) \tag{6}$$

with G, equivalent STBC encoding matrix, defined as

$$\mathbf{G} \triangleq \tilde{\mathbf{A}} + \tilde{\mathbf{B}} \left( \mathbf{I}_{p} \otimes \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right).$$

The equivalent STBC output symbol used for modulation then becomes:

$$\tilde{x}_{m,k}\left(\boldsymbol{\xi}\right) = \left(\mathbf{e}_{qM+m}^{T} \otimes \mathbf{I}_{2}\right) \mathbf{G}\tilde{\mathbf{d}}_{k}\left(\boldsymbol{\phi}\right)$$

and the respective complex symbol is

$$x_{m,k}\left(\boldsymbol{\xi}\right) = \begin{bmatrix} 1 & t \end{bmatrix} \left( \mathbf{e}_{qM+m}^{T} \otimes \mathbf{I}_{2} \right) \mathbf{G} \mathbf{d}_{k}\left(\boldsymbol{\phi}\right).$$

This expression may be put in a form explicitly dependent on the input symbol index  $0 \le p < P$  using the following identity:

$$\mathbf{G}\tilde{\mathbf{d}}_{k}\left(\boldsymbol{\phi}\right) = \sum_{p=0}^{P-1} \mathbf{G}\left(\mathbf{e}_{p} \otimes \mathbf{I}_{2}\right) \tilde{d}_{k}\left(\boldsymbol{\phi}P + p\right)$$

thus obtaining

$$x_{m,k}\left(\xi\right) = \sum_{p=0}^{P-1} \Gamma_{q,m,p} \tilde{d}_k \left(\phi P + p\right)$$

where  $\Gamma_{q,m,p} = \begin{bmatrix} 1 & t \end{bmatrix} (\mathbf{e}_{qM+m}^T \otimes \mathbf{I}_2) \mathbf{G} (\mathbf{e}_p \otimes \mathbf{I}_2).$ 

Substituting this last expression in (3) one obtains

$$r_{n}(j) = \sum_{\phi,q,m,k} \left[ \beta_{n,m,k}^{(\phi Q+q)}(j) \sum_{p=0}^{P-1} \Gamma_{q,m,p} \tilde{d}_{k}(\phi P+p) \right] + \nu_{n}(j).$$

Defining **r** and **v** as a complex-valued infinite-length vectors with elements  $\mathbf{r}_{jN+n} = r_n(j)$  and  $\mathbf{v}_{jN+n} = v_n(j)$  respectively, and **d** as a complex-valued infinite-length vector

with elements  $\mathbf{d}_{(\phi K+k)P+p} = d_k (\phi P + p)$ , the following model is obtained in matrix form:

where 
$$\boldsymbol{\Theta}_{jN+n,(\phi K+k)P+p} = \sum_{m=0}^{M-1} \sum_{q=0}^{Q-1} \beta_{n,m,k}^{(\phi Q+q)}(j) \boldsymbol{\Gamma}_{q,m,p}$$

Finally, a linear real-valued equivalent model for the received signal of space-time coded W-CDMA on a frequency selective MIMO channel is obtained as

$$\tilde{\mathbf{r}} = \tilde{\mathbf{H}}\tilde{\mathbf{d}} + \tilde{\mathbf{v}}$$

where  $\tilde{\mathbf{H}}$  is given by

$$\tilde{\mathbf{H}} = \Re(\mathbf{\Theta}) \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \Im(\mathbf{\Theta}) \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Note that the storage requirements of  $\hat{\mathbf{H}}$  are the same needed for the non-linear model which can be easily derived from (5), so the proposed model has, for non-trivial spacetime coded transmission, no performance impairments while providing a linear model which can be employed in any of many receiver structures known in literature.

As for the computational complexity, it is to be underlined that  $\Gamma$  is a sparse matrix, so that the calculation of  $\Theta$ can be significantly optimized. Moreover,  $\beta_{n,m,k}^{(\xi)}$  are promptly available at the front-end of a RAKE receiver, being the convolution of the spreading waveforms with the (estimated) channel impulse response.

#### 3. A SAMPLE APPLICATION

In this section some simulative results for a reference receiver employing a finite length version of the proposed model will be reported. The channel used in the simulations is the SCM channel specified by the 3GPP/3GPP2 Spatial Channel Model Ad-Hoc Group [7]. The chosen reference scenario is the Urban Macro with six-sector cell layout. The transmitter and receiver antennas are uniform linear arrays with  $10 \lambda$  and  $0.5 \lambda$  spacing at the transmitter and receiver respectively.

Spreading and modulation conform to the W-CDMA specifications [4]; the spreading gain  $G_k = 16$  has been set equal for all codes employed.

The reference receiver structure has been chosen as the well-known Linear Minimum Mean Square Error equalizer. A finite-length model has been obtained by first truncating the channel impulse response to .99 effective duration, then by accommodating the sufficient statistic of the desired symbols into the equalizer observation window.

The rationale for choice of modulation, transmit diversity scheme and number of codes has been to compare solutions having the same spectral efficiency and number of receiver antennas.

Figure 1 reports a comparison between schemes operating at 3.75 bps/Hz (14.4Mbps): all schemes are at full load (K=15);

- Spatial multiplex of two QPSK streams (2 transmit antennas)



Figure 1. Bit Error Rate performance of the reference receiver with 3.75 bps/Hz spectral efficiency and two receiver antennas.

- Spatial multiplex of two Alamouti QPSK streams (4 transmit antennas)
- Alamouti 16QAM stream (with diversity reception)
- Transposed rate  $\frac{3}{4}$  scheme from [3], with M = 4, P = Q = 3

As is apparent in the figure, the performance of spacetime coded schemes with diversity reception outperforms (diversity gain) that of both pure spatial multiplex and multiplexed space-time coded schemes. More advanced receiver structures might be able to gain diversity for the spatial multiplex schemes, but we regard those considerations as outside the scope of this paper.

#### 4. CONCLUSIONS

A novel, linear model for the received signal of multiantenna and space-time coded transmission on multi-input multi-output frequency selective channels has been derived. While it has been shown that the proposed model may be implemented with minor modifications to the front-end of a matched filter W-CDMA receiver, it is general enough to be adapted to other wireless medium access schemes such as OFDM.

It enables the use of well-known linear receiver structures for space-time coded transmissions and has been demonstrated by means of simulation.

### REFERENCES

[1] D. Gesbert, M. Shafi, D. Shiu, P. J. Smith and A. Naguib, "From Theory to Practice: An Overview of MIMO Space-Time Coded Wireless Systems," *IEEE Journal on Selected Areas in Communications*, Vol. 21, No. 3, April 2003.

[2] R. Kokozinski, D. Greifendorf, J. Stammen and P. Jung, "The evolution of hardware platform for mobile 'software defined radio' terminals," *IEEE PIMRC 2002*, pp. 23892393.
[3] V. Tarokh, H. Jafarkhani and A. R. Calderbank "Space-Time Block Codes from Orthogonal Designs," *IEEE Trans-*

actions on Information Theory, Vol. 45, Issue 5, July 1999.

[4] 3GPP TSG-RAN "Spreading and Modulation (FDD)," TS25.213, http://www.3gpp.org/

[5] S. M. Alamouti "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE Journal on Selected Areas in Communications*, Vol. 19, Issue 2, February 2002.

[6] G.J. Foschini "Layered Space-Time Architecture for Wireless Communication in a Fading Environment when using Multiple Antennas," *Bell Labs Technical Journal*, Vol. 1, No. 2, Autumn 1996, pp. 41-59.

[7] 3GPP TSG-RAN "Spatial Channel Model for MIMO simulations," TR25.996, http://www.3gpp.org/