JOINT ESTIMATION OF I/Q IMBALANCE AND CHANNEL RESPONSE FOR MIMO OFDM SYSTEM

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ABSTRACT
In this paper, we study joint estimation of in-phase and quadrature-phase (I/Q) imbalance and channel response for multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems using training sequence. A new class of optimal sequences, called circularly-shift orthogonal sequences, are introduced for estimating the MIMO channels. By using the newly proposed sequences, the channel coefficients can be obtained by multiplying the received vectors with an $M \times M$ unitary circulant matrix, which can be computed efficiently using FFT. Then, we develop a new method for jointly estimating the MIMO channel responses and I/Q imbalance. The proposed time-domain method needs only one OFDM training block. Simulation results show that the proposed method can accurately estimate both the I/Q imbalance and MIMO channel response.

1. INTRODUCTION
Multiple-input multiple-output orthogonal frequency division multiplexing (MIMO OFDM) has drawn a lot of attention due to its high data rate and low complexity. These advantages are based on the assumptions that the receiver has exact channel information and the system parameters at the transmitter and receiver are perfectly matched. One of these parameters is the so-called I/Q imbalance due to the mismatch of local oscillators. It was shown in [1] that I/Q imbalance can cause severe inter-carrier-interference and it leads to a serious performance degradation. By carefully designing the OFDM training blocks, several methods for estimating the I/Q imbalance have been proposed in [1]. In [2], based on the assumption that the channel frequency response is smooth, a frequency-domain approach for jointly estimating the I/Q imbalance and channel response for OFDM systems is proposed. Recently, a time-domain method for jointly estimating the I/Q imbalance and channel response in OFDM systems is proposed in [3]. The time-domain method exploits the fact that the channel length is usually much shorter than the OFDM block. Both the two methods in [2] and [3] need only one arbitrary OFDM training block and they achieve very good performance.

Channel estimation for MIMO OFDM system has been studied by many researchers [4][5][6]. The authors in [4] showed that the optimal training sequences from different transmit antennas must be orthogonal. Various conditions for the existence of the orthogonal sequences are derived in [5]. A frequency-domain approach for the channel estimation is proposed in [6]. These channel estimation methods assume that all the other system parameters are perfectly matched at the transmitter and receiver. Recently, the I/Q imbalance for the MIMO OFDM system has been investigated in [7]. The authors show that the MIMO OFDM system would suffer an error flooring due to I/Q imbalance. The compensation of the I/Q imbalance for MIMO OFDM system is studied in [8][9]. The authors proposed various methods to estimate the I/Q imbalance, but they usually need more than one training blocks.

In this paper, we first propose a special class of optimal sequences for estimating MIMO channels. Then we consider the problem of jointly estimating the channel response and I/Q imbalance. The time-domain approach in [3] is extended to the MIMO case. The proposed method has the advantages that it needs only one OFDM training block. Simulation results show that our method can accurately estimate the channel response and I/Q imbalance.

Notation: Boldfaced lower and upper case letters represent vectors and matrices respectively. The notation $A'$ denotes transpose-conjugate of $A$ and $A^r$ denotes the transpose of $A$. $W$ is the $M \times M$ normalized DFT matrix with entries $W_{kl} = \frac{1}{\sqrt{M}} e^{-j \frac{2\pi kl}{M}}$. $((x))_M$ means $x$ modulo $M$.

2. SYSTEM DESCRIPTION

2.1 Effect of I/Q Imbalance on The Received Signals
We consider the transmission of the discrete-time sequence $x(n)$ through an LTI channel $h(n)$ with additive noise $q(n)$. The received signal is given by

$$r(n) = \sum_{l=0}^{L-1} h(l)x(n-l) + q(n).$$

Suppose now there is I/Q imbalance due to the mismatch of local oscillators:

$$Osc(t) = \cos(2\pi f_c t) - j \sin(2\pi f_c t + \phi),$$

where $\varepsilon$ denotes the amplitude mismatch and $\phi$ denotes the phase mismatch of the I and O branches, and $f_c$ is the carrier frequency. Due to the I/Q imbalance, the received signal can be modeled as [1]

$$z(n) = \mu r(n) + \nu r^*(n),$$

where

$$\mu = \frac{1 + \varepsilon e^{-j\phi}}{2}$$

and

$$\nu = \frac{1 - \varepsilon e^{j\phi}}{2}.$$
Then if the interference consists not only of the vector \( x_j \) but also its complex conjugate \( x_j^* \), define

\[
\alpha_k = \frac{v_k}{\mu_k}.
\]

Then if \( \alpha_k \) is correctly estimated at the receiver, we can compensate the I/Q imbalance either in the time domain [2] or in the frequency domain [1]. It can be verified that

\[
\mu_k r_k = \frac{z_k - \alpha_k z_k^*}{1 - |\alpha_k|^2}.
\]

In Sec. 4, we will discuss how to jointly estimate the channel coefficients \( h_{k,j}(n) \) and the I/Q imbalance factor \( \alpha_k \) using training sequences.

### 3. MIMO Channel Estimation

#### 3.1 Optimal Orthogonal Sequences

We consider MIMO channel estimation using training sequences. Given \( x_i \), we want to estimate the channel response from the received vector \( r_k \). For the purpose of channel estimation, we can rewrite (5) as

\[
r_k = [ X_0 \ X_1 \ \cdots \ X_{N_t-1} ] + q_k,
\]

where \( X_{k,j} = [ h_{k,j}(0) \ h_{k,j}(1) \ \cdots \ h_{k,j}(L-1) ]^T \).

Then (11) can be rewritten as

\[
r_k = [ A_0 \ A_1 \ \cdots \ A_{N_t-1} ] + q_k
\]

where \( A_k \) is the MIMO channel matrix.

From (12), we see that the channel vector \( c_k \) is identifiable if and only if the MIMO channel matrix \( A_k \) has full column rank. Thus a necessary condition for the channel identifiability is \( A_k \) of rank \( M \times N_t \). Assume that \( q_k \) is a complex Gaussian random vector with covariance matrix \( R_{q_k} = N_0 I \). Then, a least squares estimator of \( c_k \) is given by

\[
\hat{c}_k = (A_k^H A_k)^{-1} A_k^H r_k.
\]

Define the error vector as \( e_k = c_k - \hat{c}_k \). The design of optimal sequences that minimize the mean squared error \( \sum_{k=0}^{N_r-1} E[|e_k|^2] \) is given in [4]. It was shown that the optimal training sequences from different antennas must satisfy

\[
A_k^H A_k = \delta(k-i) I.
\]

That means the training sequences from different transmit antennas must be orthogonal. The general closed-form solutions have been derived in [4].

#### 3.2 Circularly-Shift Orthogonal Sequences

In this paper, we propose a special type of orthogonal sequences for the estimation of MIMO channels. Due to the space limit, we consider the case where \( M \) is a multiple of \( N_t \). The general case is given in [10]. The training sequence sent by the \( i \)th transmit antenna is given by

\[
x_i(n) = x_0 \left( \left( n - \frac{M}{N_t} \right) \frac{1}{M} \right)_M,
\]

where \( x_0(n) \) is the training sequence sent by the 0th transmit antenna and its DFT coefficients satisfy \( |x_0(k)|^2 = \frac{1}{\sqrt{M}} \) for all \( k \). In other words, the training sequences for all transmit antennas are obtained by circularly shifting the training sequence of the 0th transmit antenna. These circularly-shift sequences satisfy the following property:
Lemma 1. When the training sequences are chosen as (15), the corresponding matrices \( A_i \) satisfy the orthogonality condition in (14).

**Proof.** Let \( s_i(k) \) be the \( M \)-point DFT coefficients of \( x_i(n) \). We have

\[
s_i(k) = s_0(k)e^{-j\frac{2\pi mk}{M}}, \quad k = 1, \ldots, M-1.
\]

Notice that for all \( i \) \( |s_i(k)| = \frac{1}{\sqrt{M}} \) because \( |s_0(k)| = \frac{1}{\sqrt{M}} \) for all \( k \). Partition the DFT matrix as \( W = [ W_0 \ W_1 ] \), where \( W_0 \) consists of the first \( L \) columns of \( W \). Since \( A_k \) is a submatrix of \( X_k \) and the circulant matrix \( X_k \) can be diagonalized by the DFT/IDFT matrices, \( A_k \) can be represented as

\[
A_k = W^H S_k W_0.
\]

where \( S_k \) is an \( M \times M \) diagonal matrix with the diagonal entries \( s_k(n) \). From (16) and (17), the \((l,m)\)th entry of the matrix \( A_k^T A_i \) is given by

\[
[A_k^T A_i]_{l,m} = \frac{1}{M} \sum_{n=0}^{M-1} e^{j\frac{2\pi}{M} (nL - m(k-i))},
\]

where \(-L \leq m-l \leq L-1\). Because \( L N_t \leq M \) due to the conditions of channel identifiability, it can be shown that both \((m-l)\) and \((k-i)\) are not a multiple of \( M \) for all \( 0 \leq k, i \leq N_t - 1 \). Using this fact and (18), one can show that

\[
[A_k^T A_i]_{l,m} = \delta(k-i)\delta(m-l).
\]

The matrix \( A_k \) satisfies (14). \( \square \)

As these sequences satisfy (14), they are optimal. They are named circularly-shift orthogonal sequences because of the circularly-shift and orthogonal properties. In what follows, we will show that using the circularly-shift orthogonal sequences, the estimate of MIMO channels can be obtained by multiplication with a circulant matrix.

Recall that for the identification of MIMO channels, we have the condition that \( M \geq LN_t \). Suppose we append \((\frac{M}{N_t} - L)\) zeros to the \( L \times 1 \) vector \( c_k,i \) to obtain the \( \frac{M}{N_t} \times 1 \) vector

\[
d_k,i = \begin{bmatrix} c_k,i \ 0 \end{bmatrix}.
\]

Then we can rewrite the received vector \( r_k \) at the \( k \)th receive antenna in (11) as

\[
r_k = \left[ \begin{array}{c} B_0 \ B_1 \ \cdots \ B_{N_t-1} \end{array} \right] \begin{bmatrix} d_{k,0} \\ d_{k,1} \\ \vdots \\ d_{k,N_t-1} \end{bmatrix} + q_k,
\]

where \( B_i \) is the \( M \times M \) submatrix consisting of the first \( \frac{M}{N_t} \) columns of the \( M \times M \) circulant matrix \( X_i \). Define the \( M \times 1 \) vector \( d_k = \left[ \begin{array}{c} d_{k,0} \\ d_{k,1} \\ \vdots \\ d_{k,N_t-1} \end{array} \right] \). Then we have

\[
r_k = Bd_k + q_k.
\]

If the training sequences are chosen as the circularly-shift orthogonal sequences in (15), then we can have the following lemma (a proof is given in [10]).

**Lemma 2.** If \( x_i \) are the circularly-shift orthogonal sequences in (15), then the matrix \( B \) is an \( M \times M \) unitary circulant matrix.

Using Lemma 2, since \( B \) is \( M \times M \) unitary, its inverse is \( B^{-1} = B^H \). The estimate of MIMO channel response can be obtained by

\[
\hat{d}_k = B^H r_k.
\]

As \( B^H \) is circulant, it can be implemented efficiently using FFT. Let us partition \( d_k \) as

\[
\hat{d}_k = \begin{bmatrix} \hat{d}_{k,0} \\ \hat{d}_{k,1} \\ \vdots \\ \hat{d}_{k,N_t-1} \end{bmatrix},
\]

where \( \hat{d}_{k,i} \) are \( \frac{M}{N_t} \times 1 \) vectors. Then from (20) the estimated channel coefficients \( \hat{h}_k,i(n) \) are given by the first \( L \) entries of \( \hat{d}_{k,i} \). The last \((\frac{M}{N_t} - L)\) entries of \( \hat{d}_{k,i} \) are nonzero due to the channel noise \( q_k \). For a moderate SNR value, these \((\frac{M}{N_t} - L)\) entries should be small. In next section, we will show how to exploit this fact for jointly estimating the MIMO channels and I/Q imbalance.
4. JOINT ESTIMATION OF I/Q IMBALANCE AND CHANNEL RESPONSE

When there is I/Q imbalance, the received vector at the $k$th receive antenna is $z_k$, not $r_k$. The channel coefficients obtained by $B^\dagger z_k$ will not be an accurate estimate due to the I/Q imbalance. Below we will show how to estimate the I/Q imbalance factor $\alpha_k$ and the MIMO channels $h_{kj}(n)$ from only one received vector $z_k$. Recall that the vector $\mu_k r_k$ can be represented as (10). Let $\hat{\alpha}_k$ be an estimate of $\alpha_k$. Then from (23) the channel estimator $\hat{d}_k$ can be obtained as

$$\hat{d}_k = B^\dagger (\mu_k r_k) = B^\dagger \frac{z_k - \hat{\alpha}_k z_k^*}{1 - |\hat{\alpha}_k|^2}.$$ \hfill (25)

When $\hat{\alpha}_k \approx \alpha_k$, the I/Q imbalance can mostly be compensated and (25) will give an accurate estimate of the MIMO channels. In other words, when $\hat{\alpha}_k = \alpha_k$, the last $(M - L)$ entries of $\hat{d}_{kj}$ defined in (24) are solely due to noise and any error in the estimation of $\alpha_k$ will increase the energy of the $(M - L)$ entries. Using this observation, we are able to estimate $\alpha_k$ accurately as we will demonstrate below. In many applications, $|\alpha_k|^2$ is small. Then the channel estimator in (25) can be approximated as

$$\hat{d}_k \approx B^\dagger (z_k - \alpha_k z_k^*).$$ \hfill (26)

Define the $(M - N_r L) \times M$ matrix $\Theta$ as

$$\Theta = \begin{bmatrix}
0 & I_{M - L} & 0 & \cdots & 0 \\
0 & 0 & I_{M - L} & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & I_{M - L} & 0
\end{bmatrix}.$$ \hfill (27)

Multiplying $\hat{d}_k$ by $\Theta$ will collect all the last $(M - N_r L)$ entries of $\hat{d}_{kj}$. From (26), we have

$$\Theta \hat{d}_k \approx \Theta B^\dagger (z_k - \alpha_k z_k^*).$$ \hfill (28)

We want to find $\hat{\alpha}_k$ such that $\|\Theta \hat{d}_k\|^2$ is minimized. From the theory of linear algebra, it is known that the optimal $\hat{\alpha}_k$ is given by

$$\hat{\alpha}_{k,opt} = \frac{\Theta(B^\dagger z_k^*)^\dagger (\Theta B^\dagger z_k)}{(\Theta B^\dagger z_k^*)^\dagger (\Theta B^\dagger z_k)}. \hfill (29)$$

Once $\hat{\alpha}_{k,opt}$ is obtained, one can get the estimate of MIMO channels, $(\mu_k d_k)$, using (25). For the compensation of I/Q imbalance, one can employ (10) to obtain $\mu_k d_k$. Notice that there is no need to compensate the factor $\mu_k$ because the factor $\mu_k$ in $(\mu_k d_k)$ will be canceled when we use $(\mu_k d_k)$ to implement the FEQ.

5. SIMULATION RESULTS

In this section, we carry out Monte-Carlo experiments to verify the performance of the proposed method. A total of 5000 random channels for each transmit and receive antenna pair are generated in the simulation process. Each transmit and receive channel pair are i.i.d. complex Gaussian random variables with 4 taps, and the variance of channel taps is normalized by $\sum_{l=0}^{M-1} E[|h_{kj}(l)|^2] = 1$ for all $k, j$. The channel noise is AWGN. The transmission and training data are both QPSK symbols. The training symbols are randomly generated at each estimation. The size of the DFT matrix is $M = 64$. To avoid inter-block interference, the CP length is $L = 1 = 3$.

First we consider the following mean squared errors (MSE) for the estimation of the channel response and I/Q imbalance respectively:

$$MSE(\text{channel}) = \frac{1}{N_r N_t L} \sum_{k=0}^{N_r-1} \sum_{j=0}^{N_t-1} \sum_{l=0}^{L-1} E[|h_{kj}(l) - \hat{h}_{kj}(l)|^2].$$

$$MSE(\text{I/Q}) = E \left\{ \sum_{k=0}^{N_r-1} \frac{V_k}{\mu_k^2} \right\}. \hfill (30)$$

Fig. 2 shows the MSEs of the estimation of the channel response and I/Q imbalance. We consider two cases: (i) $N_t = 2$, $N_r = 1$ and (ii) $N_t = 4$, $N_r = 1$. Three different mismatch parameter pairs of I/Q imbalance are considered for both cases: (i) $\epsilon = 1$, $\phi = 0$, (2) $\epsilon = 1.1$, $\phi = 10^\circ$ and (3) $\epsilon = 1.2$, $\phi = 15^\circ$. When $\epsilon = 1$ and $\phi = 0$, there is no I/Q imbalance in the system. From Fig. 2(a), we see that for all cases, the I/Q imbalance factor $\alpha_k$ can be accurately estimated. For a SNR of $30dB$, the MSE is $10^{-3}$. Notice that the accuracy of our method is independent of the values of $\epsilon$ and $\phi$. The 4 transmit antenna case is only slightly worse than the 2 transmit antenna case. Fig. 2(b) shows the corresponding MSEs of the channel estimation. Again we observe that the accuracy is independent of $\epsilon$ and $\phi$. Comparing the cases of 2TX and 4TX, it is seen that the 4TX case is worse than the 2TX case because we need to estimate 4 channels in the 4TX case rather than 2 channels in the 2TX case.

Fig. 3 shows the BER performance of OFDM systems of three cases: (i) $N_t = N_r = 1$, (ii) $N_t = N_r = 2$ and (iii) $N_t = N_r = 4$. The mismatch parameters for the three cases are: (i) $\phi_0 = 11.1^\circ$, $\phi_1 = 10^\circ$, (ii) $\phi_0 = \phi_1 = 11.1^\circ$, $\phi_0 = 5^\circ$, $\phi_1 = 10^\circ$, and (iii) $\phi_0 = \phi_1 = \phi_2 = \phi_3 = 1^\circ$, $\phi_0 = 2.5^\circ$, $\phi_1 = 5^\circ$, $\phi_2 = 7.5^\circ$, $\phi_3 = 10^\circ$. Zero-forcing FEQs are used at the receiver. For the purpose of symbol recovery, the number of antennas should satisfy $N_r \geq N_t$ and in our simulation, we let $N_r = N_t$. The BER performance of the systems with I/Q imbalance is also given in the figure. It can be found that the performance is seriously degraded and the BER suffers an error flooring. In the same figure, we also show the BER performance of the ideal case where the receiver knows the exact value of channel coefficients and $\alpha_k$. From the figure, we see that for all cases, the BER performance of the proposed method is very close to the ideal case. Note that the 4TX-4Rx case is worse than the 2TX-2Rx case and 1TX-1Rx case because of higher data rate.

6. CONCLUSION

In this paper, we proposed a method for jointly estimating the I/Q imbalance and channel response for MIMO OFDM systems. The proposed method can accurately estimate both the I/Q imbalance and channel response. The implementation complexity of this method is very low. Moreover, the proposed method can be extended for jointly estimating the carrier frequency offset, I/Q imbalance and channel response [10].

REFERENCES
Figure 2: (a) MSE of the estimated IQ imbalance factor $\alpha_k$; (b) MSE of the estimated channel

Figure 3: BER performance of MIMO OFDM systems


