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# BROADCAST CHANNEL OPTIMAL SPECTRUM BALANCING (BC-OSB) WITH PER-MODEM TOTAL POWER CONSTRAINTS FOR DOWNSTREAM DSL

*Vincent Le Nir, Marc Moonen, Mamoun Guenach, and Jan Verlinden*

K.U. Leuven, SISTA/ESAT, Kasteelpark Arenberg 10, B-3001 Leuven-Heverlee ,Belgium,
phone: + (32) 16 32 17 88, fax: + (32) 16 32 19 70, email: vincent.lenir@esat.kuleuven.be, web: www.esat.kuleuven.be
Alcatel, Research & Innovation Department, Antwerpen, Belgium

### ABSTRACT

*In this paper we investigate the problem of optimal power allocation in a BC by exploiting the duality with MAC. We first describe an algorithm for power allocation under a total power constraint. Then, the BC-OSB algorithm is devised for BC power allocation under per-modem total power constraints, where the Lagrange multipliers of the dual problem formulation are transferred into the optimization function by means of a precoding matrix, such that the MAC-BC duality can again be exploited. Simulation results are given for a VDSL2 scenario, and also for a theoretical multi-user independent flat-fading Rayleigh environment.*

## 1. INTRODUCTION

The capacity of Multiple Input Multiple Output (MIMO) systems has received significant interest since the work of Foschini and Telatar [1, 2]. For single-user MIMO, the optimal precoding and equalization are obtained through the SVD of the channel matrix (for each frequency) with optimal Power Spectral Densities (PSD's) computed from standard waterfilling (over frequencies and "virtual channels"). This work has been extended to multi-user MIMO Multiple Access Channel (MAC) by the use of an iterative waterfilling procedure ("iterative" over users, "waterfilling" over frequencies) [3].

Broadcast Channels (BC) have attracted a lot of interest in the past few years due to the lack of a full characterization of the multi-user MIMO BC capacity region. The capacity region for multi-user Single Input Single Ouptut (SISO) BC has been characterized in [4]. For these channels, one user's signal is a degraded version of the other signals. To achieve the capacity region, it is possible to employ superposition coding and successive decoding. The ordering between users is chosen by the noise variance. However, the multi-user MIMO BC is non-degraded, which makes it much more difficult to characterize the capacity regions since successive decoding is impossible [4]. Recently, it has been shown that Dirty Paper Coding (DPC) achieves the capacity region of the multi-user SISO BC and the multi-user MIMO BC as well [5, 6]. By combination with duality theory between MAC and BC, the information-theoretic characterization of the multi-user MIMO BC can be obtained in its dual multi-user MIMO MAC under a total power constraint [7].

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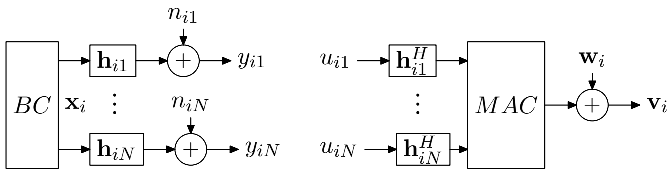


Figure 1: BC (left side) and MAC (right side) for tone *i*

In the DSL context, it is more relevant to consider a constraint on the total power used by each modem separately instead of a constraint on the total power used by all modems together. The capacity region of the multi-user MIMO BC under per-modem (or per-antenna in the wireless transmission context) total power constraints has been shown to be the dual of a multi-user MIMO MAC with an uncertain noise [8], which is an additional diagonal covariance matrix in the optimization function. This leads to a minmax optimization over transmit covariance matrices and the noise covariance matrix.

In this paper, we propose a new algorithm (referred to as BC-OSB) to find optimal covariance matrices for the BC under per-modem total power constraints which is obtained by transferring the Lagrange multipliers of the dual problem formulation into the cost function and using MAC-BC duality transformations. By the combination of the MAC-OSB algorithm [9] and the duality between MAC and BC [7], this leads to an easier maximization problem compared to the existing minmax optimization [8].

The outline of the paper is as follows. First, we recall the optimal power allocation alogrithm for multi-user MIMO BC under a total power constraint in section 2. Then, we present a new algorithm for optimal power allocation in multi-user MIMO BC under per-modem total power constraints in section 3. Simulation results and conclusions are given in section 4 and 5.

## 2. TOTAL POWER CONSTRAINT

We consider a multi-user MIMO Broadcast Channel (BC) serving N users in a downstream DSL scenario and using Discrete Multi-Tone (DMT) modulation with a cyclic prefix longer than the maximum delay spread of the channels as shown in **Figure 1**. The transmission on tone *i* can then be modelled as:

$$\mathbf{y}_i = \mathbf{H}_i\mathbf{x}_i + \mathbf{n}_i \quad \text{where} \quad \mathbf{H}_i = \begin{bmatrix} \mathbf{h}_{i1} \\ \vdots \\ \mathbf{h}_{iN} \end{bmatrix} \quad i = 1 \ldots N_c \tag{1}$$

where $N_c$ is the number of subcarriers, $\mathbf{x}_i$ is the transmitted vector on tone $i$, $\mathbf{y}_i$ the received signal vector of length $N$, $\mathbf{H}_i$ the $N\times N$ MIMO channel matrix and $\mathbf{n}_i$ the vector containing Additive White Gaussian Noise (AWGN). The dual MIMO Multiple Access Channel (MAC) for the dual uplink scenario with N users can be written as:

$$\mathbf{v}_i = \mathbf{H}_i^H\mathbf{u}_i + \mathbf{w}_i \quad \text{where} \quad \mathbf{H}_i^H = \begin{bmatrix} \mathbf{h}_{i1}^H & \dots & \mathbf{h}_{iN}^H \end{bmatrix} \tag{2}$$

where $\mathbf{u}_i = [u_{i1}\dots u_{iN}]^T$. In this paper, we assume $E[\mathbf{w}_i\mathbf{w}_i^H] = E[\mathbf{n}_i\mathbf{n}_i^H] = \mathbf{I}$, which can always be obtained after a pre-whitening step. The primal problem of finding optimal transmit vector covariance matrices in the BC under a total power constraint $P^{tot}$ is:

$$\begin{aligned} &\max_{(\mathbf{Q}_{ik})_{i=1\dots N_c,k=1\dots N}} C^{BC} \\ &\text{subject to} \sum_{k=1}^{N}\sum_{i=1}^{N_c} Trace(\mathbf{Q}_{ik}) \le P^{tot} \\ &\mathbf{Q}_{ik} \succeq 0, i=1\dots N_c, k=1\dots N \end{aligned} \tag{3}$$

where $E[\mathbf{x}_i\mathbf{x}_i^H] = \sum_{k=1}^{N}\mathbf{Q}_{ik}$ corresponds to the sum of $N$ uncorrelated transmit vector covariance matrices for the different users over tone i and $C^{BC}$ the weighted rate sum function for a given encoding order 1,...,N-1,N (i.e. user 1 is encoded first) [7]:

$$C^{BC} = \sum_{j=1}^{N} w_j \sum_{i=1}^{N_c} log_2\left[det\left(1+\frac{\mathbf{h}_{ij}\mathbf{Q}_{ij}\mathbf{h}_{ij}^H}{1+\mathbf{h}_{ij}(\sum_{k=j+1}^{N}\mathbf{Q}_{ik})\mathbf{h}_{ij}^H}\right)\right] \tag{4}$$

The $w_j$'s are the weights assigned to the different users. As we will see, the optimal detection order is actually defined by the weights (i.e. user with the largest weight is encoded first, etc.). The objective function is neither convex nor concave [7], therefore finding the optimal covariance matrices in the BC is a difficult task. Fortunately, the duality between the MAC and the BC says that it is possible to achieve the same rates in both domains under a total power constraint. As the optimal power allocation in the MAC is tractable [9], one can calculate the optimal covariance matrices in the MAC and transform them into optimal covariance matrices for the BC. The primal problem of finding power allocations in the MAC under a total power constraint $P^{tot}$ is:

$$\begin{aligned} &\max_{(\mathbf{\Phi}_i)_{i=1\dots N_c}} C^{MAC} \\ &\text{subject to} \sum_{i=1}^{N_c} Trace(\mathbf{\Phi}_i) \le P^{tot} \\ &\mathbf{\Phi}_i \succeq 0, i=1\dots N_c \end{aligned} \tag{5}$$

with $\mathbf{\Phi}_i = E[\mathbf{u}_i\mathbf{u}_i^H] = diag(\phi_{i1},\dots,\phi_{iN})$ the covariance matrix of transmitted symbols for tone $i$. If the decoding order is N,N-1,...,1 (i.e. user 1 is decoded last), the weighted rate sum in the MAC is:

$$C^{MAC} = \sum_{j=1}^{N} w_j \sum_{i=1}^{N_c} log_2\left[det\left(\mathbf{I}+\frac{\mathbf{h}_{ij}^H\phi_{ij}\mathbf{h}_{ij}}{\mathbf{I}+\sum_{k=1}^{j-1}\mathbf{h}_{ik}^H\phi_{ik}\mathbf{h}_{ik}}\right)\right] \tag{6}$$

In the MAC, the user with the largest weight is decoded last [9]. As the MAC-BC duality dictates a reverse of the decoding/encoding order, in the BC the user with the largest weight has indeed to be encoded first. The idea of dual decomposition is to solve (6) via its Lagrangian and leads to the MAC-OSB solution algorithm [9]. The Lagrangian decouples into a set of $N_c$ smaller problem, thus reducing the complexity of equation (6). The dual objective function is:

$$F^{MAC}(\lambda) = \max_{(\mathbf{\Phi}_i)_{i=1\dots N_c}} C^{MAC} + \lambda(P^{tot} - \sum_{i=1}^{N_c} Trace(\mathbf{\Phi}_i)) \tag{7}$$

with $\lambda$ the Lagrange multiplier. The dual optimization problem is:

$$\begin{aligned} &\text{minimize} \quad F^{MAC}(\lambda) \\ &\text{subject to} \quad \lambda \ge 0 \end{aligned} \tag{8}$$

Because the dual function is convex in $\lambda$, it has a unique minimum. As the duality gap is zero, this minimum corresponds to the global optimum of the primal problem in (5) [10, 11]. The search for the optimal $\lambda$ in (8) involves evaluations of the dual objective function (7), i.e. maximizations of the Lagrangian, which is decoupled over the tones for a given $\lambda$. In particular, the dual function becomes:

$$\begin{aligned} F^{MAC}(\lambda) = \max_{(\mathbf{\Phi}_i)_{i=1\dots N_c}} \sum_{i=1}^{N_c}\Bigg(\sum_{j=1}^{N} w_j log_2\Bigg[ \\ det\left(\mathbf{I}+\frac{\mathbf{h}_{ij}^H\phi_{ij}\mathbf{h}_{ij}}{\mathbf{I}+\sum_{k=1}^{j-1}\mathbf{h}_{ik}^H\phi_{ik}\mathbf{h}_{ik}}\right)\Bigg] - \lambda Trace(\mathbf{\Phi}_i)\Bigg) + \lambda P^{tot} \end{aligned} \tag{9}$$

The following algorithm (for adjusting the Lagrangian multiplier $\lambda$) leads to the optimal PSD's in the MAC domain under a total power constraint [9]:

**Algorithm 1**

- init $\lambda = 1$
- init $step = 2$
- init $b = 0$
- while $|\sum_{i=1}^{N_c} Trace(\mathbf{\Phi}_i) - P^{tot}| > tolerance$
  - Exhaustive search $\max_{(\mathbf{\Phi}_i)_{i=1\dots N_c}} F^{MAC}(\lambda)$
  - if $\sum_{i=1}^{N_c} Trace(\mathbf{\Phi}_i) - P^{tot} < 0$
    - $b = b+1$
    - $\lambda = \lambda/step$
    - $step = step - 1/2^b$
  - end if
  - $\lambda = \lambda * step$
- end while

Once the optimal covariance matrices are found in the MAC domain, we can use the formulas given in [7] to convert these into optimal covariance matrices in the BC domain. By setting $a_{ij} = (1+\mathbf{h}_{ij}(\sum_{k=j+1}^{N}\mathbf{Q}_{ik})\mathbf{h}_{ij}^H)^{-1}$ and $\mathbf{B}_{ij} = (\mathbf{I}+\sum_{k=1}^{j-1}\mathbf{h}_{ik}^H\phi_{ik}\mathbf{h}_{ik})^{-1}$, the optimal covariance matrices can be calculated recursively from user N to user 1 in the BC domain with the following equation:

$$\mathbf{Q}_{ij} = \mathbf{B}_{ij}^{-1/2}\mathbf{F}_{ij}\mathbf{G}_{ij}^H a_{ij}^{1/2}\phi_{ij}a_{ij}^{1/2}\mathbf{G}_{ij}\mathbf{F}_{ij}^H\mathbf{B}_{ij}^{-1/2} \tag{10}$$

with $\mathbf{B}_{ij}^{-1/2}\mathbf{h}_{ij}^{H}a_{ij}^{-1/2} = \mathbf{F}_{ij}\mathbf{L}_{ij}\mathbf{G}_{ij}^{H}$ and $\mathbf{L}_{ij}$ the square root of the eigenvalues coming from the Singular Value Decomposition (SVD). Note that the above procedure crucially depends on the property that the power allocation in the MAC and the corresponding transmit vector covariance matrices in the BC correspond to exactly the same total transmit power [7].

## 3. PER-MODEM TOTAL POWER CONSTRAINTS

The goal of this section is to find optimal transmit vector covariance matrices for a BC under per-modem total power constraints, that is a single total power constraint for all tones on each line. The primal problem of finding optimal transmit vector covariance matrices in the BC under a per-modem total power constraint $P_j^{tot}$ for each line is:

$$\begin{array}{l} \max\limits_{(\mathbf{Q}_{ik})_{i=1\dots N_c,k=1\dots N}} \quad C^{BC} \\ \text{subject to} \sum\limits_{k=1}^{N}\sum\limits_{i=1}^{N_c} \mathbf{Q}_{ik,jj} \le P_j^{tot}\ \forall j \\ \mathbf{Q}_{ik} \succeq 0, i = 1\dots N_c, k = 1\dots N \end{array} \tag{11}$$

with $\mathbf{Q}_{ik,jj}$ the $j^{\text{th}}$ diagonal element of the transmit vector covariance matrix for user k over tone i and $C^{BC}$ the weighted rate sum function as defined by (4). We follow a dual decomposition approach similar to section 2 using MAC-BC duality. The primal problem of finding optimal power allocations in the MAC under a per-modem total power constraint $P_j^{tot}$ is:

$$\begin{array}{l} \max\limits_{(\mathbf{\Phi}_i)_{i=1\dots N_c}} \quad C^{MAC} \\ \text{subject to} \sum\limits_{i=1}^{N_c} \phi_{ij} \le P_j^{tot}\ \forall j \\ \mathbf{\Phi}_i \succeq 0, i = 1\dots N_c \end{array} \tag{12}$$

with $\mathbf{\Phi}_i = E[\mathbf{u}_i\mathbf{u}_i^H] = diag(\phi_{i1},\dots,\phi_{iN})$ the covariance matrix of transmitted symbols for tone i and $C^{MAC}$ the weighted rate sum defined by (6). Therefore, the MAC dual objective function is:

$$\begin{array}{l} F^{MAC}(\mathbf{\Lambda}) = \max\limits_{(\mathbf{\Phi}_i)_{i=1\dots N_c}} \sum\limits_{i=1}^{N_c}\left(\sum\limits_{j=1}^{N} w_j log_2\Big[\right. \\ \left. det\left(\mathbf{I} + \frac{\mathbf{h}_{ij}^H\phi_{ij}\mathbf{h}_{ij}}{\mathbf{I}+\sum\limits_{k=1}^{j-1}\mathbf{h}_{ik}^H\phi_{ik}\mathbf{h}_{ik}}\right)\Big] - Trace(\mathbf{\Lambda\Phi}_i)\right) \\ +Trace\left(\mathbf{\Lambda} diag(P_1^{tot},\dots,P_N^{tot})\right) \end{array} \tag{13}$$

with $\mathbf{\Lambda}$ a diagonal matrix of Lagrange multipliers $diag(\lambda_1,\dots,\lambda_N)$. The dual optimization problem is:

$$\begin{array}{ll} \text{minimize} & F^{MAC}(\mathbf{\Lambda}) \\ \text{subject to} & \lambda_i \ge 0 \quad \forall i \end{array} \tag{14}$$

The search for the optimal $\mathbf{\Lambda}$ involves evaluations of the dual objective function, i.e. maximizations of the Lagrangian, which is decoupled over the tones for a given set of $\lambda_i$'s. An algorithm similar to **Algorithm 1** is straightforwardly devised, which can be used to compute optimal power allocations for the MAC under per-modem total power constraints. However, the MAC-BC duality does not preserve per-modem total power constraints [7] so that the MAC optimal power allocation cannot be converted into BC optimal covariance matrices. In the following paragraph, we therefore present a procedure to transform the objective function into an equivalent objective function with total power constraint in order to then use the MAC-BC duality. To calculate the optimal transmit vector covariance matrices in the BC, we propose to use a rescaling of the channel matrices by a precoder matrix. First, we start with the BC dual objective function:

$$\begin{array}{l} F^{BC}(\mathbf{\Lambda}) = \max\limits_{(\mathbf{Q}_{ik})_{i=1\dots N_c,k=1\dots N}} \sum\limits_{i=1}^{N_c}\left(\sum\limits_{j=1}^{N} w_j log_2\Big[\right. \\ \left. det\left(1 + \frac{\mathbf{h}_{ij}\mathbf{Q}_{ij}\mathbf{h}_{ij}^H}{1+\mathbf{h}_{ij}(\sum\limits_{k=j+1}^{N}\mathbf{Q}_{ik})\mathbf{h}_{ij}^H}\right)\Big] - \sum\limits_{k=1}^{N} Trace(\mathbf{\Lambda Q}_{ik})\right) \\ +Trace\left(\mathbf{\Lambda} diag(P_1^{tot},\dots,P_N^{tot})\right) \end{array} \tag{15}$$

Then, rescaling the channel matrices by the inverse square root of the Lagrange multiplier matrix leads to:

$$\mathbf{y}_i = \overbrace{\mathbf{H}_i\mathbf{\Lambda}^{-1/2}}^{\mathbf{H}'_i}\mathbf{x}'_i + \mathbf{n}_i \tag{16}$$

where $\mathbf{x}'_i = \mathbf{\Lambda}^{1/2}\mathbf{x}_i$. For this equivalent channel, the dual objective function in the BC becomes:

$$\begin{array}{l} F^{BC}(\mathbf{\Lambda}) = \max\limits_{(\mathbf{Q}'_{ik})_{i=1\dots N_c,k=1\dots N}} \sum\limits_{i=1}^{N_c}\left(\sum\limits_{j=1}^{N} w_j log_2\Big[\right. \\ \left. det\left(1 + \frac{\mathbf{h}'_{ij}\mathbf{Q}'_{ij}\mathbf{h}'^H_{ij}}{1+\mathbf{h}'_{ij}(\sum\limits_{k=j+1}^{N}\mathbf{Q}'_{ik})\mathbf{h}'^H_{ij}}\right)\Big] - \sum\limits_{k=1}^{N} Trace(\mathbf{Q}'_{ik})\right) \\ +Trace\left(diag(P'^{tot}_1,\dots,P'^{tot}_N)\right) \end{array} \tag{17}$$

where $\mathbf{h}'_{ij} = \mathbf{h}_{ij}\mathbf{\Lambda}^{-1/2}$, $\mathbf{Q}'_{ik} = \mathbf{\Lambda}^{1/2}\mathbf{Q}_{ik}\mathbf{\Lambda}^{1/2}$ and $diag(P'^{tot}_1,\dots,P'^{tot}_N) = \mathbf{\Lambda}^{1/2}diag(P_1^{tot},\dots,P_N^{tot})\mathbf{\Lambda}^{1/2}$. One can see in (17) that the precoder matrix $\mathbf{\Lambda}^{1/2}$ transforms the per-modem total power constraints into a total power constraint by hiding the Lagrange multipliers into the equivalent channels $\mathbf{h}'_{ij}$ and the new covariance matrices $\mathbf{Q}'_{ik}$. Therefore we obtain an optimization problem under a total power constraint that can be solved in the dual MAC. Using duality, we obtain the following received vector in the MAC:

$$\mathbf{v}_i = \mathbf{\Lambda}^{-1/2}\mathbf{H}_i^H\mathbf{u}'_i + \mathbf{w}_i \tag{18}$$

The dual objective function in the MAC becomes:

$$\begin{array}{l} F^{MAC}(\mathbf{\Lambda}) = \max\limits_{(\mathbf{\Phi}'_i)_{i=1\dots N_c}} \sum\limits_{i=1}^{N_c}\left(\sum\limits_{j=1}^{N} w_j log_2\Big[\right. \\ \left. det\left(\mathbf{I} + \frac{\mathbf{h}'^H_{ij}\phi'_{ij}\mathbf{h}'_{ij}}{\mathbf{I}+\sum\limits_{k=1}^{j-1}\mathbf{h}'^H_{ik}\phi'_{ik}\mathbf{h}'_{ik}}\right)\Big] - Trace(\mathbf{\Phi}'_i))\right) \\ +Trace\left(diag(P'^{tot}_1,\dots,P'^{tot}_N)\right) \end{array} \tag{19}$$

For a given $\mathbf{\Lambda}$, we can then compute the optimal power allocation in the MAC and then use the duality formulas of [7] to obtain the optimal covariance matrices in the BC that achieve the same rates as in the MAC and under same total power constraint. The Lagrange multipliers are then adjusted so that the per-modem total power constraints are not exceeded. The following algorithm (for adjusting the Lagrangian multiplier $\mathbf{\Lambda}$) leads to the optimal transmit vector covariance matrices under per-modem total power constraints. This algorithm will be referred to as BC-OSB (BC-Optimal Spectrum Balancing):

**Algorithm 2 BC-OSB**

$\forall j = 1 \ldots N$ jointly

- init $\lambda_j = 1$
- init $step_j = 2$
- init $b_j = 0$
- while $|\sum_{i=1}^{N_c} Trace(\mathbf{\Lambda}^{-1/2}(\sum_{j=1}^{N} \mathbf{Q}'_{ij})\mathbf{\Lambda}^{-1/2}) - diag(P_j^{tot})| > tolerance$
  - Exhaustive search $\max_{(\mathbf{\Phi}'_i)_{i=1\ldots N_c}} F^{MAC}(\mathbf{\Lambda})$
  - MAC-BC Duality $\mathbf{Q}'_{ij} = \mathbf{B}_{ij}^{-1/2}\mathbf{F}_{ij}\mathbf{G}_{ij}^{H} a_{ij}^{1/2} \phi'_{ij} a_{ij}^{1/2}\mathbf{G}_{ij}\mathbf{F}_{ij}^{H}\mathbf{B}_{ij}^{-1/2}$ $\forall i = 1 \ldots N_c$
  - if $\sum_{i=1}^{N_c} Trace(\mathbf{\Lambda}^{-1/2}(\sum_{j=1}^{N} \mathbf{Q}'_{ij})\mathbf{\Lambda}^{-1/2}) - diag(P_j^{tot}) < 0$
    - $b_j = b_j + 1$
    - $\lambda_j = \lambda_j / step_j$
    - $step_j = step_j - 1/2^{b_j}$
  - end if
  - $\lambda_j = \lambda_j * step_j$
- end while

## 4. RESULTS

In a VDSL2 transmission, the shape of the Far End Crosstalk (FEXT) can be expected to be similar to the Near End Crosstalk (NEXT). The NEXT and FEXT models for ADSL/HDSL may no longer be applicable to VDSL2 due to the much larger bandwidth that is used. In this paper, the first set of simulations results are obtained on measured channels from a France Telecom binder with 2 lines of 400 meters and 800 meters respectively. Spectral masks for VDSL2 Fiber To The exchange (FTTex) are used [12], with SNR gap $\Gamma$=0 dB (since duality does not hold if $\Gamma >$0 dB), an AWGN of -140 dBm/Hz and maximum transmit power $P_j^{tot}$=14.5 dBm per line. The frequency range is from 0 to 12 MHz with 4.3125 kHz spacing between subcarriers and 4 kHz symbol rate. The FDD band plan of VDSL2 corresponds to 2 frequency bands in the downlink scenario which are 138kHz-3.75MHz and 5.2MHz-8.5MHz.

**Table 1** shows the rates obtained in the BC for 2 users under a total power constraint $P^{tot}$=29 dBm, and for different sets of weights $(w_1, w_2) = (w_1, 1 - w_1)$. The first line corresponds to the rates R1 and R2 when user 1 is encoded first in the BC domain. The second line corresponds to the rates when user 2 is encoded first. One can see that according to the weights given to the users, different rates can be obtained in the BC. Due to the diagonal dominance of the channel matrix in this context, the difference between the possible detection orders is negligible. **Table 2** shows the rates obtained in the BC for 2 users under per-modem total power constraints. The obtained rates are equal for any detection order or weights given to the different users. This is due to the lower degree of freedom used in the optimization process compared to a total power constraint. **Figure 2** summarizes the two first tables by plotting the rates. The diagonalizing precoder of [13] achieves 135.8 Mbps for the first user and 89.30 Mbps for the second user. Therefore the diagonalizing precoder achieves most of the capacity under AWGN owing to the diagonal dominance of the channel matrix. To see the advantage of using the scheme presented in this paper compared to the diagonalizing precoder, we now select stronger crosstalk channels which are not diagonally dominant [13]. In the following simulations, we consider independent flat-fading Rayleigh channels for the different tones which follow a complex Gaussian distribution with mean 0 and power 1. The weighted rate sums are calculated in **Table 3** and **Table 4** under a total power constraint and per-modem total power constraints respectively. A main observation is that the weights indeed define the detection order. The user with the smallest weights has to be detected first and the user with the largest weights has to be detected last in order to achieve the largest weighted rate sum. On **Figure 3** the different rates are plotted, leading to four different rate regions with total power constraint and per-modem total power constraints. The rates obtained on this channel with the diagonalizing precoder [13] are R1=177.69 Mbps and R2=178.20 Mbps.

| R1/R2 (Mbps) | w1=0.0 | w1=0.1 | w1=0.2 | w1=0.3 | w1=0.4 | w1=0.5 | w1=0.6 | w1=0.7 | w1=0.8 | w1=0.9 | w1=1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| User 1 first | 0/95.93 | 120.9/94.99 | 127.3/93.92 | 131.6/92.53 | 133.7/91.41 | 136.2/89.37 | 137.9/87.18 | 139.1/85.05 | 140.4/80.80 | 141.5/74.44 | 142.4/0 |
| User 2 first | 0/95.93 | 120.9/94.99 | 127.3/93.93 | 131.6/92.53 | 133.7/91.41 | 136.2/89.37 | 137.9/87.18 | 139.1/85.05 | 140.4/80.80 | 141.5/74.44 | 142.4/0 |

Table 1: Rates under total power constraint in the BC

| R1/R2(Mbps) | w1=0.0 | w1=0.1 | w1=0.2 | w1=0.3 | w1=0.4 | w1=0.5 | w1=0.6 | w1=0.7 | w1=0.8 | w1=0.9 | w1=1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| User 1 first | 0/89.96 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.2/0 |
| User 2 first | 0/89.96 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.0/89.53 | 136.2/0 |

Table 2: Rates under per-modem total power constraint in the BC

| w1R1+w2R2 | w1=0.0 | w1=0.1 | w1=0.2 | w1=0.3 | w1=0.4 | w1=0.5 | w1=0.6 | w1=0.7 | w1=0.8 | w1=0.9 | w1=1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| User 1 first | 190.0 | 183.7 | 181.4 | 181.3 | 181.7 | 182.5 | 183.5 | 185.1 | 187.1 | 189.7 | 193.7 |
| User 2 first | 193.8 | 189.8 | 187.2 | 185.2 | 183.6 | 182.5 | 181.6 | 181.2 | 181.3 | 183.5 | 189.7 |

Table 3: Weighted rate sum under total power constraint in the BC

## 5. CONCLUSION

In this paper we have investigated the problem of optimal power allocation in a BC by exploiting the duality with MAC.

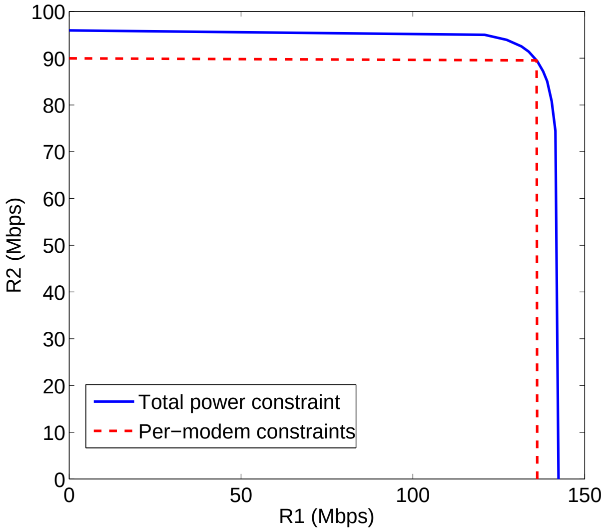


Figure 2: Rate region of BC-OSB in a VDSL2 system

| w1R1+w2R2 | w1=0.0 | w1=0.1 | w1=0.2 | w1=0.3 | w1=0.4 | w1=0.5 | w1=0.6 | w1=0.7 | w1=0.8 | w1=0.9 | w1=1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| User 1 first | 193.2 | 178.6 | 179.5 | 180.5 | 181.5 | 182.5 | 183.4 | 184.4 | 185.4 | 186.3 | 193.7 |
| User 2 first | 193.8 | 186.4 | 185.4 | 184.4 | 183.4 | 182.5 | 181.4 | 180.4 | 179.4 | 178.4 | 192.9 |

Table 4: Weighted rate sum under per-modem total power constraint in the BC

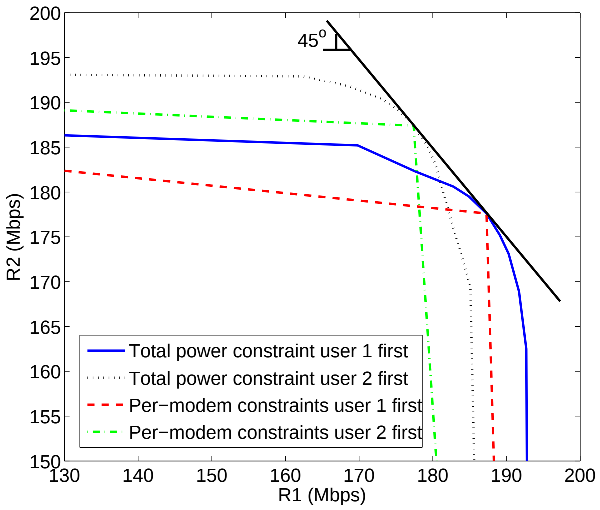


Figure 3: Rate region of BC-OSB in a complex gaussian channels

We have first described an algorithm for power allocation under a total power constraint. Then, the BC-OSB algorithm has been devised for BC power allocation under per-modem total power constraints, where the Lagrange multipliers of the dual problem formulation are transferred into the optimization function by means of a precoding matrix, such that the MAC-BC duality can again be exploited. Simulation results were given for a VDSL2 scenario, and also for a theoretical multi-user independent flat-fading Rayleigh environment.

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