A RATE-DISTORTION APPROACH TO OPTIMAL COLOR IMAGE COMPRESSION

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ABSTRACT

Most image compression systems deal today with color images although the coding theory and algorithms are still based on gray level imaging. The performance of such algorithms has not been analyzed and optimized so far for color images, especially when the selection of color components is considered. In this work we introduce a rate-distortion approach to color image compression and employ it to find the optimal color components and optimal bit allocation for the compression. We show that the DCT (Discrete Cosine Transform) can be used to transform the RGB components into an efficient set of color components suitable for subband coding. The optimal rates can be used to design adaptive quantization tables in the coding stage with results superior to fixed quantization tables. Based on the presented results, our conclusion is that the new approach can improve presently available methods for color compression.

1. INTRODUCTION

It is well known that natural images are characterized by high correlation between their RGB components [2], [4], [14]. This data redundancy has to be considered in order to reduce the volume of information that has to be stored or transmitted for a given image. Most of the techniques for color image compression reduce the redundancies between the colors components by transforming the colors primaries into a decorrelated color space such as YIQ or YUV [7], [12] or by performing the KLT (Karhunen-Loeve Transform) on the color components in some color space [5], [13]. The choice of the YUV or YIQ color space is the most common, but it is usually not optimal as demonstrated in this work in the context of subband transform coding systems.

1.1 Subband transforms

Subband transforms include the Discrete Cosine Transform (DCT), Discrete Fourier Transform (DFT) as well as wavelet tree decompositions, wavelet packets and filter banks. The most familiar systems based on subband transform coding are the JPEG [12] and JPEG2000 [7] standards for image coding. Other examples include the EZW wavelet based algorithm [9] for images and for example Uniform DFT Filter Banks [8] for speech coding and communication tasks [1], [6].

Subband transforms are a generalization of block transforms. If we consider a non-expansive transform (which transforms an input signal into an output signal of the same

size), the input sequence x is divided into m-dimensional vectors $\mathbf{x}[k]$ and each is transformed to form the (m-dimensional) output vector $\mathbf{y}[k]$. However, in contrast to a simple block transform, multiple input vectors are used to form one output vector according to:

$$\mathbf{y}[k] = \sum_{i \in \mathbb{Z}} \mathbf{A}^{H}[i]\mathbf{x}[k-i], \tag{1}$$

where A[i] is a series of matrices and H stands for conjugate transpose. If $A[i] \neq 0$ only for i = 0, (1) describes a block transform. The inverse transform, if exists, is of the form:

$$\mathbf{x}[k] = \sum_{i \in \mathbb{Z}} \mathbf{S}[i]\mathbf{y}[k-i]. \tag{2}$$

When referring to the b^{th} subband of the transform, we mean the sequence of the b^{th} components of each output vector $\mathbf{y}[k]$, which we denote $y_b[n]$.

1.2 Rate-Distortion theory of subband coders

1.2.1 Rate Distortion of the PCM scheme

The rate-distortion performance of a scalar quantizer with independently coded samples for a stochastic source x with variance σ_x^2 can be modeled as [10]:

$$d(R) = g(R)\sigma_x^2 2^{-2R},$$
 (3)

where d() is the MSE (Mean Square Error) distortion, R is the rate in bits per sample and g() is a weak function of the source. For large enough R, $g(R) \cong \varepsilon^2$, where ε^2 is a constant dependent upon the distribution of x and therefore:

$$d(R) = \varepsilon^2 \sigma_x^2 2^{-2R}. \tag{4}$$

The scheme that performs scalar quantization with independent coding of the source samples is called PCM (Pulse Code Modulation). An example of such a system is a uniform scalar quantizer with entropy coded output.

1.2.2 Rate-Distortion of a general subband transform coder

Consider an encoder that first transforms an N samples source signal x into a set of subbands by a subband transform and then each subband coefficients are coded independently by the PCM scheme, while the decoder reconstructs the signal \hat{x} from the dequantized transform coefficients. The MSE

of the coding system for the signal x can be expressed as [10]:

$$d_{x} = E\left[\frac{1}{N}\sum_{k}(\mathbf{x}[k] - \hat{\mathbf{x}}[k])^{2}\right] = \sum_{b=0}^{B-1} \eta_{b}G_{b}d_{b}, \quad (5)$$

where d_b is the MSE distortion of subband b, η_b denotes the ratio between the number of coefficients in subband b and the total number of samples in the source N and B is the total number of subbands. G_b are the energy gains equal to the L_2 squared norms of the subband synthesis vectors [10]. Since the transform is assumed to be non-expansive, the following equation holds:

$$\sum_{b=0}^{B-1} \eta_b = 1.$$
(6)

Substituting the PCM MSE of (4) for d_b , we get:

$$d_x = \sum_{b=0}^{B-1} \eta_b G_b \sigma_b^2 \varepsilon^2 2^{-2R_b},\tag{7}$$

where σ_b^2 is the variance of the subband indexed b ($b \in [0, B-1]$) and R_b is the rate allocated to it. Equation (7) holds for orthonormal as well as non-orthonormal transforms, since for an orthonormal transform we can substitute $G_b = 1$ for all $b \in [0, ..., B-1]$.

2. EXTENSION OF THE RATE-DISTORTION TO COLOR IMAGES

Denote each pixel in a color image in the RGB domain by a 3x1 vector $\mathbf{x} = [R \ G \ B]^T$. We first apply a color component transform (CCT) to the image, denoted by a matrix \mathbf{M} to obtain at each pixel a new vector of 3 components C1, C2, C3, denoted $\widetilde{\mathbf{x}} = [C1 \ C2 \ C3]^T$ and related to \mathbf{x} by:

$$\widetilde{\mathbf{x}} = \mathbf{M}\mathbf{x}.$$
 (8)

Then each component in the C1,C2,C3 color space is subband transformed, quantized and its samples are independently encoded (e.g. entropy coded). This description corresponds to such image compression algorithms as JPEG [12] and JPEG 2000 [7], when applied to a color image up to and including the quantization stage (note that after the quantization stage, JPEG or JPEG2000 encode the transform coefficients not independently, but using the correlation between them, so the above description does not apply).

We denote by $\tilde{\mathbf{x}}_{rec}$ the reconstructed image in the C1C2C3 domain after inverse quantization, and by \mathbf{x}_{rec} the reconstructed image in the RGB domain, when $\tilde{\mathbf{x}}_{rec} = \mathbf{M}\mathbf{x}_{rec}$ similarly to (8). Now we can define the error covariance matrix in the RGB domain \mathbf{Er} :

$$\mathbf{Er} = E\left[(\mathbf{x} - \mathbf{x}_{rec})(\mathbf{x} - \mathbf{x}_{rec})^T \right], \tag{9}$$

and the error covariance matrix in C1C2C3 domain Er:

$$\widetilde{\mathbf{Er}} = E\left[(\widetilde{\mathbf{x}} - \widetilde{\mathbf{x}}_{\mathbf{rec}}) (\widetilde{\mathbf{x}} - \widetilde{\mathbf{x}}_{\mathbf{rec}})^T \right], \tag{10}$$

where $E[\cdot]$ stands for statistic mean. It is easy to show that:

$$\widetilde{\mathbf{Er}} = E \left[(\mathbf{Mx} - \mathbf{Mx_{rec}}) (\mathbf{Mx} - \mathbf{Mx_{rec}})^T \right] = \mathbf{MErM^T}$$

$$\Rightarrow \mathbf{Er} = \mathbf{M}^{-1} \widetilde{\mathbf{ErM}}^{-T}.$$
(11)

The average MSE between the original and reconstructed images in the RGB domain is then simply:

$$MSE = \frac{1}{3}trace(\mathbf{Er}) = \frac{1}{3}trace(\mathbf{M}^{-1}\widetilde{\mathbf{Er}}\mathbf{M}^{-T})$$
$$= \frac{1}{3}trace(\widetilde{\mathbf{Er}}(\mathbf{M}\mathbf{M}^{T})^{-1}).$$
(12)

Assuming that the errors in the three color components C1, C2, C3 (that occur due to the quantization inherent in the compression process) are uncorrelated, i.e.,

$$E[(Ci-Ci_{rec})(Cj-Cj_{rec})] = 0, \quad i, j \in \{1,2,3\}, \quad i \neq j$$
(13)

(where Ci_{rec} is the reconstructed Ci component), it is clear that $\widetilde{\mathbf{Er}}$ is diagonal and therefore the expression for the average MSE of (12) simplifies to:

$$MSE = \frac{1}{3} \sum_{i=1}^{3} \widetilde{\mathbf{Er}}_{ii} \left((\mathbf{M} \mathbf{M}^{T})^{-1} \right)_{ii}, \tag{14}$$

where $\widetilde{\mathbf{Er}}_{ii}$ denotes the MSE of color component Ci. Using (7) for this MSE, one can easily derive the following expression for the average MSE:

$$MSE = \frac{1}{3} \sum_{i=1}^{3} \sum_{b=0}^{B-1} \eta_b G_b \sigma_{bi}^2 \varepsilon_i^2 e^{-aR_{bi}} \left((\mathbf{M} \mathbf{M}^T)^{-1} \right)_{ii}. \quad (15)$$

 R_{bi} stands for the rate allocated for the subband b of component i and σ_{bi}^2 is this subband's variance. Also a = 2ln2.

The expression obtained can be used to find the optimal subband rates allocation for minimal MSE for a given color components transform, to find the optimal color components transform for a given rates allocation, or to find both optimal rates allocation and color transform. In the next section we minimize the MSE function.

3. FINDING THE OPTIMAL RATES AND COLOR COMPONENTS TRANSFORM

We would like to minimize the MSE of (15) subject to the constraint of some total rate allocation *R* for the image:

$$\sum_{i=1}^{3} \sum_{b=0}^{B-1} \eta_b R_{bi} = R. \tag{16}$$

Using the Lagrange multipliers method, we thus have to minimize the function

$$L(\{R_{bi}\}, \mathbf{M}, \lambda) = \frac{1}{3} \sum_{i=1}^{3} \sum_{b=0}^{B-1} \eta_b G_b \sigma_{bi}^2 \varepsilon_i^2 e^{-aR_{bi}} \left((\mathbf{M} \mathbf{M}^T)^{-1} \right)_{ii} + \lambda \left(\sum_{i=1}^{3} \sum_{b=0}^{B-1} \eta_b R_{bi} - R \right),$$
(17)

where λ denotes the Lagrange multiplier. It can be shown [3], that minimizing for R_{bi} and M yields the following equations for the optimal rates (extensions considering downsampling of the color components and non-negativity constraints can be found in [3]):

$$R_{bi} = \frac{1}{a} ln \left(\frac{\varepsilon_i^2 G_b \sigma_{bi}^2 \left((\mathbf{M} \mathbf{M}^T)^{-1} \right)_{ii}}{\left(\prod_{k=1}^3 \left(G M_k \varepsilon_k^2 (\mathbf{M} \mathbf{M}^T)_{kk}^{-1} \right) \right)^{\frac{1}{3}}} \right) + \frac{R}{3}. \quad (18)$$

 GM_k here is the weighted geometric mean of the subband variances of component k (corrected by the energy gains G_b):

$$GM_k = \prod_{b=0}^{B-1} (G_b \sigma_{bk}^2)^{\eta_b}.$$
 (19)

We can see that the equations for optimal rates allocation depend on the color components transform (CCT) M. The optimal color component transform is the one minimizing the following target function:

$$\tilde{f}(\mathbf{M}) = \prod_{k=1}^{3} \left((\mathbf{M} \mathbf{M}^{T})^{-1} \right)_{kk} GM_{k}.$$
 (20)

Taking the energy gains in GM_k out of the target function, since it is not dependent on M, the target function simplifies to:

$$f(\mathbf{M}) = \prod_{k=1}^{3} ((\mathbf{M}\mathbf{M}^{T})^{-1})_{kk} \prod_{b=0}^{B-1} (\sigma_{bk}^{2})^{\eta_{b}}.$$
 (21)

We will now concentrate on the $\tilde{f}(\mathbf{M})$ target function minimization. As can be seen, no constraints are needed here for the matrix minimizing this function to be invertible due to the $\prod_{k=1}^{3} \left((\mathbf{M}\mathbf{M}^T)^{-1} \right)_{kk}$ part.

3.1 Minimization of the CCT target function

The target function of (20) can be written as:

$$\tilde{f}(\mathbf{M}) = \frac{\prod_{k=1}^{3} \left((\mathbf{M} \mathbf{M}^{T})^{-1} \right)_{kk} \mathbf{m_k}^{T} \mathbf{\Lambda} \mathbf{m_k}}{\prod_{k=1}^{3} G_{T_k}}, \quad (22)$$

where G_{T_k} is the subband transform coding gain for the color component Ck (defined as the ratio of the MSE of the PCM scheme and the MSE of the subband transform coder for the same rate), $\mathbf{m_k}$ is the k^{th} row of \mathbf{M} as column vector and $\boldsymbol{\Lambda}$ is the covariance matrix in the RGB image domain:

$$\mathbf{\Lambda} \triangleq E\left[(\mathbf{x} - \boldsymbol{\mu}_{\mathbf{x}}) (\mathbf{x} - \boldsymbol{\mu}_{\mathbf{x}})^T \right], \quad \boldsymbol{\mu}_{\mathbf{x}} \triangleq E\left[\mathbf{x} \right].$$
 (23)

Denoting the numerator of the right hand side of (22) by $g(\mathbf{M})$:

$$g(\mathbf{M}) \triangleq \prod_{k=1}^{3} ((\mathbf{M}\mathbf{M}^{T})^{-1})_{kk} \mathbf{m_{k}}^{T} \mathbf{\Lambda} \mathbf{m_{k}},$$
 (24)

it can be shown that the KLT minimizes this target function, but not the full $\tilde{f}(\mathbf{M})$ [3]. Referring to the problem of minimizing the full target function, an approximated solution, named Generalized KLT (GKLT) can be proposed under the assumption of a unitary \mathbf{M} ($\mathbf{M}\mathbf{M}^T = \mathbf{I}$) [3].

4. DCT COLOR COMPONENTS TRANSFORM

The one-dimensional DCT transform matrix

$$\mathbf{M_{DCT}} = \begin{pmatrix} 0.5774 & 0.5774 & 0.5774 \\ 0.7071 & 0.0000 & -0.7071 \\ 0.4082 & -0.8165 & 0.4082 \end{pmatrix}$$
 (25)

can be applied to the $[R \ G \ B]^T$ vector as the color components transform. It has been shown [11] that the DCT can

be used as an approximation of the KLT transform. However, we have stated earlier that the KLT is *not* optimal from the minimal MSE point of view and it does not minimize the $f(\mathbf{M})$ target function of (20). Thus, the DCT is not of interest as an approximation of the KLT, but due to a different property: although the DCT is not a solution to (20), it usually achieves very close values of the target function to those of the solution and the optimal transform, i.e., the one minimizing $f(\mathbf{M})$ is sometimes very similar to the DCT. The performance of the DCT vs. other transforms is demonstrated in the next section.

5. NEW COLOR COMPRESSION ALGORITHM

In this section we propose a new algorithm for color image compression based on the 2D DCT block transform.

The stages of the algorithm are:

- 1. Apply the CCT to the RGB color components of a given image to obtain new color components C1, C2, C3.
- 2. Apply the two-dimensional block DCT to each color component *Ci*.
- Quantize each subband of each color component independently using uniform scalar quantizers. The quantization step sizes are chosen so that optimal subband rates are achieved as presented in subsection 5.1.
- 4. Apply lossless coding of the quantized DCT coefficients similarly to JPEG [12]: differential coding for the DC coefficients and zigzag scan, run-length coding and Huffman coding (combined with variable-length integer codes) for the AC coefficients.
 - Optional down-sampling of some of the color components can be performed between stages 1 and 2.

5.1 Determining the quantization steps

Consider a stochastic source X with distribution $f_X(x)$, uniformly quantized to \hat{X} with (small) step size of Δ and then entropy coded. The entropy of \hat{X} is approximately [10]:

$$H(\hat{X}) \cong h(X) - \log_2 \Delta.$$
 (26)

Here h(X) is the entropy of the continuous variable X. The rate of \hat{X} is measured by its entropy: $R = H(\hat{X})$ and thus using (26) we get:

$$\Delta = 2^{h(X) - R} \Longrightarrow \frac{\Delta 1}{\Delta 2} = 2^{-(R1 - R2)} \tag{27}$$

when 2 quantization steps $\Delta 1$ and $\Delta 2$ and two rates are considered. Using (27) the following algorithm is proposed:

- 1. Calculate the optimal rates R_{bi}^* . The calculation should consider down-sampling of the color components if employed and non-negativity constraints for the rates [3].
- 2. Set some initial quantization steps Δ_{bi} and calculate the resulting rates R_{bi} .
- 3. Update the quantization steps according to:

$$\Delta_{bi}^{new} = \Delta_{bi} 2^{-(R_{bi}^* - R_{bi})}$$

until the optimal rates R_{bi}^* are sufficiently close, i.e., $E(|R_{bi}^* - R_{bi}|) < \varepsilon$ for some small constant ε .

5.2 Performance of the algorithm with various CCTs

In this section we present results of images compression using the new algorithm with different CCTs as shown in Table 1. 'Opt Trans' denotes there the optimal transform - the minimizer of (20). We down-sample the C2 and C3 components by a factor of 2 in each direction. We use the PSNR (Peak Signal to Noise Ratio) as the performance criterion.

It can be concluded from Table 1 that the optimal CCT indeed yields the best results on average, but the DCT is close behind. Both transforms are superior to the RGB to YUV transform and the KLT, which fails for low inter-color correlation images in the RGB domain, such as Peppers and Baboon, however, works well for images such as Lena, where the RGB components inter-color correlations are high [3]. We, therefore, propose using the DCT as CCT. Additional results are given in the next subsection.

5.3 Performance of the algorithm vs. JPEG

Similar to the PSNR, we define the PSPNR (Peak Signal to Perceptible Noise Ratio):

$$PSPNR = 10\log_{10} \frac{255^2}{WMSE},$$
 (28)

where *WMSE* (Weighted Mean Square Error) for each color component is calculated similarly to (5) as:

$$WMSE = \sum_{b=0}^{B-1} \eta_b W_b G_b d_b.$$
 (29)

Here W_b denotes the visual perception weight of subband b. We have taken the WMSE suggested in [10] for JPEG2000, so that the subbands in (29) are of the DWT (Discrete Wavelet Transform). We consider 256x256 or similar size images displayed on a screen as $12\text{cm} \times 12\text{cm}$ size images and a viewing distance of about 50 cm. The PSPNR measure used is the mean PSPNR on the three color components. The results for the Lena, Peppers and Baboon images are displayed in Fig. 1. The new algorithm outperforms JPEG by slightly more than 0.5dB PSNR for Lena and Baboon and by 1.7dB for Peppers. Its PSPNR gain is, however, above 2dB for all the images. The performance gain can also be seen visually: JPEG introduces color artifacts that are absent or less pronounced in the images of the new algorithm.

6. SUMMARY

We have introduced a Rate-Distortion model for color image compression using subband transform coders. Based on the model, a target function for an optimal CCT and optimal rates allocation are derived. The performance of various color components transforms for image compression has been studied and it has been shown that the DCT can be used as a sub-optimal CCT, close to the optimal adaptive CCT and superior to the commonly used RGB to YUV transform or KLT. This solution also has an advantage over adaptive CCTs, since it is a fixed image independent transform. An algorithm for designing optimal quantization tables has been introduced and implemented in the context of the new compression algorithm for color images. Both quantitative (MSE and WMSE) and visual results have been presented, showing that the proposed compression algorithm outperforms baseline JPEG. Our conclusion is that in addition to the theoretical aspects of the new Rate-Distortion model, it can also serve as a tool for improving color image compression systems compared to presently available algorithms.

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	Image	YUV	DCT	KLT	Opt Trans	CR	Rate (bpp)
	Lena	30.019	30.372	30.355	30.285	37.75	0.636
	Peppers	30.013	30.144	29.475	30.148	30.05	0.799
	Baboon	30.010	30.468	28.595	30.540	13.33	1.800
	Girl	30.015	30.359	30.343	30.450	43.87	0.547
	Tree	30.018	30.295	30.601	30.649	13.76	1.744
	Landscape	30.019	30.382	30.195	30.145	13.26	1.810
AND .	Jelly Beans	30.019	30.294	30.294	30.314	49.04	0.489
	Mean	30.016	30.330	29.980	30.362		

Table 1: PSNR for DCT compression with optimal rates and several CCTs at the same CR (compression ratio).

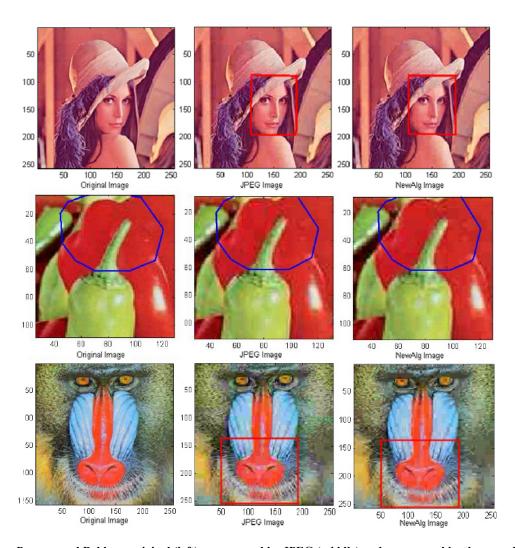


Figure 1: Lena, Peppers and Babbon: original (left), compressed by JPEG (middle) and compressed by the new algorithm (right). PSNR for Lena (at 0.469bpp) is 28.447dB (JPEG) and 29.015dB (new), with PSPNR of 37.679dB and 39.846dB, respectively. PSNR for Peppers (at 0.731bpp) is 28.273dB (JPEG) and 29.995dB (new), with PSPNR of 35.429dB and 37.844dB, respectively. PSNR for Babbon (at 0.287bpp) is 21.469dB (JPEG) and 22.028dB (new), with PSPNR of 29.693dB and 32.065dB, respectively.