ZERO-FORCING EQUALIZATION OF SPACE-TIME BLOCK CODED TRANSMISSIONS OVER FREQUENCY SELECTIVE CHANNELS

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ABSTRACT

Recently, a polyphase approach was proposed for the analysis of space-time block coded (STBC) transmissions over frequency selective channels. Using this approach, we study the zero-forcing (ZF) equalization of these systems. In this study, we consider only the two transmit antennas case. For the receiver, the single and multiple receive antennas cases are both considered. For the case of single receive antenna, the ZF receiver is a unique infinite impulse response (IIR) filter whereas for the case of multiple receive antennas, there exist finite impulse response (FIR) ZF receivers. We will derive the general form of the ZF receiver. Then the optimal ZF solution that minimizes the output noise variance will be derived. Simulation results show that the performance of STBC system with two transmit and one receive antenna is better than the conventional system with one transmit and one receive antenna. When there are multiple receive antennas, it is found that there can be significant improvement if we optimize the ZF receiver.

1. INTRODUCTION

Space-time block coding (STBC) was first proposed by Alamouti in [1] for the communication system with two transmit antennas. It was shown that when the channel is flat fading, STBC system can have a very good performance by only using linear processing at the receiver. Later STBC was extended to the case of more antennas by Tarokh et al in [2][3]. The authors proved that for flat fading channels, linear processing operation at the receiver can achieve the performance of a maximum likelihood detector. However for broadband communication systems, the channel is usually frequency selective. For these channels, the orthogonality of STBC would be destroyed. The resulting inter-symbol interference (ISI) effect can degrade the system performance significantly. Therefore, new equalization techniques that are different from those in [1][2][3] are required to combat the ISI effect.

In the past, many methods have been proposed for the equalization of STBC transmission over frequency selective channel. In [4], the authors used a matched filter followed by the linear equalizer (LE) or the decision feedback equalizer (DFE) to recover the transmitted signal. The finite impulse response linear equalizer (FIR LE) and decision feedback equalizer (FIR DFE) are studied in [5] and [6]. In [6], the sufficient conditions for the existence of the FIR ZF receiver have been addressed. However, the discussions in these studies are based on the assumption that the number of the receive antenna must be larger or equal to that of the transmit antenna. In fact, the redundancy in STBC can be used to help recover the transmitted signals. Recently, the authors in [7] proposed a polyphase approach to study STBC transmission over frequency selective channels. It was shown that symbol recovery is possible even if the number of receive antenna is fewer than that of transmit antenna. Both the widely linear (WL) FIR LE and DFE were derived and it was demonstrated that the performance of the STBC system is better than that of the conventional SISO system. These equalization techniques all need channel state information at the receiver. On the other hand, the direct equalization without channel state information is considered in [10] and [11].

In this paper, we adopt the polyphase method in [7] to analyze the STBC transmission over frequency-selective channel. We will focus on the two transmit antennas case. Firstly when there is only one receive antenna, the zero-forcing (ZF) receiver is derived. The ZF receiver is a unique infinite impulse response (IIR) filter. We will also explore the stability issue of the IIR ZF receiver. We will give an example to show that even when the two channels from the two transmit antennas have zeros inside the unit circle, the IIR ZF receiver can be unstable. Secondly we will consider the case when there are more than one receive antenna. In this case, the ZF receiver is not unique. We will first derive the general form of all ZF receivers and then derive the optimal receiver to minimize the output noise variance. Simulation results show that for the one receive antenna case, the STBC system with an IIR ZF receiver has a better performance than the conventional SISO system. For the multiple receive antenna case, by optimizing the FIR ZF receiver, we can greatly improve the performance.

Notations and Definitions: Boldfaced letters are used to denote vectors and matrices. \( A^* \) and \( A^\dagger \) denote the complex conjugate and Hermitian of \( A \) respectively. \( A(z) \) is a polynomial matrix given by \( A(z) = \sum_{l=-N}^{N} A_l z^{-l} \), where \( A_l \) is the order of the polynomial matrix. The notation \( A^\dagger(z) \) represents \( A(z) = \sum_{l=-N}^{N} A_l^\dagger z^{-l} \). A polynomial matrix \( A(z) \) is said to be unimodular if det\( A(z) \) is a constant independent of \( z \). One important property of a unimodular polynomial matrix \( A(z) \) is that its inverse \( A^{-1}(z) \) is also a polynomial matrix.

2. SYSTEM DESCRIPTION AND POLYPHASE FORMULATION

In this paper, we consider the Alamouti’s STBC systems with two transmit and \( M \) receive antennas [1]. The method can be extended to the case of more antennas using STBC in [3]. In this section, the polyphase formulation proposed in [7] is described. We will derive the result in the \( z \) domain rather...
than the time domain. Assume that \( s(n) \) are the modulation symbols. After the space-time encoding scheme, two \( 2 \times 1 \) signal blocks are formed. In the first block, the two modulation symbols \( s(2n) \) and \( s(2n+1) \) are transmitted from Antenna 0 and Antenna 1 while in the second block the two signals \(-s(2n+1)\) and \( s(2n)\) are transmitted from Antenna 0 and Antenna 1 respectively. Letting \( s_0(n) = s(2n) \) and \( s_1(n) = s(2n+1) \), the output signals \( x_0(n) \) and \( x_1(n) \) of Antenna 0 and Antenna 1 are respectively

\[
x_0(n) = \begin{cases} s_0(n) \frac{1}{2}, & \text{if } n \text{ is even;} \\ -s_1(n) \frac{1}{2}, & \text{if } n \text{ is odd.} \end{cases} \tag{1}
\]

and

\[
x_1(n) = \begin{cases} s_1(n) \frac{1}{2}, & \text{if } n \text{ is even;} \\ s_0(n) \frac{1}{2}, & \text{if } n \text{ is odd.} \end{cases} \tag{2}
\]

In the \( z \)-domain, we can write

\[
X_0(z) = S_0(z^2) - z^{-1}S_1(z^2), \tag{3}
\]

\[
X_1(z) = S_1(z^2) + z^{-1}S_0(z^2). \tag{4}
\]

The channel from the \( i \)th transmit antenna to the \( i \)th receive antenna is modeled as linearly time-invariant FIR filter \( H_{i,j}(z) \) given by

\[
H_{i,j}(z) = h_{i,j}(0) + h_{i,j}(1)z^{-1} + \cdots + h_{i,j}(L)z^{-L}, \tag{5}
\]

where \( L \) is the channel order. Let its polyphase representation be

\[
H_{i,j}(z) = E_{i,j,0}(z^2) + z^{-1}E_{i,j,1}(z^2), \tag{6}
\]

for \( i = 0, 1, \ldots, M-1 \) and \( j = 0, 1 \). The received signal \( Y_i(z) \) at the \( i \)th receive antenna is given by

\[
Y_i(z) = H_0(z)X_0(z) + H_{1i}(z)X_1(z) + Q_i(z), \tag{7}
\]

where \( Q_i(z) \) is the \( z \)-transform of the noise of the \( i \)th receive antenna. Let us decompose \( Y_i(z) \) into its polyphase representation \( Y_i(z) = Y_{i0}(z^2) + z^{-1}Y_{i1}(z^2) \). Define the \( 4 \times 1 \) vectors

\[
y_i(z) = \begin{bmatrix} Y_{i0}(z) & Y_{i1}(z) & Y_{i0}'(z) & Y_{i1}'(z) \end{bmatrix}^T, \tag{8}
\]

\[
s(z) = \begin{bmatrix} S_0(z) & S_1(z) & S_0'(z) & S_1'(z) \end{bmatrix}^T, \tag{9}
\]

\[
q_i(z) = \begin{bmatrix} Q_{i0}(z) & Q_{i1}(z) & Q_{i0}'(z) & Q_{i1}'(z) \end{bmatrix}^T. \tag{10}
\]

Then from (3) ~ (6), the received signals can be arranged in the matrix form

\[
y_i(z) = H_i(z)s(z) + q_i(z), \tag{11}
\]

where

\[
H_i(z) = \begin{bmatrix} A_i(z) & B_i(z) \\ B_i^*(z) & A_i^*(z) \end{bmatrix}. \tag{12}
\]

The matrices \( A_i(z) \) and \( B_i(z) \) are respectively given by

\[
A_i(z) = \begin{bmatrix} E_{i0,0}(z) & E_{i1,0}(z) \\ E_{i0,1}(z) & E_{i1,1}(z) \end{bmatrix}, \tag{13}
\]

\[
B_i(z) = \begin{bmatrix} z^{-1}E_{i1,1}(z) & -z^{-1}E_{i0,1}(z) \\ E_{i1,0}(z) & -E_{i0,0}(z) \end{bmatrix}. \tag{14}
\]

By collecting all the signals \( y_i(z) \) from the \( M \) receive antennas, we can have

\[
y(z) = T(z)s(z) + q(z), \tag{15}
\]

where

\[
T(z) = \begin{bmatrix} H_0^T(z) & H_0^T(z) & \cdots & H_{M-1}^T(z) \end{bmatrix}^T, \tag{16}
\]

\[
q(z) = \begin{bmatrix} q_{00}^T(z) & q_{01}^T(z) & \cdots & q_{M-1,0}^T(z) \end{bmatrix}^T, \tag{17}
\]

\[
y(z) = \begin{bmatrix} y_{00}^T(z) & y_{01}^T(z) & \cdots & y_{M-1,0}^T(z) \end{bmatrix}^T. \tag{18}
\]

The problem is to recover the transmitted signal \( s(z) \) from the observation \( y(z) \). In the following, we consider the cases of one receive antenna and \( M \) receive antennas \( (M \geq 2) \).

### 3. One Receive Antenna Case

If one receive antenna is employed at the receiver, the received signal in (11) is reduced to a \( 4 \times 1 \) vector

\[
y_0(z) = H_0(z)s(z) + q_0(z). \tag{19}
\]

As \( H_0(z) \) is a \( 4 \times 4 \) matrix, the ZF receiver (if exists) is given by

\[
F(z) = H_0^{-1}(z), \tag{20}
\]

Since \( H_0(z) \) is a square matrix, its inverse \( F(z) \) is unique \(^1\), and in general its implementation needs an IIR filter. When the channel is flat fading, the two channel transfer functions are \( H_{00}(z) = h_{00}(0) \) and \( H_{01}(z) = h_{01}(0) \) respectively. From (5) and (6), we have \( E_{0,0}(z) = E_{0,1}(z) = 0 \), \( E_{0,0}(0) = h_{00}(0) \), and \( E_{0,1}(0) = h_{01}(0) \). Substituting these results into (8), the matrix \( H_0(z) \) becomes

\[
H_0(z) = \begin{bmatrix} h_{00}(0) & 0 & 0 & 0 \\ h_{00}(0) & h_{01}(0) & -h_{00}(0) & 0 \\ 0 & 0 & h_{00}(0) & h_{01}(0) \\ h_{01}(0) & -h_{00}(0) & 0 & 0 \end{bmatrix}, \tag{21}
\]

which is a constant orthogonal matrix. The zero-forcing receiver reduces to the well-known Alamouti receiver.

### A Note on the Stability Issue

The IIR zero-forcing receiver \( F(z) \) in (13) may have poles on or outside the unit circle. In this case, there does not exist any causal stable implementation. The stability of \( F(z) \) depends on the two channel transfer functions \( H_{00}(z) \) and \( H_{01}(z) \). Numerical experiments found that in general \( F(z) \) can be unstable even when both \( H_{00}(z) \) and \( H_{01}(z) \) have zeros inside the unit circle, and vice versa. How the stability of \( F(z) \) depends on \( H_{00}(z) \) and \( H_{01}(z) \) is still unknown. However, for the special case of \( H_{00}(z) = (1 - az^{-1}) \) and \( H_{01}(z) = (1 - bz^{-1}) \) for real \( a \) and \( b \), it can be shown that the IIR ZF receiver is stable if and only if the two channels of two transmit antennas have zeros inside the unit circle. Substituting \( E_{0,0}(z) = 1, E_{0,1}(z) = -a, E_{0,0}(z) = 1 \), and \( E_{0,1}(z) = -b \) into the expression of \( H_0(z) \) in (10), one can find that the determinant of \( H_0(z) \) is given by

\[
\det H_0(z) = ((a-b)^2 - 4) + 2(a+b)^2 z^{-1} - [(a^2 + b^2)^2 - (a-b)^2] z^{-2}. \tag{22}
\]

\(^1\)When there are more than two transmit antennas, we know that there does not exist any full rate STBC. In this case, if we apply the polyphase approach to find the corresponding \( H_0(z) \), we will find that \( H_0(z) \) is a tall matrix and the ZF receiver is not unique.
One can show that the zeros of \( \text{det}H_0(z) \) are inside the unit
circle if and only if \( a^2 + b^2 < 2 \). Therefore for this special
case, the ZF receiver \( H_0^{-1}(z) \) is casual stable whenever the
two inverse \( H_{00}(z) \) and \( H_{01}^{-1}(z) \) are causal stable. It is
interesting to see that even when one of the two inverses is
unstable (e.g. \( a = 1.1, b = 0.8 \)), the block version \( H_0^{-1}(z) \) is
stable. However, when the channel order is larger, stabi-
licity of \( H_{00}(z) \) and \( H_{01}^{-1}(z) \) does not guarantee the stability of
\( H_0^{-1}(z) \). One example is given by
\[
H_{00}(z) = 1 - 0.707z^{-1}, H_{01}(z) = (1 - 0.8z^{-1})(1 + 0.6z^{-1}).
\]
In this case, because the zeros of \( H_{00}(z) \) and \( H_{01}(z) \) are both
inside the unit circle, the inverse of \( H_{00}^{-1}(z) \) and \( H_{01}^{-1}(z) \) is
stable. However, one can show that the four poles of \( H_0(z) \) are
given by \(-0.9250 + 0.5765i, -0.9250 - 0.5765i, 0.2312 + 0.6469i \)
and \( 0.2312 - 0.6469i \). It is easy to verify that ampli-
tude of the first two poles are 1.09 > 1, which are outside
the unit circle. Therefore, even if the zeros of the two chan-
nels \( H_{00}(z) \) and \( H_{01}(z) \) are inside the unit circle, \( H_0(z) \) is
not guaranteed to be stable.

4. MULTIPLE RECEIVE ANTENNA CASE (M ≥ 2)
When the number of receive antennas is \( M ≥ 2 \), \( T(z) \) is a
\( 4M \times 4 \) tall matrix whose left inverse is not unique. In par-
ticular, for almost all cases, there exist FIR ZF receivers. Below
we will derive the general form of all causal stable ZF
receivers. To do this, we apply the Smith form decomposition
to \( T(z) \) [9]:
\[
T(z) = U(z)A(z)W(z),
\]
where \( U(z) \) and \( W(z) \) are \( 4M \times 4 \) and \( 4 \times 4 \) unimu-
lar matrices and \( A(z) \) is diagonal. It can be shown that except
for some very rare degenerated cases, the diagonal matrix
\( A(z) \) is a constant matrix; that is,
\[
A(z) = \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix},
\]
where \( A \) is a \( 4 \times 4 \) constant diagonal matrix. Partition
\( U^{-1}(z) \) as
\[
U^{-1}(z) = \begin{bmatrix} U_0(z) \\ U_1(z) \end{bmatrix}.
\]
Then all the ZF receivers can be expressed as
\[
F(z) = K(z) + P(z)U_1(z),
\]
where \( K(z) = W^{-1}(z)A^{-1}U_0(z) \) and \( P(z) \) is an arbitrary
polynomial matrix of size \( 4 \times 4(M - 1) \). One can prove
that the ZF condition can be achieved since \( K(z)T(z) = I \)
and \( U_1(z)T(z) = 0 \). Because \( U(z) \) and \( W(z) \) are unimu-
lar, the matrices \( W^{-1}(z), U_0(z) \), and \( U_1(z) \) are causal FIR.
Hence \( F(z) \) is causal stable if and only if \( P(z) \) is causal sta-
ble. Moreover \( F(z) \) will be FIR if \( P(z) \) is FIR. As the ZF
receiver is not unique, one can find \( P(z) \) so that the output
noise variance is minimized. Let \( P(z) \) be FIR of order \( N_P \).
The noise vector \( e(n) \) at the output of \( F(z) \) is shown in Fig. 1:
\[
e(n) = \theta(n) + \sum_{k=0}^{N_P} P(k)\nu(n-k),
\]
where \( \theta(n) \) is a \( 4 \times 1 \) vector given by \( \theta(n) = \sum_{l=0}^{N_p} K(l)q(n-l) \),
and \( \nu(n) \) is a \( 4(M - 1) \times 1 \) vector given by \( \nu(n) =
\sum_{l=0}^{N_p} U_1(l)q(n-l) \) respectively. \( q(n) \) is the \( 4M \times 1 \) noise
vector at the receiver in (11).

When the channel and noise are given, the vectors \( \theta(n) \)
and \( \nu(n) \) are fixed. Our goal is to find \( P(z) \) such that
\( E\{ |e(n)|^2 \} \) is minimized. This problem can be solved
by applying the orthogonality principle. The optimal \( P(z) \)
should be chosen such that
\[
E\{ |e(n)|^2 \} = 0, \text{ for } k = 0, 1, \ldots, N_P.
\]
Solving the above equations, we get the optimal \( 4 \times 4(N_P + 1)/(M - 1) \)
matrix \( P \) as
\[
P = \begin{bmatrix} P(0) & P(1) & \cdots & P(N_P) \\ \\ -E\{ \theta(n)\eta^*(n)\} (E\{ \eta(n)\eta^*(n)\})^{-1} \end{bmatrix},
\]
where \( \eta(n) \) is a \( 4(N_P + 1)/(M - 1) \) \times 1 \) vector given by
\[
\eta(n) = [v^T(n) \ v^T(n-1) \ \cdots \ v^T(n-N_P)]^T.
\]
Define the \( 4M(N_U + N_P + 1) \times 1 \) vector \( \mu(n) \), \( 4 \times 4(N_U +
N_P + 1) \) matrix \( K \), and the \( 4(M - 1) \times 4(N_P + N_U + 1) \)
matrix \( U \) as
\[
\mu(n) = [q^T(n) \ q^T(n-1) \ \cdots \ q^T(n-N_U-N_P)]^T,
\]
\[
K = [K(0) \ K(1) \ \cdots \ K(N_U)] \quad 0 \quad \cdots \quad 0,
\]
and
\[
U = \begin{bmatrix} U_1(0) \ \cdots \ U_1(N_U) \ 0 \ \cdots \ 0 \\ 0 \ U_1(0) \ \cdots \ U_1(N_U) \ \cdots \ 0 \\ \vdots \ \ddots \ \ddots \ \ddots \ \vdots \\ 0 \ \cdots \ 0 \ \cdots \ U_1(N_U) \end{bmatrix},
\]
where \( r \) is a design parameter. Then
\[
\theta(n) = K\mu(n), \quad \eta(n) = U\mu(n).
\]
Then substituting (23) into (21), we can have the optimal
solution \( P \) given by
\[
P = -KR_{\mu\mu}U^T(UR_{\mu\mu}U^T)^{-1},
\]
where \( R_{\mu\mu} \) is the autocorrelation matrix of \( \mu(n) \). To guar-
nantee the existence of the inverse matrix \( (UR_{\mu\mu}U^T)^{-1} \), U
must be a tall matrix. Therefore, we must have \(4(M-1)r \geq 4M(N_p + N_U + 1)\) so that the design parameter \(r\) satisfies \(r \geq \frac{M}{M-1}(N_p + N_U + 1)\). After getting the optimal \(P\), the polynomial matrix \(P(z)\) is correspondingly given by

\[
P(z) = \sum_{k=0}^{N_p} P_k z^{-k}.
\] (25)

The optimal solution \(P\) depends on the statistics of the channel noise \(q_i(n)\), and the channel transfer function \(T(z)\). Given the channel noise and channel information, we can design an optimal ZF receiver using (24) and (25). With the increase of the order \(N_p\), simulation results in next section show that the system performance can be significantly improved. Assume that the channel noise \(q_i(n)\) is a zero-mean complex random variable and satisfies

\[
E\{q_i(n)q_j^*(m)\} = \mathcal{N}_0 \delta_{ij} \delta(n-m).
\] (26)

Then the correlation matrix \(R_{\mu\mu}\) reduces to \(R_{\mu\mu} = \mathcal{N}_0 I\) and the optimal solution is simplified as

\[
P = -KU^\dagger(UU^\dagger)^{-1}.
\] (27)

5. SIMULATION RESULTS

In this section, we carry out Monte Carlo numerical experiment to verify the bit error rate (BER) performance. The transmitted signal \(\lambda(n)\) is QPSK. The noise is assumed to be AWGN. Four frequency selective channels of order 4 are employed in the simulations and their impulse responses are given in Table 1 in next page.

Firstly, we show the BER comparison between the STBC system with 2Tx-1Rx antennas and the SISO system with 1Tx-1Rx antennas. The zero-forcing receivers for both systems are IIR. For the two transmit antenna case, the two channels \(H_{00}(z)\) and \(H_{01}(z)\) are Channels A and B respectively. For the one transmit antenna case, we average the BER of the SISO system for Channels A and B. These results are shown in Fig. 3. For comparison, we also plot the BER curve of the STBC system when the two channels are flat fading channels with the same signal power gain as Channels A and B. From the figure, we find that the performance of STBC is always better than the SISO system. The performance of STBC in the frequency-selective channel is much worse than the flat-fading channel. This is due to the intersymbol interference generated by the frequency-selective channel. For the two receive antenna case, four channels \(H_{00}(z), H_{01}(z), H_{10}(z)\) and \(H_{11}(z)\) are Channels A, B, C and D in Table 1 respectively. Optimal ZF FIR receivers with different \(N_p\) (order of \(P(z)\)) are compared. These results are shown in Fig. 4. For comparison, we also plot the curve when \(P(z) = 0\). It is seen that by optimizing \(P(z)\), the performance can be enhanced with the increase of \(N_p\).

6. CONCLUSIONS

The ZF equalization of STBC systems in frequency-selective channels is studied. For the one receiver antenna case, the ZF receiver is IIR. In the case of multiple receive antennas, we derive the general form of the ZF receiver. We also derive the optimal FIR ZF receiver to minimize the output noise variance. Simulation results show that by optimizing FIR ZF receiver, the BER performance can be improved significantly.
Table 1: Impulse response of Channels A, B, C, D

<table>
<thead>
<tr>
<th>Channel</th>
<th>h(0)</th>
<th>h(1)</th>
<th>h(2)</th>
<th>h(3)</th>
<th>h(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.5569+0.3465i</td>
<td>-0.0766-0.6431i</td>
<td>-0.7776-0.5116i</td>
<td>-0.5902-0.5360i</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.0357+0.1165i</td>
<td>-0.0392-0.1390i</td>
<td>-0.5136+0.1286i</td>
<td>0.2479-0.6957i</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.0953-0.0095i</td>
<td>0.0513+0.2267i</td>
<td>-0.2354-0.2471i</td>
<td>0.2483+0.2604i</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0.1710-0.0719i</td>
<td>0.0583+0.1624i</td>
<td>-0.3234+0.0586i</td>
<td>0.2163+0.1243i</td>
<td></td>
</tr>
</tbody>
</table>

REFERENCES


