Network Planning for Multi-radio Cognitive Wireless Networks

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Abstract—We propose a general network planning framework for multi-radio multi-channel cognitive wireless networks. Under this framework, data routing, resource allocation, and scheduling are jointly designed to maximize a network utility function. We treat such a cross-layer design problem with fixed radio distributions across the nodes and formulate it as a large-scale convex optimization problem. A primal-dual method together with the column-generation technique is proposed to efficiently solve this problem. Simulation studies are carried out to assess the performance of the proposed cross-layer network planning framework. It is seen that the proposed approach can significantly enhance the overall network performance.

I. INTRODUCTION

In the emerging multi-radio multi-channel wireless networks, each node is equipped with several network interface cards (NIC) and able to access multiple orthogonal channels simultaneously. Recent studies indicate that employing the multi-radio multi-channel transmission technologies in multi-hop wireless networks has the potential of significantly improving the network performance [1], [2], [3]. While so far most works in the literature treat issues related to practical and efficient protocol design for such networks, in this paper, we investigate the achievable capacity of such a network by performing joint design of several key network functionalities across different layers. We propose a general modelling and solution framework to address the above problem. It is formulated as a large-scale convex optimization problem. We develop a solution to this based on the primal-dual approach [4] together with the column-generation method, which converges in finite steps. In each step of the algorithm, the dual solution provides a non-trivial upper bound of the achievable capacity of the given network; and a near-optimal solution of the joint data routing, resource allocation and scheduling problem is obtained by solving the primal problem. Although the proposed algorithm is centralized, it provides important benchmarks for the capacity limits of any distributed algorithms for any multi-radio wireless network, and may potentially inspire efficient distributed or semi-distributed protocols.

Related works: Current IEEE 802.11 standards provide multiple orthogonal frequency channels, which can be used simultaneously within a neighborhood. Intuitively, the throughput can be increased by putting more radios on each node to fully make use of the channels. Most existing work in the literature can be broadly divided into two categories. The works in the first category consider the practical and efficient protocol design for multi-radio wireless networks [1]; while those in the second category considers the achievable capacity region [2], [3]. Our work falls into the second category but differs from other works in several aspects. First, we consider a more general cross-layer framework which includes not only the data routing and scheduling, but also resource allocation in the physical layer (e.g., power control). Secondly, the proposed scheme can be employed for any convex utility functions while the methods developed in [2], [3] are applicable only to linear utility functions. Moreover, the proposed column-generation approach is very efficient and typically able to obtain near-optimal solution in a few iterations. Furthermore, we consider more general multicast and the scenario with network coding.

The remainder of this paper is organized as follows. In Section 2, we present the general modelling framework for the cross-layer design problem in multi-radio cognitive wireless networks. This problem is formulated as a constrained optimization problem, where the constraints are imposed by both the data routing in the network layer and the resource limitation in the physical layer. The emerging network coding techniques are also be incorporated into our problem formulation. A primal-dual approach together with the column-generation method is developed to solve the problem. Simulation results are given in Section 3. Section 4 contains the conclusions.

II. CROSS-LAYER OPTIMIZATION FOR MULTI-RADIO NETWORKS

We consider a multi-radio wireless network consisting of a collection of nodes located on the plane. Each node can transmit, receive or relay information to any other nodes in the network. Each node is assumed to be equipped with several NICs and able to access multiple orthogonal channels. A directed graph $G = (N, E)$ is used to model the system. Nodes are labelled by $N = \{1, 2, \ldots, N\}$ and links are labelled by $E = \{1, 2, \ldots, L\}$. The network topology is represented by a $N \times L$ indicator matrix $D$, 

$D_{ij} = 1$ if node $i$ can access channel $j$, otherwise $D_{ij} = 0$.
such that
\[ D(n, l) = \begin{cases} 
1, & \text{if the } n^{th} \text{ node is the start node of the } l^{th} \text{ link,} \\
-1, & \text{if the } n^{th} \text{ node is the end node of the } l^{th} \text{ link,} \\
0, & \text{otherwise.}
\end{cases} \]

We consider the general multicast data transmission over the whole network, which includes the unicast and broadcast as special cases. Assume there are \( S \) multiple data sessions needed to be transmitted in the network. Denote \( s^i \) and \( T^j \) as the source node and \( j^{th} \) destination node in the \( i^{th} \) multicast session respectively. Let \( r = \{r^i\}_{i=1}^S \) be the multicast data rates for all sessions. Denote \( f^l \) as the flow rate of the \( i^{th} \) session \((1 \leq i \leq S)\) over the \( l^{th} \) link \((1 \leq l \leq L)\). Given \( r \), denote \( \mathcal{F}(r) \) as the set containing all possible network flow rates that can support \( r \). Let \( \mathcal{C}(I) \) be the set containing all achievable rates \( c = \{c_l\}_{l=1}^L \) that the physical layer can support on links \( l \in E \). \( \mathcal{C}(I) \) is determined by the radio distribution vector \( I = \{I_i\}_{i=1}^N \), which indicates the radio distribution over different nodes, where \( I_i, 1 \leq i \leq N \), is the number of radios at the \( i^{th} \) node. Note that in this section, we assume \( I \) is given. In Section 3, we consider the problem of optimizing \( I \). The network optimization problem can then be formulated as:

\[
\max \ U(r, f), \\
\text{s.t. } f \in \mathcal{F}(r), r \succeq 0, c \in \mathcal{C}(I), \sum_{i=1}^S f^i \leq c_l, \forall l, (2)
\]

where \( U(r, f) \) is a utility function that is assumed to be concave; \( r \succeq 0 \) means \( r^i \geq 0, \forall i \). The first constraint enforces the dependence between the achievable rates \( r \) and the data flows \( f \). The third constraint indicates the relationship between the achievable link capacity \( c \) and radio allocation together with the resource allocation, scheduling and data routing schemes. The fourth constraint states that the sum of the flow rate on each link is bounded by the link capacity. We refer to (2) as the general joint optimization problem (GJO) for future discussion. Furthermore, we omit the index \( I \) in \( \mathcal{C}(I) \) hereafter to simplify the notation.

In the following section, a primal-dual approach together with the column-generation method is proposed to solve the general network planning problem in (2).

A. Primal-dual Approach with Column-generation

The set \( \mathcal{F}(r) \) and \( \mathcal{C} \) in (2) are determined by the specific network and physical layer model. Since time-sharing is allowed in the system, \( \mathcal{C} \) is a convex set [5]. Furthermore, \( \mathcal{F}(r) \) is also convex if we assume data routing is based on the multicommodity flow model or multicast routing with network coding, as shown in the next section. These two constraints are coupled only through the constraints \( \sum_{i=1}^S f^i \leq c_l, \forall l \). Thus (2) is a convex optimization problem. Since \( \mathcal{C} \) is a convex hull, it is fully determined by its vertices. However, because joint resource allocation and scheduling is considered, \( \mathcal{C} \) may be too complex to be described in polynomial time. We therefore employ the column-generation method [6] to solve this problem.

Instead of trying to exactly describe the convex hull \( \mathcal{C} \), we use another convex hull \( \mathcal{C}' \subseteq \mathcal{C} \) to approximate \( \mathcal{C} \). Assume we have several feasible link capacities vectors \( c_k \in (\mathcal{C}', 1 \leq k \leq K) \). Denote \( \mathcal{V}' = \{k\}_{k=1}^K \). The corresponding convex hull \( \mathcal{C}' = \{c = \sum_{k=1}^K \alpha_k c_k, \alpha \geq 0, \sum_{k=1}^K \alpha_k = 1\} \) is fully characterized by the set \( \mathcal{V}' \). We thus transform the original problem (2) into the following problem:

\[
\max_{c \in \mathcal{C}} U(r, f), \text{s.t. } f \in \mathcal{F}(r), r \succeq 0, c = \sum_{k=1}^K \alpha_k c_k, \\
\sum_{k=1}^K \alpha_k = 1, \sum_{i=1}^S f^i \leq c_l, \forall l, \alpha_k \geq 0, \forall k. (3)
\]

We refer to (3) as the restricted primal problem for future discussion. Since \( \mathcal{C}' \subseteq \mathcal{C} \), the solution to this problem provides a lower bound \( U_{\text{lower}}(I) \) for the original problem. This problem can be solved via the dual decomposition method in a distributed manner [7]. We may also obtain an upper bound of the original problem by considering the dual of the original problem in (2), given by,

\[
\max_{r \succeq 0} \left\{ U(r, f) - \sum_{i=1}^L \lambda_i \sum_{i=1}^S f^i \mid f \in \mathcal{F}(r) \right\} + \max_{c \in \mathcal{C}} \sum_{l=1}^L \lambda_l c_l, (4)
\]

where \( \{\lambda_i \geq 0\}_{i=1}^L \). In addition, since we solve the restricted primal problem via the dual decomposition method, it is natural to use the dual variables of the constraints \( \sum_{i=1}^S f^i \leq c_l \) in the computation of the dual problem. We thus obtain a lower bound \( U_{\text{lower}}(I) \) and an upper bound \( U_{\text{upper}}(I) \) of the original optimization in (2). The gap between the two bounds indicates the accuracy of the current solution. If the two bounds coincide, we then obtain the optimal solution to (2). If they are different, we will add the vertex generated by solving the problem \( \max_{c \in \mathcal{C}} \sum_{l=1}^L \lambda_l c_l \) into the subset \( \mathcal{V}' \). The iteration will continue until the gap between the two bounds is below some pre-defined threshold \( \eta \). The algorithm is summarized as follows.

Algorithm 1: (Column-generation for cross-layer design of multi-radio networks)

1) Generate a set \( \mathcal{V}' \) containing several feasible link rates \( c_k \).
2) Let \( \mathcal{C}' \) be the convex hull of \( \mathcal{V}' \). We can obtain the lower bound \( U_{\text{lower}} \) and the dual factors \( \{\lambda_i\}_{i=1}^L \) by solving the restricted primal problem (3) via the dual decomposition method.
3) Obtain the upper bound \( U_{\text{upper}} \) and the new vertex by solving the dual problem (4).
4) Stop if \( U_{\text{lower}} - U_{\text{upper}} < \eta \) or the time-delay constraint in the practical system is violated. Otherwise, add the new vertex into \( \mathcal{V}' \) and go to step 2.

We have the following proposition if (3) is a linear programming problem.

Proposition 1: Denote the optimal solution of problem (3) as \( \{r, f\} \). If problem (3) is a linear programming, i.e., \( U(r, f) \) is a linear function of \( r \) and \( f \), \( \mathcal{F}(r) \) is a polyhedra, it follows that \( U_{\text{upper}} = U_{\text{lower}} - \sum_{l=1}^L \lambda_l \sum_{i=1}^S f^i + \).
max_{c \in C} \sum_{i=1}^L \lambda_i c_i.

Proof: If (3) is a linear programming problem, then the strong duality holds, i.e., the optimal primal solution coincides with the dual solution [4], thus we have \( \max_{r \geq 0} \left\{ U(r,f) - \sum_{i=1}^L \lambda_i \sum_{j=1}^S f_{ij} \mid f \in F_r \right\} = U(\tilde{r},\tilde{f}) - \sum_{i=1}^L \lambda_i \sum_{j=1}^S \tilde{f}_{ij} \). It follows from (4) that \( U_{\text{upper}} = U_{\text{lower}} - \sum_{i=1}^L \lambda_i \sum_{j=1}^S \tilde{f}_{ij} + \max_{c \in C} \sum_{i=1}^L \lambda_i c_i. \)

The above proposition can be applied under quite a few performance metrics [5], which is summarized as follows. We can then efficiently compute the upper bound by solving only the problem \( \max_{c \in C} \sum_{i=1}^L \lambda_i c_i. \)

**Throughput and transport capacity:** The total throughput of a wireless network is a very important performance metric. We can perform the cross-layer design to maximize the total system throughput by setting the utility function in (2) as the sum of the rate constraints in all sessions, i.e., \( U(r,f) \triangleq \sum_{i=1}^L \bar{r}_i \). Alternatively, we can maximize the transport capacity [5] of a given network by setting \( U(r,f) \triangleq \sum_{i=1}^L \sum_{j=1}^S f_{ij} \), where \( d_{l,i} \) is the distance between the transmitter and receiver of the \( i \)-th link.

**Minimum end-to-end rate:** Only considering the throughput maximization may lead to unfair allocations of end-to-end rates. In order to guarantee the fairness, we may consider to maximize the minimum end-to-end communication rates by setting \( U(r,f) \triangleq \max x \min \{r^l\} \). Thus (2) is a nonlinear optimization problem since the minimum rate is a nonlinear function of the end-to-end rates. However, it can be transformed into the following linear programming problem:

\[
\max \quad r, \quad s.t. \quad \tau \leq r^l, \quad \forall i, f \in F_r, \quad r \geq 0, \quad c \in C(I), \quad \sum_{s \in S} f_{i} \leq c_i, \quad \forall l. \quad (5)
\]

Thus Proposition 1 can still be applied.

We have shown how a primal-dual approach together with the column-generation method can be employed to solve the joint optimization problem. It remains to show how data routing, resource allocation, and scheduling are performed for this specific multi-radio network, i.e., how to characterize \( F(r) \) and \( C(I) \) in (2) under different models. In what follows, we provide the detailed network and physical layer models.

### B. Network Models for Multi-radio Networks

We consider two models in the network layer to characterize \( F(r) \) and \( C(I) \) in (2). The first model is the typical data routing based on the multicommodity flow model; and the second one is data routing (or subgraph selection) for multicast with network coding.

**Network flow model:** We first consider a multicommodity flow model to describe the packet routing across the network [8]. The data flows are assumed to be lossless and they satisfy the flow conservation law. Denote \( e_{l_i}^{i,j} \) as the data flow from the source node \( s^i \) to the \( j \)-th destination node \( T_j^i \). Note that in this section, \( s^i = i \) and \( T_j^i = j \) since we only consider the unicast based on the multicommodity flow model. In the next section, \( s^i \) and \( T_j^i \) may differ from \( i \) and \( j \) because we consider the multicast with network coding. \( \mathcal{I}(n) \) is defined as the set of the incoming links to the \( n \)-th node and \( \mathcal{O}(n) \) is defined as the outgoing links from the \( n \)-th node. Then the constraints in the network layer can be modelled as:

\[
\begin{align*}
\sum_{l \in \mathcal{O}(s^i)} f_l &= \sum_{l \in \mathcal{O}(s^i)} e_l^{i,j}, \quad \forall i, l, \quad e_l^{i,j} \geq 0, \quad \forall i, j, l, \\
\sum_{l \in \mathcal{O}(n)} e_l^{i,j} &= \sum_{l \in \mathcal{I}(n)} e_l^{i,j}, \quad \forall i, j, \quad \forall n \in \mathcal{N}\{s^i, T_j^i\}. \quad (6)
\end{align*}
\]

The first constraint reflects the fact the source data rate is equal to the sum of the flow rates leaving from the source node \( s^i \). The total data flow on each link in the \( i \)-th session is equal to the sum of data flows to each destination node, which is shown in the second constraint. The fourth constraint describes the network flow conservation law. The set \( F(r) \) in (2) is fully characterized by these constraints.

**Networks with network coding:** Network coding allows a node in the network to perform algebraic operations on the received data. It has been shown that networking coding can significantly improve the network performance [9]. We consider network coding for multicast without intra-session network coding. In this case, the problem of establishing multicast connections to maximize the network performance subject to network and physical constraints can be decoupled into two subproblems, i.e., subgraph selection (determining the amount of flow over each link) and code selection (determining the code over each link) problems [10]. Denote \( x_{l}^{i,j} \) as the conceptual flow rate on link \( l \) in the \( i \)-th multicast session from the source node \( s^i \) to its \( j \)-th destination \( T_j^i \). We use the term “conceptual flow” because it differs from the actual data flow on the link. For example, as shown in Fig. 1, the conceptual data flows \( X_1 \) and \( X_2 \) are encoded into \( X_1 \oplus X_2 \) at node 4 and then be transmitted. Note that \( “\oplus” \) denotes the modulo 2 addition in this example, which characterizes the advantage of employing network coding technologies, i.e., it allows the actual data flow on the link be the maximum, instead of the summation, of the conceptual flows [9]. Thus we can obtain the constraints imposed by the multicast routing with network coding as follows:

\[
\begin{align*}
x_l^i &\leq \sum_{l \in \mathcal{I}(T_j^i)} x_l^{i,j}, \quad \forall i, j, l, \quad x_l^{i,j} \geq 0, \quad \forall i, j, l, \\
\sum_{l \in \mathcal{I}(n)} x_l^{i,j} &= \sum_{l \in \mathcal{O}(n)} x_l^{i,j}, \quad \forall i, j, \quad \forall n \in \mathcal{N}\{s^i, T_j^i\}. \quad (7)
\end{align*}
\]

The first constraint shows the fact that the \( i \)-th session multicast rate is no higher than the sum of all conceptual flow rates. The second constraint describes the relationship between the conceptual flow rates and the actual flow rates, i.e., the maximum of the conceptual flow rates is no higher than the actual data flow rate on the link.
fourth constraint follows from the law of flow conservation for conceptual flows.

C. Communication Models of Multi-radio Networks

We now present the communication models of multi-radio networks in order to fully characterize the convex region $C$ in (2). Important physical-layer issues such as the power control, transmission scheduling, and medium access schemes will be addressed.

In a wireless network, the capacity of any individual link depends on the allocated resource such as the power and bandwidth as well as the media access scheme. Since the multi-radio system is mainly targeted for IEEE 802.11-type environment [1], we assume each node can access several orthogonal channels and the bandwidth of each channel has been pre-defined.

Power control and transmission scheduling: Let $H_{l,k}$ denotes the effective power loss between the transmitter of the $l^{th}$ link and the receiver of the $k^{th}$ link, which is governed by the $\beta^{th}$ power path-loss law, given by

$$H_{l,k} = G_{l,k}d_{l,k}^{-\beta},$$

(8)

where $d_{l,k}$ is the distance between the start node of the $l^{th}$ link and the end node of the $k^{th}$ link, $\beta$ is the path-loss exponent; and $G_{l,k}$ is a constant representing the radio propagation properties of the environment and other physical-layer effects such as coding gain or spreading gain.

Denote $p_{l,m}$ as the transmit power for the $l^{th}$ link over the $m^{th}$ channel, and $\sigma_l$ as the thermal noise power at its receiver. The signal-to-interference-and-noise-ratio (SINR) is give by

$$\gamma_{l,m}(p) = \frac{G_{l,m}p_{l,m}}{\sigma_l + \sum_{j \neq l} G_{j,m}p_{j,m}},$$

(9)

where $p$ is the vector consisting of the transmit power of all links, i.e., $p = \{p_{l,m}\}$, $\forall l, m$. Therefore, each link can be viewed as a single-user Gaussian channel with the capacity given by

$$c_{l,m} = W_m \log(1 + \gamma_{l,m}(p)),$$

(10)

where $W_m$ is the system bandwidth for the $m^{th}$ orthogonal channel. However, this capacity may not achievable in practice. We thus employ the following discrete model,

$$c_{l,m} = c_{l,m}^\nu, \text{ if } \gamma_{l,m}^\nu < \gamma_{l,m}(p) < \gamma_{l,m}^{\nu+1}, \nu = 1, 2, \ldots,$$

(11)

where $c_{l,m}^\nu$ and $\gamma_{l,m}^\nu$ denote the $\nu^{th}$ discrete rate level and the corresponding SINR target, respectively.

Media access scheme: We consider the simple media access scheme in which the transmission rate is fixed and the transmitters adjust powers for the data transmission. In this model, a set of links can be active in the same time slot only if each link exceeds its SINR target $\gamma_{l,m}$ and the corresponding data rate $c_{l,m}$ is given by (11). The system variables are defined as follows:

- $a_{n,m}$ is the indicator variable such that
  $$a_{n,m} = \begin{cases} 1, & \text{if the } n^{th} \text{ node send or receive signal over the } m^{th} \text{ channel,} \\ 0, & \text{otherwise.} \end{cases}$$

- $v_{l,m}$ is the indicator variable such that
  $$v_{l,m} = \begin{cases} 1, & \text{if the } l^{th} \text{ link is active over the } m^{th} \text{ channel,} \\ 0, & \text{otherwise.} \end{cases}$$

- $p_{l,m}$ is the power transmitted by $m^{th}$ channel over the $l^{th}$ link, assume it is upper bounded by $p_{l,max}$.
- $P_{n,max}$ is the maximum power of the $n^{th}$ node.

Assuming half-duplex mode, i.e., each transceiver can only transmit or receive at a time. Then the medium access of the multi-radio network is subject to three types of constraints. First, a node cannot transmit or receive in the same channel simultaneously, which is called the primary conflict, given by

$$a_{n,m} = \sum_{l \in \mathcal{O}(n)} v_{l,m} + \sum_{l \in \mathcal{Z}(n)} v_{l,m},$$

$$a_{n,m} \leq 1, \forall n, m, v_{l,m} \in \{0, 1\}, \forall l, m. \quad (12)$$

Secondly, the total number of channels accessed by one node is bounded by the number of radios of that node, which is called the radio conflict, given by,

$$\sum_{m=1}^{M} a_{n,m} \leq I_n. \quad (13)$$

Furthermore, a transmission can be corrupted from the neighboring nodes, which is called the transmission conflict, given by

$$G_{l,i}p_{l,m} + (1 - v_{l,m})Z_l \geq \gamma_{l,m}^i(\sigma_l + \sum_{j \neq l} G_{l,j}p_{j,m}),$$

$$\sum_{l \in \mathcal{O}(n)} \sum_{m} p_{l,m} \leq P_{n,max}, \forall n. \quad (14)$$

where $Z_l$ is a constant which is sufficiently large to guarantee $Z_l > \gamma_{l,m}^i(\sigma_l + \sum_{j \neq l} G_{l,j}p_{j,max}), \forall m$. Finally, the capacity of the $l^{th}$ link is given by

$$c_{l} = \sum_{m=1}^{M} c_{l,m}^i. \quad (15)$$

Therefore, the convex set $C$ in (2) is given by:

$$C = \text{Convex hull of } \{c | c \text{ satisfies the constraints in (12), (13), (14), (15)}\}. \quad (16)$$

Thus the optimization problem posed in (4) can be reformulated as

$$\max \sum_{l=1}^{L} \lambda_l c_{l}, \text{ s.t. } c \text{ satisfies the constraints in (12), (13), (14), (15)}. \quad (17)$$

Although we only consider this simple MAC scheme
in this paper, we can easily extend this framework to incorporate other MAC schemes such as the discrete rate selection and power control [5].

III. SIMULATION RESULTS

We provide several numerical examples to show the performance of the proposed method. We assume the power limit of each node is 100 mW. $G_{1,m}$ and $\beta$ in (8) are set as $2 \cdot 10^{-4}$ and 3 respectively, corresponding to a UMTS indoor scenario [11]. The noise factor in (9) are set as $3.34 \cdot 10^{-12}$. We assume the bandwidth of each orthogonal channel is the same and each node can simultaneously access at most 4 channels.

We consider a string topology with 9 points. Without loss of generality, the relative distance between adjacent nodes is set as 1 and the data rate supported by each orthogonal channel is normalized as 1.

We first compare the network performance between the proposed joint optimization approach and the traditional separate-layer approach. We assume each node in the network is equipped with one radio. In the cross-layer design approach, routing, scheduling and power control are jointly considered. In the separate-layer approach, each radio will perform the scheduling to evenly distribute the capacity to the links associated with it, e.g., if there are two links associated with a node, we schedule the radio to set the capacity of each link to be 1/2. The first and second nodes are the source nodes and the incoming traffic of each source node is transported to all the other nodes. Minimum end-to-end flow is set as the utility function. The performance of the primal-dual column-generation algorithm as a function of iteration number is shown in Fig. 2. It is seen that the proposed method quickly converges to the optimal solution. At each iteration, while the primal solution provides a feasible solution to the original problem, the solution to the dual problem gives an upper bound of the optimal solution. Furthermore, it is seen that the joint approach yields much better performance than that of the separate-layer approach (more than 100% performance improvement is observed in this case).

IV. CONCLUSIONS

We have proposed a general modelling and solution framework for joint optimization of a multi-radio network with fixed radio distribution. In this framework, data routing, resource allocation, and scheduling are jointly designed to optimize the network performance. It is formulated as a convex optimization problem and a primal-dual method together with the column-generation approach is employed to solve this problem. Simulation results are provided to show the impact of the proposed methods.

REFERENCES