ANALYTICAL EXIT FUNCTIONS FOR THE PERFORMANCE EVALUATION OF ITERATIVE MIMO RECEIVERS WITH CHANNEL ESTIMATION ERROR

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ABSTRACT

Analytical extrinsic information transfer (EXIT) functions provide an accurate and flexible method to analyze and calculate the performance of iterative linear multiple input-multiple output (MIMO) receivers through their decomposition into independent elementary blocks, such as the linear combiner or the demapper. These analytical functions have been shown to be very accurate to evaluate the performance of linear MIMO receivers with perfect channel knowledge in Rayleigh-fading channels.

This contribution extends this analysis to MIMO receivers with channel estimation, deriving new analytical transfer functions and adapting the performance evaluation algorithm. Bit error rate (BER) results are provided for training-based and soft decision directed expectation-maximization (EM) channel estimation techniques, showing the validity and accuracy of this analysis method.

1. INTRODUCTION

Multiple input-multiple output (MIMO) techniques enable high-rate data transfers and improved link quality through the use of multiple antennas at both transmitter and receiver [1]. When the number of transmit antennas grows or a forward error correcting (FEC) coding scheme is used, the optimal detection of the transmitted information bits becomes prohibitively complex. Iterative receivers, based on the turbo principle, can approach the optimal performance limits of these coded MIMO systems with reduced complexity, by transferring extrinsic soft information between the outer soft-input soft-output (SISO) decoder and the inner SISO MIMO detector [2].

The performance of MIMO receivers depends on the accuracy of the channel matrix estimate, which can be obtained using training-based or semi-blind techniques. Several analysis of channel estimation effects and iterative estimator algorithms have been conducted for turbo MIMO receivers, mainly based on adaptive filter theory and the EM algorithm [3, 4].

EXIT charts have been shown to be powerful semi-analytical tools for analyzing and calculating the performance of iterative MIMO receivers [5, 6]. Nevertheless, the EXIT transfer chart of each MIMO detector depends on the channel state, leading to lower accuracy when Rayleigh-fading channels are used and the mean of output mutual information values is used to calculate an unique EXIT chart for each signal to noise ratio (SNR) [5]. This limitation may be overcome if the MIMO detector is decomposed into elementary blocks, such as the linear combiner

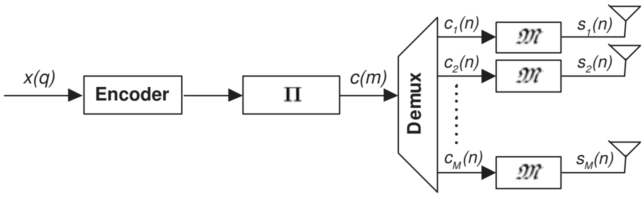

Figure 1: Diagram of a MIMO transmitter with coding and interleaving.

(LC) or the non-linear soft symbol demapper, and analytical EXIT transfer functions are used to describe their behavior [7]. This method provides a more flexible and accurate tool for the analysis and BER performance evaluation of turbo MIMO receivers. Its validity has been shown in [7] for quasi-static Rayleigh-fading MIMO channels, where only the EXIT function of the MIMO detector needs to be calculated online for each channel state, which is perfectly known at the receiver.

This contribution extends these analytical EXIT functions to channel estimation-based MIMO receivers. A new transfer function is derived for minimum mean square error-parallel interference cancellation (MMSE-PIC) receivers with channel estimation error and the performance evaluation algorithm of [7] is adapted. The accuracy and validity of these functions are shown for classical training-only based and soft decision-directed EM channel estimation algorithms. The results shown here can be extended to other iterative linear receivers, such as turbo-equalizers or multiuser detectors.

2. SYSTEM MODEL

The considered theoretical system model has, in the general case, M transmit and N receive antennas, with $N \geq M$, denoted as $M \times N$. Figure 1 shows the structure of the transmitter. The information symbol bits $x(q)$ are encoded, interleaved and demultiplexed. The resulting bits $c_k(n)$ are independently mapped onto a generic constellation of B points, modulated and transmitted simultaneously by M antennas.

Assuming symbol-synchronous receiver sampling and ideal timing, the received N-vector, using matrix notation, is given by

$$\mathbf{r}(n) = \mathbf{H}\mathbf{s}(n) + \boldsymbol{\eta}(n), \qquad (1)$$

where $\mathbf{s}(n) = [s_1(n), s_2(n), ..., s_M(n)]^T$ denotes the vector of transmitted symbols with $\mathrm{E}[|s_i(n)|^2] = 1$, $\boldsymbol{\eta}(n) =$

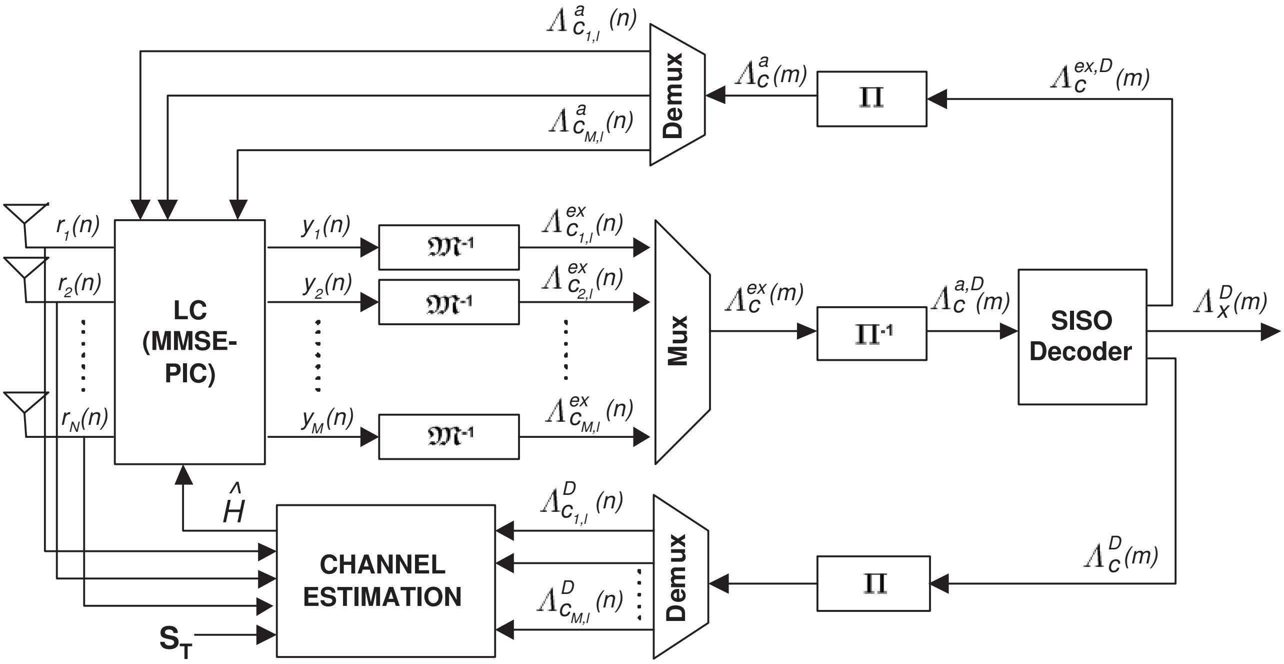

Figure 2: Diagram of a generic interference cancelation-based iterative MIMO receiver with channel estimation.

$[\eta_1(n), \eta_2(n), ..., \eta_N(n)]^T$ is the vector of independent and identically distributed (i.i.d.) complex Gaussian noise samples with covariance matrix $E\{\eta(n)\eta^H(n)\} = \mathbf{I}_N N_0$, $\mathbf{r}(n) = [r_1(n), r_2(n), ..., r_N(n)]^T$ is the vector of received symbols and $n = 1, ..., L$, where L represents the number of symbols in a frame. $\mathbf{H}$ denotes the $N \times M$ channel matrix, which is assumed constant for a frame.

Figure 2 shows the structure of the iterative receiver. The received symbols $\mathbf{r}(n)$ are processed by the SISO MIMO detector, whose outputs are the soft symbol estimates $\mathbf{y}(n) = [y_1(n), y_2(n), ..., y_M(n)]^T$. A MIMO detector that carries out parallel interference cancelation (PIC) and minimum mean squared error (MMSE) combining operations will be assumed for the rest of this paper, which can be summarized as:

$$\mathbf{r}'_k(n) = \mathbf{r}(n) - \sum_{\substack{m=1 \\ m \neq k}}^{M} \hat{\mathbf{h}}_m \tilde{s}_m = \mathbf{r}(n) - [\hat{\mathbf{H}}\tilde{\mathbf{s}}(n) - \hat{\mathbf{h}}_k \tilde{s}_k(n)] \quad (2)$$

and

$$y_k(n) = \mathbf{w}_k^H \mathbf{r}'_k(n), \quad (3)$$

where k denotes the detected branch, $\hat{\mathbf{h}}_k$ is the kth column of channel matrix estimate $\hat{\mathbf{H}}$ and $\tilde{\mathbf{s}}(n)$ is an $M \times 1$ vector of soft symbols $\tilde{s}_k(n)$, which are derived from a priori log-likelihood ratio (LLR) metrics $\Lambda^a_{c_{k,l}}(n)$ fed back from the decoder [8].

The spatial combining matrix $\mathbf{w}_k$ is calculated from the estimated channel matrix $\hat{\mathbf{H}}$ as follows:

$$\mathbf{w}_k = \left(\hat{\mathbf{H}}\bar{\mathbf{V}}\hat{\mathbf{H}}^H + (1 - \bar{v}_k)\hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H + \mathbf{I}N_0\right)^{-1}\hat{\mathbf{h}}_k, \quad (4)$$

where matrix $\bar{\mathbf{V}} = diag(\bar{v}_1, ..., \bar{v}_M)$ represents the mean of symbol variance matrices $\mathbf{V}(n) = diag(v_1(n), ..., v_M(n))$ with [6, 7]:

$$v_k(n) = E\{|s_k(n) - \tilde{s}_k(n)|^2\} = E\{|s_k(n)|^2\} - |\tilde{s}_k(n)|^2.$$

As can be seen in the figure, a channel estimate $\hat{\mathbf{H}}$ is provided to the MIMO detector by a channel estimation block, whose inputs are the received symbols, the training symbol matrix $\mathbf{S}_T$ and soft information from decoded information bits. For the sake of simplicity, only the a posteriori probability (APP) LLR metrics $\Lambda^D_c(m)$ will be considered for iterative channel estimation, as in [3, 4].

The symbol estimates $\mathbf{y}(n)$ are soft-demapped [8], providing the extrinsic LLRs of the coded bits $\Lambda^{ex}_{c_{k,l}}$, which become the input of the SISO decoder, after multiplexing and de-interleaving operations. The soft decoder delivers the following metrics:

- APP LLRs of the uncoded bits $\Lambda^D_x(m)$, whose signs define the finally detected bit values.
- Extrinsic LLRs of coded bits $\Lambda^{ex,D}_c(n)$, which are fed back to the MMSE-PIC for interference cancelation.
- APP LLRs of the coded bits $\Lambda^D_c(m)$ for channel re-estimation.

3. ANALYTICAL EXIT FUNCTIONS FOR BER PERFORMANCE EVALUATION

The EXIT transfer function-based performance evaluation method of [7] divides a generic front-end (FE) into two elementary blocks, a LC and a non-linear demapper. In this paper the FE is the MIMO detector, which will be decomposed into three elementary devices: the MMSE-PIC linear combiner, the non-linear demapper and the channel estimation block. In [7], where the channel is perfectly known at the receiver, the FE is represented by the following parametric transfer functions:

$$\Gamma_k = F_k(I^R_{in}; \mathbf{H}, N_0),$$

$$I^R_{out,k} = G(\Gamma_k, I^R_{in}) \quad (5)$$

and

$$I_{out}^{R} = \frac{1}{M}\sum_{k=1}^{M} I_{out,k}^{R}\,. \qquad (6)$$

Function F_k describes the LC behaviour for a certain channel state and SNR, giving an effective signal to interference and noise ratio (SINR) value Γ_k for each branch depending on the input mutual information (MI) I_{in}^{R}. Function G characterizes the soft demapper, as described in [7], and its output is the MI at each demapped branch $I_{out,k}^{R}$. Parting from these analytical functions and the EXIT transfer function of the SISO decoder, which does not depend on any system parameter, the authors in [7] introduced a performance evaluation algorithm that reduces drastically the simulation time and shows good accuracy in quasi-static Rayleigh-fading MIMO channels. Only F_k must be calculated online for each channel realization, while the rest of the functions are generated off-line.

4. ANALYTICAL EXIT FUNCTIONS WITH CHANNEL ESTIMATION ERRORS

Only one of the aforementioned EXIT transfer functions, F_k, needs to be changed if channel estimation error is included in the analysis method. Function F_k will now be defined by a new parameter, the estimated channel error matrix $\tilde{\mathbf{H}}$:

$$\Gamma_k = F_k(I_{in}^{R}; \mathbf{H}, N_0, \tilde{\mathbf{H}}), \qquad (7)$$

where $\tilde{\mathbf{H}} = \mathbf{H} - \hat{\mathbf{H}}$ is the channel estimation error. These variables will be generated for each channel realization by the channel estimation block, whose generation function H_{est} can be represented for the jth iteration as:

$$\hat{\mathbf{H}}^{(j)} = H_{est}(\mathbf{H}, No, \mathbf{S}_T, I_{in}^{R,(j)}), \qquad (8)$$

where $I_{in}^{R,(j)}$ is the input MI at the jth iteration. Therefore, the randomly generated channel estimate depends on the channel state, the noise, the transmitted training sequence and the mutual information statistics fed back from the decoder, if iterative channel estimation is used.

4.1 Analytical transfer function of MMSE-PIC with channel estimation error

The output SINR Γ_k at the kth branch of a generic MMSE receiver can be defined as [9]:

$$\Gamma_k = \frac{E\{s_k s_k^H\}}{tr(E\{e_k e_k^H\})} - 1 = \frac{1}{E\{e_k e_k^H\}} - 1, \qquad (9)$$

where $e_k = s_k - y_k$. Parting from Equations (1-3) and omitting the symbol index n, e_k can be written as:

$$e_k = s_k - \mathbf{w}_k^H \left(\mathbf{h}_k s_k + \sum_{\substack{m=1\\m\neq k}}^{M} \mathbf{h}_m s_m - \sum_{\substack{m=1\\m\neq k}}^{M} \hat{\mathbf{h}}_m \tilde{s}_m + \eta \right)$$

$$= s_k - \mathbf{w}_k^H \left(\mathbf{h}_k s_k + \sum_{\substack{m=1\\m\neq k}}^{M} \mathbf{h}_m (s_m - \tilde{s}_m) + \sum_{\substack{m=1\\m\neq k}}^{M} \tilde{\mathbf{h}}_m \tilde{s}_m + \eta \right),$$

where $\tilde{\mathbf{h}}_m$ is the mth column of the channel estimation error matrix $\tilde{\mathbf{H}}$.

Assuming $E\{s_k s_k^H\} = 1$, $E\{(s_k - \tilde{s}_k)(s_k - \tilde{s}_k)^H\} = v_k$ and $E\{(\tilde{s}_k \tilde{s}_k^H)\} = E\{(s_k \tilde{s}_k^H)\} = 1 - v_k$, the error variance can be expressed as

$$E\{e_k e_k^H\} = 1 - \mathbf{w}_k^H \mathbf{h}_k - \mathbf{h}_k^H \mathbf{w}_k + \mathbf{w}_k^H \mathbf{R}_{rr} \mathbf{w}_k,$$

where

$$\mathbf{R}_{rr} = \mathbf{h}_k \mathbf{h}_k^H + N_0 \mathbf{I}_M + \sum_{\substack{m=1\\m\neq k}}^{M} \mathbf{h}_m v_m \mathbf{h}_m^H + \underbrace{\sum_{\substack{m=1\\m\neq k}}^{M} \tilde{\mathbf{h}}_m (1 - v_m) \tilde{\mathbf{h}}_m^H}_{\mathbf{T}}. \qquad (10)$$

As can be seen, the effect of wrong channel estimate is twofold: the combining vector $\mathbf{w}_k^H$ is not matched to the actual channel $\mathbf{H}$ and a new error term $\mathbf{T}$ appears in Equation (10) due to the wrong cancelation of detected symbols.

4.2 Adaptation of the performance evaluation algorithm

The performance evaluation algorithm of [7] can be extended to Rayleigh-fading iterative receivers with channel estimation, independently of the estimation technique:

Performance Evaluation Algorithm

(0) Generate $\mathbf{H}$.
(1) Initialization: $j = 1$; $I_{out}^{D,(0)} = 0$.
(2) Get FE input MI. $I_{in}^{R,(j)} = I_{out}^{D,(j-1)}$.
(3) Generate estimate $\hat{\mathbf{H}}^{(j)} = H_{est}(\mathbf{H}, No, \mathbf{S}_T, I_{in}^{R,(j)})$.
(4) Compute $\bar{\mathbf{V}}$ from $I_{in}^{R,(j)}$, as in [7].
(5) Calculate $\mathbf{w}_k$ vectors from (4) and Γ_k values from (9).
(6) Compute $I_{out,k}^{R,(j)}$ and $I_{out}^{R,(j)}$ via (5) and (6).
(7) Obtain the decoder's output $I_{out}^{D,(j)} = f^D(I_{out}^{R,(j)})$.
(8) Calculate $BER^{(j)} = f_{BER}(I_{out}^{D,(j)})$.
(9) Return to step (2) with $j = j + 1$.

Algorithm 1: BER performance evaluation algorithm for each channel realization of an iterative MIMO receiver with channel estimation.

As can be seen in Algorithm 1, a new step has been included, numbered as (3), where a new channel estimate is generated for each channel realization and turbo iteration. The rest of the algorithm works as detailed in [7], transfering MI values between the MIMO detector and the outer soft decoder. Functions f^D and f_{BER} represent the EXIT transfer function and the BER estimation function of the decoder, respectively. $I_{out}^{D,(j)}$ is the output MI of the decoder and j represents the iteration index.

5. APPLICATION EXAMPLES

Function $H_{est}(\mathbf{H}, No, \mathbf{S}_T, I_{in}^{R})$ calculates a channel estimate for each channel realization based on the information fed back from the decoder as APP LLRs and the statistics of the training process. Two classical channel estimation techniques will be considered: training-based least squares (LS) and soft decision-directed EM.

5.1 Training-based LS channel estimation

If a training symbol matrix $\mathbf{S}_T$ of dimensions $M \times L_T$ is sent before data transmission and LS channel estimation is applied, the estimated channel is:

$$\hat{\mathbf{H}} = \mathbf{R}\mathbf{S}_T^H \left(\mathbf{S}_T \mathbf{S}_T^H\right)^{-1},$$

where $\mathbf{R}$ is an $N \times L_T$ matrix with the received training signal. $\hat{\mathbf{H}}$ is an unbiased estimate of $\mathbf{H}$, the estimation error is uncorrelated among the N receivers and the covariance matrix for each row is [10]:

$$E\{\tilde{\mathbf{h}}_n^H \tilde{\mathbf{h}}_n\} = N_0\left(\mathbf{S}_T \mathbf{S}_T^H\right)^{-1}.$$

If $\mathbf{S}_T$ is formed by orthogonal training sequences, i.e, $\mathbf{S}_T\mathbf{S}_T^H = L_T \mathbf{I}_M$, the elements of the error matrix $\tilde{\mathbf{H}}$ become i.i.d. complex random variables with mean zero and variance N_0/L_T.

The estimate generation function $H_{est}^{(1)}(\mathbf{H}, No, \mathbf{S}_T)$, which does not depend on I_{in}^R if only training symbols are used for channel estimation, must then create an estimate $\hat{\mathbf{H}}$ according to the aforementioned statistics for each channel realization of the performance evaluation method described in Algorithm 1.

5.2 EM channel estimation

Training-only based channel estimation techniques do not profit from the iterative nature of turbo receivers. Many algorithms have been developed to re-estimate the channel from hard and soft decision statistics fed back from the SISO decoder. The classical EM channel estimation technique [3, 4] has been chosen here to show how iterative estimation can be included in the analytical EXIT function-based performance evaluation method. Based on [3, 4], the EM channel estimate obtained as

$$\hat{\mathbf{H}}^{(j+1)} = \bar{\mathbf{R}}_{rs}^{(j)} \left[\bar{\mathbf{R}}_s^{(j)}\right]^{-1}.$$

If the iteration index j is omitted, the correlation matrices $\bar{\mathbf{R}}_{rs}^{(j)}$ and $\bar{\mathbf{R}}_s^{(j)}$ become:

$$\bar{\mathbf{R}}_{rs} = \sum_{n=1}^{N_s} \mathbf{r}(n)\bar{\mathbf{s}}^H(n) = \mathbf{H}\mathbf{R}'_s + \theta,$$

$$\bar{\mathbf{R}}_s(i,k) = \begin{cases} N_s & ;\ i = k \\ \sum_{n=1}^{N_s} \bar{\mathbf{s}}_i(n)\bar{\mathbf{s}}_k^*(n) & ;\ i \neq k \end{cases} \tag{11}$$

and

$$\mathbf{R}'_s = \sum_{n=1}^{N_s} \mathbf{s}(n)\bar{\mathbf{s}}^H(n)\ ,\ \theta = \sum_{n=1}^{N_s} \eta(n)\bar{\mathbf{s}}^H(n), \tag{12}$$

where $\bar{\mathbf{s}}(n)$ are the soft symbol estimates obtained from the APP LLRs fed back to the channel estimation block [8], while θ is the matrix of weighted noise samples with autocovariance $N_0\mathbf{R}''_s$, where

$$\mathbf{R}''_s = \sum_{n=1}^{N_s} \bar{\mathbf{s}}(n)\bar{\mathbf{s}}^H(n). \tag{13}$$

The estimated channel $\hat{\mathbf{H}}$ is a biased estimate of $\mathbf{H}$ and can be written as:

$$\hat{\mathbf{H}} = \mathbf{H}\mathbf{R}'_s\bar{\mathbf{R}}_s^{-1} + \theta\bar{\mathbf{R}}_s^{-1}. \tag{14}$$

The estimation function $H_{est}(\mathbf{H}, No, \mathbf{S}_T, I_{in}^{(j,R)})$ must generate an estimate $H^{(j)}$ for all the iterations $j > 1$ following Equation (14), while the training-based function $H_{est}(\mathbf{H}, No, \mathbf{S}_T)$ is used at the first iteration. Thus, matrices of Equations (11-13) must be calculated from the output statistics of the decoder. A very simple approach has been followed which calculates the aforementioned matrices according to the following approximations:

$$\hat{\bar{\mathbf{R}}}_s(i,k) = \begin{cases} N_s & ;\ i = k \\ N_s(1-\sigma_p)\mathbf{w}_i^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{w}_i & ;\ i \neq k \end{cases} \tag{15}$$

$$\hat{\mathbf{R}}'_s(i,k) = \begin{cases} N_s\sigma_p & ;\ i = k \\ N_s(1-\sigma_p)\mathbf{h}_k^H\mathbf{w}_i & ;\ i \neq k \end{cases} \tag{16}$$

$$\hat{\mathbf{R}}''_s(i,k) = \begin{cases} N_s\sigma_p & ;\ i = k \\ N_s(1-\sigma_p)\mathbf{w}_i^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{w}_i & ;\ i \neq k\ . \end{cases} \tag{17}$$

The value of $\sigma_p = E\{\bar{\mathbf{s}}_k\bar{\mathbf{s}}_k^H\}$ has been calculated off-line for each constellation alphabet when generating the EXIT transfer function of the decoder. The approximations of Equations (15-17) have been tested for quaternary phase shift keying (QPSK) modulation with several different channel realizations.

6. RESULTS

A system with $M = 4$ transmit and $N = 4$ receive antennas has been chosen to validate the performance evaluation method. Simulations with QPSK modulation have been conducted to compare the classical Monte-Carlo (MC) simulation and the EXIT-based analytical performance evaluation method with channel estimation error. Up to 10000 data blocks of 2048 coded bits have been simulated with a quasi-static Rayleigh-fading MIMO channel. Walsh codes of 4×8 and 4×16 symbols have been sent as training symbols and perfect timing and demodulation have been assumed at the receiver. A non-recursive non-systematic convolutional code with generator polynomials $\{5,7\}_8$ and the common log-map BCJR algorithm have been selected for FEC encoding and decoding, respectively, with a random interleaver.

Figure 3 shows the BER comparison of EXIT and MC techniques for the iterative MMSE-PIC receiver with LS channel estimation and a 4×8 training matrix. As can be seen, the EXIT-based analysis gives slightly optimistic and quite accurate results for training-based channel estimation. MC simulation with perfect channel estimation has been included in the figures as reference. Figure 4 shows the same comparison for a training matrix of dimensions 4×16 symbols with similar results.

Figure 5 extends the comparison to EM-based channel estimation with the techniques and simplifications of section 4.3. The BER estimation accuracy is shown for a system with iterative channel estimation and initial training matrix of dimensions 4×8. These results show that this method and the simplifications assumed can be used to estimate or predict the BER performance of iterative channel estimation-based MIMO receivers. For the case of EM channel estimation, further analysis is required to extend the aforementioned assumptions to other modulations and MIMO detectors.

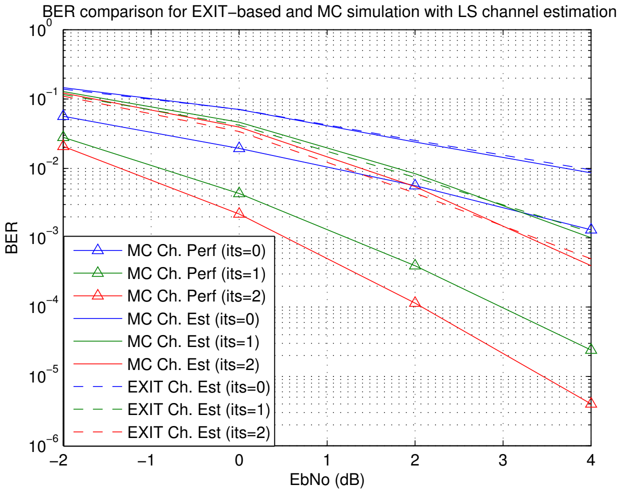

Figure 3: Comparison of EXIT fuction-based and MC simulation with LS channel estimation and a 4×8 training matrix.

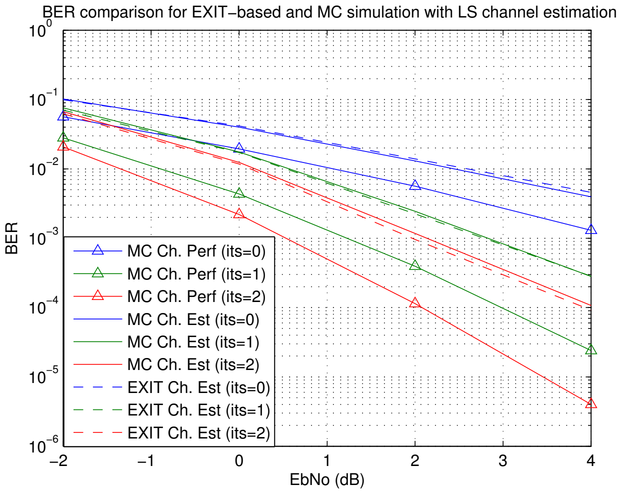

Figure 4: Comparison of EXIT fuction-based and MC simulation with LS channel estimation and a 4×16 training matrix.

7. CONCLUSION AND FURTHER RESEARCH

This paper has shown an effective and complexity-reduced method to include channel estimation errors on analytical EXIT function-based performance evaluation methods. An analytical transfer function has been derived for an MMSE-PIC receiver with channel estimation error. An algorithm has been shown which allows to evaluate the performance of different channel estimation techniques in iterative linear turbo receivers and comparative results have been provided for QPSK transmission with trained LS and soft decision-directed EM channel estimation.

As future work lines, these results can be extended to other modulations or different turbo-based applications, such as multiuser detection or turbo-equalization. Other interesting work lines include the analysis of other channel estimation techniques, based on soft or hard decisions, and the use of the different LLR metrics available at the decoder.

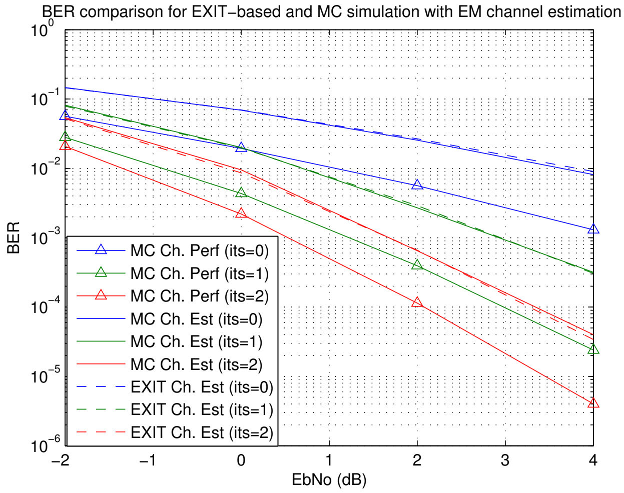

Figure 5: Comparison of EXIT fuction-based and MC simulation with soft decision-directed EM channel estimation and an initial 4×8 training matrix.

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