OPTIMAL VARIABLE STEP-SIZE NLMS ALGORITHMS FOR FEEDFORWARD ACTIVE NOISE CONTROL

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ABSTRACT

The paper improves the feedforward active noise control system with online secondary path modeling developed by Akhtar, Abe, and Kawamata by deriving optimal variable step-size parameters for the adaptation algorithms of the secondary path modeling filter and of the control filter. It is shown that the adaptation algorithms equipped with the optimal variable step-size parameters improve the convergence speed of the system and the estimation accuracy of the optimal control filter.

1. INTRODUCTION

Active noise control (ANC) systems equipped with the Filtered-x Least Mean Square (FX-LMS) adaptation algorithm cannot prescind from the online estimation of the secondary path [1]. Two different approaches can be adopted for the secondary path modeling. A first approach involves the injection of an auxiliary white random noise in the ANC system and it uses a system identification method to model the secondary path [2], [3], [4], [5], [6]. The second approach estimates the secondary path directly from the output of the control filter, without the injection of additional noise [1]. It has been shown in [7] that the first approach is superior for convergence speed of both the control filter and the secondary path modeling filter, for speed of response to modifications in the primary noise and the secondary path, and for independence between the primary noise attenuation and the online secondary path identification.

The injection of an auxiliary noise for estimating the secondary path was first proposed in [2], where two adaptive filters were used for adapting the control filter and for identifying the secondary path, respectively. The system of [2] suffers the slow convergence of the control filter and of the secondary path modeling filter and the low estimation accuracy of the optimal values of these filters. Indeed, with the injection of an auxiliary noise, the signal at the error microphone has two components: (1) the auxiliary noise filtered by the secondary path, (2) the residual noise of the ANC system. In the ANC system of [2], the first component disturbs the adaptation of the control filter, while the second component disturbs the identification of the secondary path. In order to solve this problem the use of a third adaptive filter was proposed in [3], [4], [5]. In [3] and [4] the third adaptive filter is used to improve the convergence performance and the estimation accuracy of the secondary path. In fact, this adaptive filter acts as a noise suppressor that removes the residual noise from the error signal of the secondary path modeling filter. The third adaptive filter is used for the same purpose also in [5], but a cross-update strategy is employed for removing also the auxiliary noise from the error signals of the control filter and of the noise suppresser. More recently, improved convergence performances were obtained with the ANC structure proposed in [6]. The ANC system of [6] uses again only two adaptive filters, one for adapting the control filter and one for modeling the secondary path, but an improved convergence speed of the control filter is obtained by introducing the delay compensation scheme of [8], and by removing the auxiliary noise from the error signal of the control filter.

This paper improves the ANC system of [6] by deriving optimal variable step-size parameters for the adaptation algorithms of the secondary path modeling filter and of the control filter. The transient and the steady-state behavior of the adaptation algorithms used for the secondary path modeling filter and the control filter can be tuned by acting on the step-size parameter of the adaptive filter. By reducing the step-size parameter we can improve the estimation accuracy of the secondary path modeling filter or of the control filter at steady state, but the convergence speed of the ANC system reduces as well. A classical approach proposed in the literature to meet these conflicting requirements is that of the variable step-size parameter methods. In these methods, the step-size parameter is varied in accordance with the state of the adaptive filter and its distance from the steady-state condition. Different criteria have been developed for estimating this distance and for controlling the step-size parameter. In particular, by minimizing at each iteration the mean-square-deviation of the adaptive filter, an optimal step-size parameter was obtained for the Normalized Least Mean Square (NLMS) algorithm in [9]. This optimal step-size parameter was applied in [10] to the secondary path modeling filter in order to theoretically compare the ANC systems of [2], [3], and [5]. However, in [10] the practical estimation of the optimal step-size parameter was not discussed. An heuristically motivated variable step-size parameter was also proposed for the secondary path modeling filter in [6]. In [6] small step-size parameter values are used in the early phases of the adaptation, when the residual noise is large and it disturbs considerably the secondary path modeling filter adaptation. On the contrary, larger step-size parameter values are used when the residual noise reduces. In this paper, using the theory of [9] we derive optimal step-size parameters for both the secondary path modeling filter and the control filter, and we discuss suitable practical estimators for all quantities involved in the computation of the optimal step-size parameters. In contrast to the variable step-size parameter of [6], we show that the optimal value of the step-size parameter for the secondary path modeling filter has a decreasing behavior with the convergence of the ANC system. We also show that the adaptation algorithm equipped with the optimal variable step-size parameters is capable to improve the convergence speed of the ANC system and the estimation accuracy of the control filter.

The paper is organized as follows. Section 2 provides a brief overview of the ANC system of [6] and of the variable step-size there proposed. Section 3 derives the optimal step-size parameters for the secondary path modeling filter and for the control filter and discusses how these optimal step-size parameters can be practically estimated. Section 4 provides simulation results for the ANC system equipped with the optimal step-size parameters and compares them with those of the system equipped with the variable step-size parameter of [6].

Throughout the paper small boldface letters are used to denote vectors, the symbol $*$ denotes the linear convolution, $E[\cdot]$ denotes the mathematical expectation, and $\|\cdot\|$ denotes the Euclidean norm.

2. BACKGROUND THEORY

Figure 1 shows the block diagram of the ANC system with secondary path modeling proposed in [6]. The definition of all quantities in Figure 1, together with the definition of other quantities used in the following of the paper can be found in Table 1.

The active noise control system of [6] exploits the delay compensation scheme of [8] in order to improve the convergence prop-

Table 1: Quantities used for the ANC system description.

Quantity	Description
$p(n)$	impulse response of the primary path;
$\mathbf{p}(n)$	vector collecting the samples of $p(n)$;
$s(n)$	impulse response of the secondary path;
$\mathbf{s}(n)$	vector collecting the samples of $s(n)$;
$\hat{\mathbf{s}}(n) = [s_0(n), s_1(n), \ldots, s_{M-1}(n)]^T$	coefficient vector of the secondary path modeling filter, an FIR filter of memory length M;
$\mathbf{w}(n) = [w_0(n), w_1(n), \ldots, w_{N-1}(n)]^T$	coefficient vector of the noise control filter, an FIR filter of memory length N;
$x(n)$	reference signal;
$\mathbf{x}_N(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^T$	data vector with the last N samples of $x(n)$;
$y(n) = \mathbf{w}^T(n)\mathbf{x}_N(n)$	output of the *actual* noise control filter;
$\nu(n)$	internally generated zero mean, unit variance, white Gaussian noise;
$G(n)$	amplification factor of the white Gaussian noise;
$v(n) = G(n)\nu(n)$	auxiliary noise injected in the system;
$d(n) = p(n) * x(n)$	primary disturbance signal;
$y'(n) = s(n) * y(n)$	canceling signal;
$v'(n) = s(n) * v(n)$	modeling signal;
$e(n) = d(n) - y'(n) + v'(n)$	error microphone signal;
$\mathbf{v}(n) = [v(n), v(n-1), \ldots, v(n-M+1)]^T$	data vector with the last M samples of $v(n)$;
$\hat{v}'(n) = \hat{\mathbf{s}}^T(n)\mathbf{v}(n)$	output of the adaptive secondary path modeling filter;
$f(n) = e(n) - \hat{v}'(n)$	estimation error of the secondary path;
$\mathbf{y}(n) = [y(n), y(n-1), \ldots, y(n-M+1)]^T$	data vector with the last M samples of $y(n)$;
$\hat{y}'(n) = \hat{\mathbf{s}}^T(n)\mathbf{y}(n)$	internal estimate of the canceling signal;
$\hat{d}(n) = f(n) + y'(n)$	internal estimate of the primary disturbance signal;
$\mathbf{x}_M(n) = [x(n), x(n-1), \ldots, x(n-M+1)]^T$	data vector with the last M samples of $x(n)$;
$\hat{x}'(n) = \hat{\mathbf{s}}^T(n)\mathbf{x}_M(n)$	reference signal filtered $\hat{\mathbf{s}}(n)$;
$\hat{\mathbf{x}}'(n) = [\hat{x}'(n), \hat{x}'(n-1), \ldots, \hat{x}'(n-N+1)]^T$	data vector with the last N samples of $\hat{x}'(n)$;
$\hat{d}'(n) = \mathbf{w}^T(n)\hat{\mathbf{x}}'(n)$	output of the *dummy* adaptive noise control filter;
$g(n) = \hat{d}(n) - \hat{d}'(n)$	estimation error of the noise control filter.

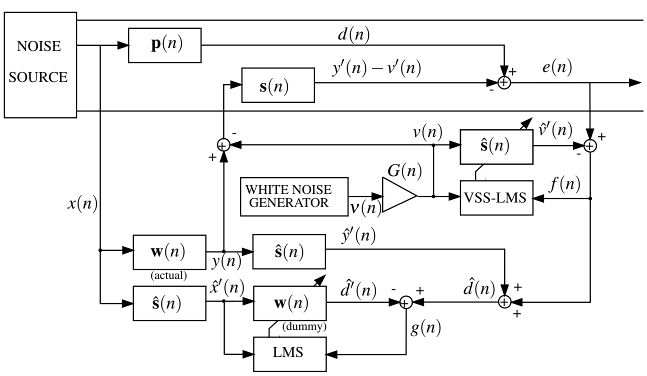


Figure 1: The ANC system with secondary path modeling of Akhtar, Abe, and Kawamata.

erties of the noise control filter and to avoid the use of a noise suppresser in the estimation of the secondary path. The adaptation algorithm for the noise control filter was called in [8] "modified filtered-x algorithm" and it adapts $\mathbf{w}(n)$ with the following rule:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_w g(n)\hat{\mathbf{x}}'(n), \quad (1)$$

where μ_w is a fixed step-size parameter.

The secondary path is estimated from the zero mean white Gaussian auxiliary noise $v(n)$ injected in the secondary path. In [6] the secondary path modeling filter $\hat{\mathbf{s}}(n)$ is adapted with a variable step-size LMS algorithm with the following adaptation rule:

$$\hat{\mathbf{s}}(n+1) = \hat{\mathbf{s}}(n) + \mu_s(n) f(n)\mathbf{v}(n), \quad (2)$$

where $\mu_s(n)$ is the variable step-size parameter. This parameter is varied between a minimum value $\mu_{s_{\min}}$ and a maximum value $\mu_{s_{\max}}$ (with $\mu_{s_{\min}}$ and $\mu_{s_{\max}}$ determined experimentally) on the basis of the ratio $\rho(n)$ between the power of the error signal $f(n)$ and the power of the error microphone signal $e(n)$,

$$\rho(n) = \frac{P_f(n)}{P_e(n)}, \quad (3)$$

with

$$P_f(n) = \lambda P_f(n-1) + (1-\lambda) f^2(n), \quad (4)$$

$$P_e(n) = \lambda P_e(n-1) + (1-\lambda) e^2(n), \quad (5)$$

and λ a forgetting factor close to 1. The variable step-size parameter $\mu_s(n)$ is computed as follows:

$$\mu_s(n) = \rho(n)\mu_{s_{\min}} + (1-\rho(n))\mu_{s_{\max}}. \quad (6)$$

The choice of this variable step-size parameter was heuristically motivated in [6] with the fact that in the early phases of adaptation of the ANC system (when $y'(n)$ is close to zero) the convergence of the secondary path model is degraded by the large disturbance at the error microphone. Thus, a small step-size $\mu_s(n)$ should be chosen. With the convergence of the active noise control filter the disturbance reduces and a larger step-size can be used in the secondary path model adaptation. It is shown in [6] that $\rho(n) \simeq 1$ in the early phases of the ANC system adaptation, while $\rho(n) \simeq 0$ when the ANC system is converged.

In [6] the auxiliary noise power was kept constant ($G(n) = G$, with G constant) and it was chosen by compromising between the contrasting needs of fast convergence of the secondary path (which benefits from a large auxiliary noise power) and of low steady state residual noise (which requires a low auxiliary noise power). A scheduling strategy for the auxiliary noise was later introduced in [11]. This scheduling strategy provides a large auxiliary noise power in the early phases of adaptation of the ANC system, and a much lower

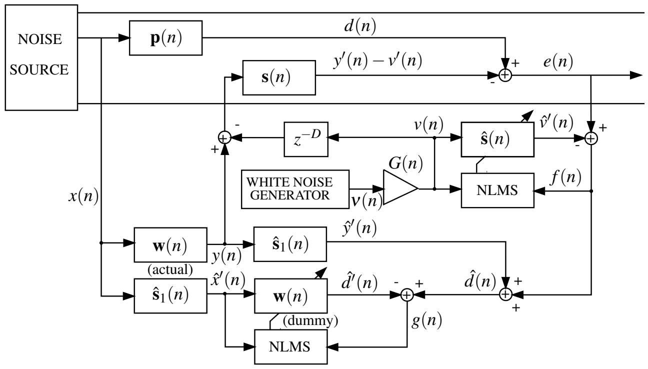


Figure 2: The ANC system modified with the introduction of the delay coefficient technique.

power at steady state. In particular, the auxiliary noise power is varied between a maximum value $\sigma^2_{v_{\max}}$ and a minimum value $\sigma^2_{v_{\min}}$ (with $\sigma^2_{v_{\max}}$ and $\sigma^2_{v_{\min}}$ determined experimentally) by varying the gain $G(n)$ with the following rule:

$$G(n) = \sqrt{\rho(n)\sigma^2_{v_{\max}} + (1-\rho(n))\sigma^2_{v_{\min}}}, \quad (7)$$

where $\rho(n)$ is defined in (3).

A self tuning power scheduling for the auxiliary noise was proposed by the authors of this paper in [12]. This auxiliary noise power scheduling keeps approximately constant the ratio R between the power of the residual noise $d(n) - y'(n)$ and the power of auxiliary noise at the error microphone $v'(n)$. It sets

$$G(n) = \sqrt{\frac{P_e(n)}{(R+1)P_s(n)}} \quad (8)$$

where $P_e(n)$ is an estimate of the power of $e(n)$, given by (5), and $P_s(n)$ is an exponentially smoothed estimate of $\hat{\mathbf{s}}^T(n)\hat{\mathbf{s}}(n)$,

$$P_s(n) = \lambda P_s(n-1) + (1-\lambda)\hat{\mathbf{s}}^T(n)\hat{\mathbf{s}}(n). \quad (9)$$

3. OPTIMAL VARIABLE STEP-SIZE PARAMETERS

Ideal optimal variable step-size parameters

In order to derive the optimal variable step-size parameters we replace the adaptation rules of (1) and (2) with the following NLMS adaptations

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_w(n)\frac{g(n)\hat{\mathbf{x}}'(n)}{\hat{\mathbf{x}}'^T(n)\hat{\mathbf{x}}'(n)}, \quad (10)$$

$$\hat{\mathbf{s}}(n+1) = \hat{\mathbf{s}}(n) + \mu_s(n)\frac{f(n)\mathbf{v}(n)}{\mathbf{v}^T(n)\mathbf{v}(n)}, \quad (11)$$

where $\mu_w(n)$ and $\mu_s(n)$ are variable step-size parameters.

Let $\mathbf{w}_o(n)$ be the minimum-mean-square (MMS) solution of the ANC problem of memory length N [1], and $\mathbf{s}(n)$ the M-sample impulse response of the secondary path. When the ANC system is convergent for $n \to +\infty$, $\mathbf{w}(n) \to \mathbf{w}_o(n)$ and $\hat{\mathbf{s}}(n) \to \mathbf{s}(n)$. At time n the errors in the noise control filter and in the secondary path modeling filter, respectively, are given by

$$\mathbf{m}(n) = \mathbf{w}_o(n) - \mathbf{w}(n), \quad (12)$$

$$\mathbf{r}(n) = \mathbf{s}(n) - \hat{\mathbf{s}}(n). \quad (13)$$

By following the approach of [9], it can be proved that the optimal step-size parameters that maximize the convergence speed of the ANC system are given by the following equations:

$$\mu_w(n) = \frac{E[g(n)\mathbf{m}^T(n)\hat{\mathbf{x}}'(n)]}{E[g^2(n)]}, \quad (14)$$

$$\mu_s(n) = \frac{E[f(n)\mathbf{r}^T(n)\mathbf{v}(n)]}{E[f^2(n)]}. \quad (15)$$

Since $f(n) = e(n) - \hat{v}'(n) = d(n) - y'(n) + \mathbf{r}^T(n)\mathbf{v}(n)$ and $v(n)$ is uncorrelated with $x(n)$ and $d(n)$, (15) can be rewritten as follows:

$$\mu_s(n) = \frac{E[(\mathbf{r}^T(n)\mathbf{v}(n))^2]}{E[f^2(n)]}. \quad (16)$$

Moreover, by assuming $\mathbf{r}(n)$ and $\mathbf{v}(n)$ to be independent, (16) can also be written as

$$\mu_s(n) = \frac{E[(\mathbf{r}^T(n)\mathbf{r}(n)\mathbf{v}^T(n)\mathbf{v}(n))/M]}{E[f^2(n)]}. \quad (17)$$

Practical estimation of the optimal variable step-size parameters

In (14) and (17), the expectations $E[g^2(n)]$ and $E[f^2(n)]$ can be evaluated with exponentially smoothed estimates of the power of $g(n)$ and of $f(n)$. $E[g^2(n)]$ can be evaluated as follows:

$$P_g(n) = \lambda P_g(n-1) + (1-\lambda)g^2(n), \quad (18)$$

and $E[f^2(n)]$ with (4).

Let us first discuss the evaluation of (17). For its evaluation we have to estimate the Euclidean norm of the system error $\mathbf{r}(n)$, i.e., the system distance $\|\mathbf{r}(n)\|^2 = \|\mathbf{s}(n) - \hat{\mathbf{s}}(n)\|^2$. An accurate estimate of $\mathbf{r}(n)$ can be obtained with the delay coefficient technique [9]. The delay coefficient technique derives from the observation that the system error tends to have a uniform distribution on its coefficients [9]. By delaying by D samples the internally generated signal $v(n)$ sent to the secondary path, as shown in Figure 2, the steady-state value of the first D coefficients of the secondary path modeling filter becomes 0, and the instantaneous value of the first D delay coefficients can be used for estimating the system distance.

For applying the delay coefficient technique we consider from now on a secondary path modeling filter $\hat{\mathbf{s}}(n)$ of memory length $M + D$. The collection of the first D coefficients of this filter is indicated with $\hat{\mathbf{s}}_0(n)$ and it is used for estimating the system distance at time n as follows [9]:

$$\|\mathbf{r}(n)\|^2 \simeq \frac{M}{D}\hat{\mathbf{s}}_0^T(n)\hat{\mathbf{s}}_0(n) \quad (19)$$

The collection of the last M coefficients of $\hat{\mathbf{s}}(n)$ is indicated with $\hat{\mathbf{s}}_1(n)$ and it is the true estimate of the secondary path impulse response. Thus, as shown in Figure 2, $\hat{\mathbf{s}}_1(n)$ is used for computing the delay compensation signal $\hat{y}'(n)$ and the filtered reference signal $\hat{x}'(n)$. $\hat{\mathbf{s}}_0(n)$ and $\hat{\mathbf{s}}_1(n)$ are adapted with the NLMS algorithm as follows:

$$\hat{\mathbf{s}}_0(n+1) = \hat{\mathbf{s}}_0(n) + \mu_s(n)\frac{f(n)\mathbf{v}_0(n)}{\mathbf{v}^T(n)\mathbf{v}(n)}, \quad (20)$$

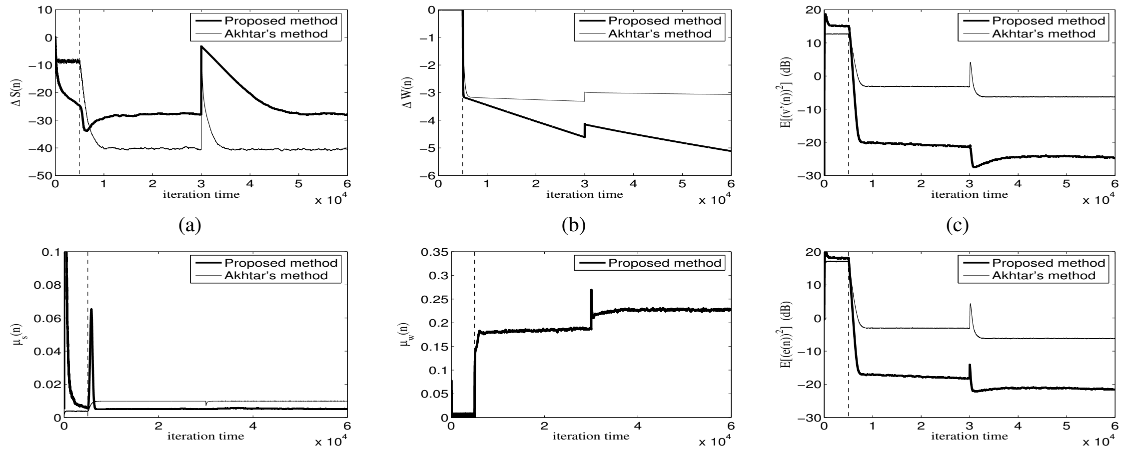
$$\hat{\mathbf{s}}_1(n+1) = \hat{\mathbf{s}}_1(n) + \mu_s(n)\frac{f(n)\mathbf{v}_1(n)}{\mathbf{v}^T(n)\mathbf{v}(n)}, \quad (21)$$

where $\mathbf{v}(n) = [v(n), v(n-1), \ldots, v(n-D-M+1)]^T$, $\mathbf{v}_0(n) = [v(n), v(n-1), \ldots, v(n-D+1)]^T$, $\mathbf{v}_1(n) = [v(n-D), v(n-D-1), \ldots, v(n-D-M+1)]^T$. In (20) and (21) the optimal variable step-size of (17) is approximated as follows:

$$\mu_s(n) = \begin{cases} \frac{\hat{N}_S(n)}{P_f(n)} & \text{when } \frac{\hat{N}_S(n)}{P_f(n)} > \mu_{s_{\min}} \\ \mu_{s_{\min}} & \text{otherwise} \end{cases} \quad (22)$$

where $\hat{N}_S(n)$ is an exponentially smoothed estimate of $E[(\mathbf{r}^T(n)\mathbf{r}(n)\mathbf{v}^T(n)\mathbf{v}(n))/M]$ obtained by using the delay coefficient technique,

$$\hat{N}_S(n) = \lambda\hat{N}_S(n-1) + (1-\lambda)(\hat{\mathbf{s}}_0^T(n)\hat{\mathbf{s}}_0(n)\mathbf{v}^T(n)\mathbf{v}(n))/D, \quad (23)$$



(a) (b) (c) (d) (e) (f)

Figure 3: Performance comparison between the proposed method and the Akhtar's method with a sudden change in the acoustic paths.

and $\mu_{s_{\min}}$ is the minimum value of the step-size parameter, used for avoiding the adaptation freezing in case of a sudden change of the secondary path impulse response [9].

The estimation of the numerator of $\mu_w(n)$ in (14) is more difficult. We cannot apply the delay coefficient technique for estimating the system distance $\|\mathbf{m}(n)\|^2$. Indeed, the control filter acts as a predictor of the input signals [13] and, even if we introduce a delay in the error microphone signal, the steady-state value of the first coefficients is never zero. A reasonable heuristic estimate of the system error $\mathbf{m}(n)$ is given by:

$$\hat{\mathbf{m}}(n) = \hat{\lambda}\hat{\mathbf{m}}(n-1) + (1-\hat{\lambda})\frac{g(n)\hat{\mathbf{x}}'(n)}{\hat{\mathbf{x}}'^T(n)\hat{\mathbf{x}}'(n)}, \qquad (24)$$

i.e., the exponentially smoothed value of $\mathbf{w}(n+1)-\mathbf{w}(n)$ when $\mu_w(n)=1$. The validity of this choice has been confirmed by the results of extensive simulations. The forgetting factor $\hat{\lambda}$ in our experiments was usually taken much lower than the forgetting factor λ used for $P_f(n)$, $P_g(n)$ and $\hat{N}_s(n)$, and it was chosen in the range $[0.6, 0.9]$. By estimating $\mathbf{m}(n)$ with (24), we can approximate the optimal step-size $\mu_w(n)$ as follows:

$$\mu_w(n) = \frac{\hat{N}_w(n)}{P_g(n)}, \qquad (25)$$

where $\hat{N}_w(n)$ is the exponentially smoothed estimate of $E[g(n)\mathbf{m}^T(n)\hat{\mathbf{x}}'(n)]$ given by:

$$\hat{N}_w(n) = \lambda\hat{N}_w(n-1) + (1-\lambda)g(n)\hat{\mathbf{m}}^T(n)\hat{\mathbf{x}}'(n). \qquad (26)$$

It should be noted that no good result can be obtained with the ANC system equipped with the estimated optimal step-size parameters of (22) and (25) without a sufficiently high auxiliary noise during the initial convergence of the ANC system. Thus, in order to not penalize the residual noise at steady state the auxiliary noise power scheduling of (8) has been used in all experimental results.

4. EXPERIMENTAL RESULTS

In this section we provide some experimental results for the ANC system equipped with the optimal step-size parameters of (22) and (25) using the auxiliary noise power scheduling of (8). The resulting system performances are compared with those of the ANC system equipped with the variable step-size parameter of (6) and the auxiliary noise power scheduling of (7). In the authors opinion, the last system is the most performant ANC system presently available in the literature.

We consider the same experimental conditions of the second experiment of [6]. The sampling frequency is 2 kHz. The reference signal $x(n)$ is a multi-tonal signal with frequencies 100 Hz, 200 Hz, 300 Hz and 400 Hz and variance 2.0. The signal is corrupted with a zero-mean white Gaussian noise till a 30 dB SNR. The control filter and the secondary path modeling filter have memory lengths $N = 32$ and $M = 16$, respectively.

We consider two set of experiments. In the first set of experiments we study the behavior of the ANC system in presence of a strong variation in the impulse response of the primary and secondary acoustic paths. In particular, in the first 30000 simulation samples we assume to have impulse responses of the primary path $\mathbf{p}_1(n)$ and of the secondary path $\mathbf{s}_1(n)$ which have been obtained by truncating the impulse responses reported in the companion disk of [1] to $L = 48$ and $M = 16$ taps, respectively. In the second 30000 simulation samples we assume to have impulse response of the primary path $\mathbf{p}_2(n)$ and of the secondary path $\mathbf{s}_2(n)$ given by:

$$\mathbf{p}_2(n) = [\frac{2}{3}p_1(0), \frac{2}{3}p_1(1)+\frac{1}{3}p_1(0), \ldots, \frac{2}{3}p_1(L)+\frac{1}{3}p_1(L-1)]^T,$$

$$\mathbf{s}_2(n) = [\frac{2}{3}s_1(0), \frac{2}{3}s_1(1)+\frac{1}{3}s_1(0), \ldots, \frac{2}{3}s_1(M)+\frac{1}{3}s_1(M-1)]^T.$$

In our experimental set-up we assume that the secondary path cannot be modelled off-line by switching off the primary noise source, but it must be modelled online with the noise source $x(n)$ active. Thus, we consider two phases of operation of the ANC system. In the first phase we keep inactive the control filter and we adapt the secondary path modeling filter to obtain a first estimate of the secondary path. In the second phase, we operate the ANC system by adapting both the secondary path modeling filter and the control filter. The duration of the first phase was tuned to the minimum duration that guarantees a stable operation of the ANC system.

The performance comparison of the ANC systems is done on the basis of different performance measures: the power of the error microphone signal ($E[e^2(n)]$), the power of the residual noise ($E[(d(n)-y'(n))^2]$), the relative modeling error of the secondary path, defined as $\Delta S(n) = 10\log_{10}\left[\frac{\|\mathbf{s}(n)-\hat{\mathbf{s}}(n)\|^2}{\|\mathbf{s}(n)\|^2}\right]$, and the relative modeling error of the control filter, defined as $\Delta W(n) = 10\log_{10}\left[\frac{\|\mathbf{w}_o(n)-\mathbf{w}(n)\|^2}{\|\mathbf{w}_o(n)\|^2}\right]$, with $\mathbf{w}_o(n)$ the MMS optimal control filter [1], which has been a priori determined.

In the ANC system equipped with the optimal step-size parameters, the parameters have been set as follows: number of delay coefficients $D = 8$, the minimum step-size parameter for $\mu_s(n)$ set to 0.005, $\lambda = 0.99$, $\hat{\lambda} = 0.8$, and desired ratio $R = 1$. Thus, in every condition we enforce the power of $v'(n)$, to be equal to the power of $d(n)-y'(n)$. In the ANC system equipped with the variable step-size parameter of (6), the parameter settings are the same of [11], i.e., $\mu_{s_{\min}} = 10^{-3}$, $\mu_{s_{\max}} = 10^{-2}$, $\mu_w = 10^{-5}$, and $\lambda = 0.99$. Moreover, in (7) we choose $\sigma^2_{v_{\max}} = 4$ and $\sigma^2_{v_{\min}} = 0$. With this choice,

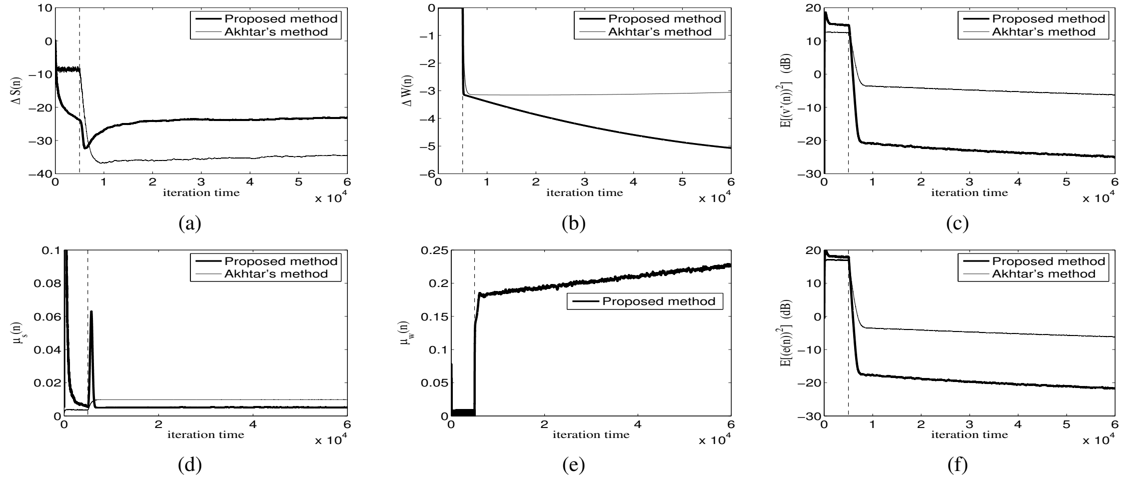


Figure 4: Performance comparison between the proposed method and the Akhtar's method with a slow change in the acoustic paths.

during the initialization phase the auxiliary noise power is almost equal to that obtained with the power scheduling of (8).

Figures 3 provides the performance comparison of the two systems in case of a strong variation of the impulse response of the acoustic paths. In these figures, plot (a) diagrams the system distance $\Delta S(n)$, plot (b) the system distance $\Delta W(n)$, plot (c) the evolution of the power of the auxiliary noise at the error microphone $v'(n)$, plot (d) the evolution of the step-size $\mu_s(n)$, plot (e) the evolution of the step-size $\mu_w(n)$, plot (f) the evolution of the power of the error microphone signal $e(n)$. The plots have been obtained with ensemble averages over 100 runs of the system.

From there figures in the first 30000 samples we notice a strong performance improvement with the proposed method in the convergence speed of the algorithm, in the estimation accuracy of the optimal controller, and in the power of error microphone signal. The lower accuracy in the estimation of the secondary path modeling filter is caused by the lower power of the auxiliary noise with the proposed power scheduling compared with that of [11]. However the estimation accuracy of $s(n)$ is still very good and it does not affect the estimation of the optimal controller. After the first 30000 samples, because of the higher power of auxiliary noise, the system equipped with the variable step-size parameter of (6) responds more rapidly to the variation of the secondary path. Nevertheless, the system equipped with the optimal step-size parameters still provides a better accuracy in the estimation of the optimal controller and a much lower power of the error microphone signal $e(n)$.

In the second set of experiments we study the behavior of the ANC system in presence of a slow variation in the impulse response of the primary and secondary acoustic paths. In particular we assume that the impulse responses of the primary and secondary paths evolve with a linear law from $\mathbf{p}_1(n)$ to $\mathbf{p}_2(n)$ and from $\mathbf{s}_1(n)$ to $\mathbf{s}_2(n)$, respectively. Figure 4 provides the performance comparison of the two systems. The plots are the same of the previous experiment and they have been obtained with ensemble averages over 100 runs. Despite the variation in the primary and secondary paths, in the system equipped with the optimal step-size parameters the control filter converges rapidly to the optimal control filter. On the contrary, the system equipped with the variable step-size parameter of (6) has more difficulties in tracking the variation of the optimal controller.

Similar experimental results have been obtained also with tonal and broadband input noises.

5. CONCLUSION

In this paper we have discussed the choice of optimal step-size parameters for the adaptation algorithms of the noise control filter and of the secondary path modeling filter of the ANC system described in [6]. It should be noted that with our method, the number of parameters to be tuned is strongly reduced in comparison to other similar solutions proposed in the literature [2], [3], [4], [5], [6].

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