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CORRELATION AND KULLBACK MATCHING APPROACHES: APPLICATION TO BLIND CHANNEL ESTIMATION IN OSTBC SYSTEMS

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ABSTRACT

In many practical parameter estimation problems, the statistical properties of the sources can be exploited to improve the quality of the estimates. In this paper we consider the correlation and Kullback matching criteria (CM and KM respectively), which are applied to the problem of blind channel estimation under orthogonal space-time block coded (OSTBC) transmissions. Specifically, it is shown that the special OSTBC structure provides straightforward closed form solutions, which reduce to the extraction of the principal eigenvector of the observation correlation matrix modified by the code matrices and a set of weighting factors. Additionally, we prove that the KM technique is equivalent to the CM approach for low SNRs, and to the relaxed blind maximum likelihood (ML) decoder for high SNRs. Finally, the performance of the proposed techniques is illustrated by means of several simulation examples.

1. INTRODUCTION

There is a large number of parameter estimation problems where the statistical properties of the inputs can be exploited to obtain accurate parameter estimates. Two well-known criteria based on this idea are the correlation and Kullback matching approaches (CM and KM respectively). On one hand, the CM approach amounts to minimizing the Euclidian distance between the empirical and theoretical correlation matrices of the observations. This technique has been applied in blind channel estimation and equalization problems [1–3], and under certain assumptions, it asymptotically provides the unbiased estimator with minimum variance [1, 2]. On the other hand, the KM criteria stems from the field of information geometry [4, 5], and it amounts to minimizing the Kullback-Leibler divergence between the empirical and theoretical pdfs of the observations, which is closely related with the maximum-likelihood (ML) estimation problem [5–8]. However, in general both criteria result in non-linear optimization problems, which must be solved by means of numerical methods.

In this paper we apply the CM and KM approaches to the problem of blind channel estimation under orthogonal space time block coded (OSTBC) transmissions [9], showing that the special OSTBC structure permits the solution of both problems in closed form. Specifically, the CM and KM criteria reduce to the extraction of the main eigenvector of a matrix, which is obtained from the correlation matrix of the observations, the OSTBC code matrices, and a set of weighting factors. In the CM case, the weights are fixed and given by the eigenvalues of the source correlation matrix, whereas in the KM case the weights not only depend on the source eigenvalues, but also on the signal to noise ratio (SNR). Additionally, it is shown that, in the case of uncorrelated and equipower sources, both techniques reduce to the relaxed blind ML decoder.

The analysis of the KM weights permits a straightforward interpretation of the proposed techniques. On one hand, in the high noise regime, the KM technique is equivalent to the CM criterion, i.e., the channel state information is extracted from the previous knowledge of the source correlation matrix. On the other hand, in the low noise regime the KM technique is asymptotically equivalent to the relaxed blind ML decoder, which means that the channel is not extracted from the estimates of the correlation matrices, which can be inaccurate due to the finite sample problem, but from the congruence of the data model. This suggests that the best results should be provided by the KM criterion, which is corroborated by means of some numerical examples.

2. OSTBC DATA MODEL AND MAIN ASSUMPTIONS

2.1 Notation and OSTBC Data Model

Throughout this paper we will use bold-faced upper case letters to denote matrices, bold-faced lower case letters for column vector, and light-faced lower case letters for scalar quantities. The superscript $\hat{(\cdot)}$ will denote estimated matrices, vectors or scalars. The trace and Frobenius norm of matrix $\mathbf{A}$ will be denoted as $\mathrm{Tr}(\mathbf{A})$ and $\|\mathbf{A}\|$, respectively, $\mathrm{vec}(\mathbf{A})$ will denote the columnwise vectorized form of matrix $\mathbf{A}$, and $\mathrm{diag}(\mathbf{a})$ will denote the diagonal matrix defined by vector $\mathbf{a}$. Finally, the identity and zero matrices of the required dimensions will be denoted as $\mathbf{I}$ and $\mathbf{0}$, respectively.

Let us assume a flat fading multiple-input multiple-output (MIMO) system with n_T transmit and n_R receive antennas, and affected by a zero mean i.i.d. complex Gaussian noise with variance σ^2. The MIMO channel, which remains constant during the transmission of N OSTBC blocks, will be represented by the matrix $\mathbf{H} \in \mathbb{C}^{n_T \times n_R}$. Thus, using the notation in [10–12], and assuming an OSTBC transmitting M symbols during L time slots (transmission rate $R = M/L$), the observations associated to the n-th OSTBC block can be represented by the following real data model

$$\tilde{\mathbf{y}}[n] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{s}[n] + \tilde{\mathbf{n}}[n], \tag{1}$$

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where $\tilde{\mathbf{y}}[n] \in \mathbb{R}^{2Ln_R \times 1}$ contains the real and imaginary parts of the observations, $\mathbf{s}[n] = [s_1[n], \ldots, s_{M'}[n]]^T$ contains the M' ($M' = 2M$ in the general case of complex OSTBCs) real information symbols transmitted in the n-th block, $\tilde{\mathbf{n}}[n] \in \mathbb{R}^{2Ln_R \times 1}$ is a real i.i.d. Gaussian noise vector with variance $\sigma^2/2$, and $\tilde{\mathbf{W}}(\mathbf{H}) \in \mathbb{R}^{2Ln_R \times M'}$ is the equivalent channel, whose k-th column is given by $\tilde{\mathbf{w}}_k(\mathbf{H}) = \tilde{\mathbf{D}}_k \tilde{\mathbf{h}}$, with $\tilde{\mathbf{h}} = \mathrm{vec}(\begin{bmatrix}\Re^T(\mathbf{H}) & \Im^T(\mathbf{H})\end{bmatrix}^T)$,

$$\tilde{\mathbf{D}}_k = \underbrace{\begin{bmatrix} \tilde{\mathbf{C}}_k & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \tilde{\mathbf{C}}_k \end{bmatrix}}_{2Ln_R \times 2n_T n_R}, \qquad \tilde{\mathbf{C}}_k = \underbrace{\begin{bmatrix} \Re(\mathbf{C}_k) & -\Im(\mathbf{C}_k) \\ \Im(\mathbf{C}_k) & \Re(\mathbf{C}_k) \end{bmatrix}}_{2L \times 2n_T},$$

and where $\mathbf{C}_k \in \mathbb{C}^{L \times n_T}$ ($k = 1, \ldots, M'$) are the OSTBC code matrices. These matrices fulfill the following orthogonality property

$$\tilde{\mathbf{W}}^T(\mathbf{H})\tilde{\mathbf{W}}(\mathbf{H}) = \|\mathbf{H}\|^2 \mathbf{I}, \quad \forall \mathbf{H}, \tag{2}$$

which, under perfect channel state information (CSI), reduces the complexity of the maximum likelihood (ML) receiver to a matched filter followed by a symbol by symbol detector.

$$\hat{\mathbf{s}}_{\mathrm{ML}}[n] = \frac{\tilde{\mathbf{W}}^T(\mathbf{H})\tilde{\mathbf{y}}[n]}{\|\mathbf{H}\|^2}.$$

2.2 Assumption on the Correlation of the Sources

During this paper we will assume that the correlation matrix $\mathbf{\Lambda_s} = E[\mathbf{s}[n]\mathbf{s}^T[n]]$ of the sources is known and diagonal with elements $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{M'}$. However, we must note that this is not a restrictive condition. In the case of correlation matrices of the general form $\mathbf{R_s} = \mathbf{Q_s}\mathbf{\Lambda_s}\mathbf{Q_s}^T$ (with $\mathbf{Q_s}$ an orthogonal matrix), the data model in (1) can be rewritten as $\tilde{\mathbf{y}} = \tilde{\mathbf{W}}'(\mathbf{H})\mathbf{s}'[n] + \tilde{\mathbf{n}}[n]$, where $\tilde{\mathbf{W}}'(\mathbf{H}) = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{Q_s}$ is the equivalent channel of a modified OSTBC [10,12], and $\mathbf{s}'[n] = \mathbf{Q_s}^T\mathbf{s}[n]$ is a rotated information vector with $E\left[\mathbf{s}'[n]\mathbf{s}'^T[n]\right] = \mathbf{\Lambda_s}$.

3. PROPOSED APPROACHES

In this section, the problem of blind channel estimation from second-order statistics (SOS) is solved by means of the correlation and Kullback matching (CM and KM) criteria. Interestingly, due to the orthogonality property (2), both criteria lead to closed form solutions obtained by solving an eigenvalue (EV) problem. Due to the space limitation, here we only consider the problem of estimating the channel up to a real scale factor. However, as it will be shown in a forthcoming paper [13], accurate estimates of the channel norm and the noise variance can be easily obtained as well by the proposed CM and KM criteria.

3.1 Correlation Matching (CM)

The correlation matching criterion for the estimate of the channel $\hat{\mathbf{H}}$ amounts to minimizing the Euclidian distance between the theoretical correlation matrix of the observations

$$\mathbf{R}_{\tilde{\mathbf{y}}} = \tilde{\mathbf{W}}(\hat{\mathbf{H}})\mathbf{\Lambda_s}\tilde{\mathbf{W}}^T(\hat{\mathbf{H}}) + \frac{\sigma^2}{2}\mathbf{I}, \tag{3}$$

which depends on the parameter $\hat{\mathbf{H}}$, and its finite sample estimate[1]

$$\hat{\mathbf{R}}_{\tilde{\mathbf{y}}} = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{\mathbf{y}}[n]\tilde{\mathbf{y}}^T[n].$$

Therefore, the optimization problem is

$$\arg\min_{\hat{\mathbf{H}}} \left\| \hat{\mathbf{R}}_{\tilde{\mathbf{y}}} - \tilde{\mathbf{W}}(\hat{\mathbf{H}})\mathbf{\Lambda_s}\tilde{\mathbf{W}}^T(\hat{\mathbf{H}}) - \frac{\sigma^2}{2}\mathbf{I} \right\|^2,$$

which after some straightforward but tedious algebra can be rewritten as

$$\arg\max_{\hat{\tilde{\mathbf{h}}}} \left(\hat{\tilde{\mathbf{h}}}^T \mathbf{\Phi}^{\mathrm{CM}} \hat{\tilde{\mathbf{h}}} - \frac{\sigma^2}{2}\|\hat{\tilde{\mathbf{h}}}\|^2 \mathrm{Tr}(\mathbf{\Lambda_s}) - \frac{1}{2}\|\mathbf{\Lambda_s}\|^2 \|\hat{\tilde{\mathbf{h}}}\|^4 \right), \tag{4}$$

where

$$\mathbf{\Phi}^{\mathrm{CM}} = \sum_{k=1}^{M'} \rho_k^{\mathrm{CM}} \tilde{\mathbf{D}}_k^T \hat{\mathbf{R}}_{\tilde{\mathbf{y}}} \tilde{\mathbf{D}}_k, \tag{5}$$

and the weights ρ_k^{CM} are directly given by the source eigenvalues, i.e., $\rho_k^{\mathrm{CM}} = \lambda_k$. Thus, solving (4) w.r.t. $\hat{\tilde{\mathbf{h}}}$ yields the EV problem

$$\mathbf{\Phi}^{\mathrm{CM}} \hat{\tilde{\mathbf{h}}} = \beta^{\mathrm{CM}} \hat{\tilde{\mathbf{h}}},$$

where $\beta^{\mathrm{CM}} = \frac{\sigma^2}{2}\mathrm{Tr}(\mathbf{\Lambda_s}) + \|\mathbf{\Lambda_s}\|^2 \|\hat{\tilde{\mathbf{h}}}\|^2$ is the largest eigenvalue of $\mathbf{\Phi}^{\mathrm{CM}}$. Therefore, the CM estimate of the normalized channel $\hat{\tilde{\mathbf{h}}}/\|\hat{\tilde{\mathbf{h}}}\|$ is obtained as the eigenvector associated to the largest eigenvalue of $\mathbf{\Phi}^{\mathrm{CM}}$, whereas the channel energy $\|\hat{\tilde{\mathbf{h}}}\|$ can be easily recovered from the eigenvalue β^{CM}.

3.2 Kullback Matching (KM)

In this subsection, the blind channel estimation problem is analyzed from an information geometric point of view. Specifically, we propose to minimize the Kullback-Leibler (KL) divergence between the empirical $\hat{p}(\tilde{\mathbf{y}}[n])$ and theoretical $p(\tilde{\mathbf{y}}[n])$ pdfs of the observations

$$D(\hat{p}|p) = \int_{\tilde{\mathbf{y}}[n]} \hat{p}(\tilde{\mathbf{y}}[n]) \log \frac{\hat{p}(\tilde{\mathbf{y}}[n])}{p(\tilde{\mathbf{y}}[n])} d\tilde{\mathbf{y}}[n],$$

which is closely related to the ML estimation of the parameters [4,5].

Assuming that the observations follow a zero-mean Gaussian distribution, the KL divergence can be rewritten in closed form as [6,7]

$$D(\hat{p}|p) = \frac{1}{2}\mathrm{Tr}\left(\mathbf{R}_{\tilde{\mathbf{y}}}^{-1}\hat{\mathbf{R}}_{\tilde{\mathbf{y}}} - \mathbf{I}\right) - \frac{1}{2}\log\left|\det\left(\mathbf{R}_{\tilde{\mathbf{y}}}^{-1}\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}\right)\right|.$$

Furthermore, although the Gaussian assumption is only strictly correct in the asymptotic cases of $\sigma^2 \to \infty$ or $M' \to \infty$ independent sources, it has been recently proven [8] that, under multilevel constellations, the above criterion provides (asymptotically as $\sigma^2 \to 0$) the optimum second-order estimator [7].

In the OSTBC case, the KM criterion can be easily simplified. In particular, taking (2) and (3) into account, we ob-

[1] We assume that the MIMO channel remains constant during the transmission of N OSTBC blocks.

tain

$$\mathbf{R}_{\tilde{\mathbf{y}}}^{-1} = \left(\frac{\sigma^2}{2}\right)^{-1}\mathbf{I} - \left(\frac{\sigma^2}{2}\right)^{-2}\tilde{\mathbf{W}}(\hat{\mathbf{H}})\mathbf{\Gamma}^{\mathrm{KM}}\tilde{\mathbf{W}}^T(\hat{\mathbf{H}}),$$

where $\mathbf{\Gamma}^{\mathrm{KM}} = \mathrm{diag}([\rho_1^{\mathrm{KM}}, \ldots, \rho_{M'}^{\mathrm{KM}}])$, and

$$\rho_k^{\mathrm{KM}} = \frac{\lambda_k}{1+\lambda_k \frac{\|\hat{\mathbf{H}}\|^2}{\sigma^2/2}}, \qquad k = 1, \ldots, M'. \tag{6}$$

Thus, the problem of minimizing $D(\hat{p}|p)$ can be rewritten as

$$\arg\max_{\hat{\mathbf{h}}} \left[\left(\frac{\sigma^2}{2}\right)^{-2} \hat{\mathbf{h}}^T \mathbf{\Phi}^{\mathrm{KM}} \hat{\mathbf{h}} - \sum_{k=1}^{M'} \log\left(\lambda_k \|\hat{\mathbf{h}}\|^2 + \frac{\sigma^2}{2}\right)\right],$$

where

$$\mathbf{\Phi}^{\mathrm{KM}} = \sum_{k=1}^{M'} \rho_k^{\mathrm{KM}} \tilde{\mathbf{D}}_k^T \hat{\mathbf{R}}_{\tilde{\mathbf{y}}} \tilde{\mathbf{D}}_k, \tag{7}$$

and analogously to the CM case, the solution $\hat{\hat{\mathbf{h}}}$ is obtained from the EV problem

$$\mathbf{\Phi}^{\mathrm{KM}}\hat{\hat{\mathbf{h}}} = \beta^{\mathrm{KM}}\hat{\hat{\mathbf{h}}},$$

where β^{KM} is the largest eigenvalue of $\mathbf{\Phi}^{\mathrm{KM}}$.

4. DISCUSSION AND RELATED TECHNIQUES

Here we present a brief analysis of the CM and KM estimators, and compare them with the blind channel estimation techniques in [14]. In particular, we must note the following:

- Unlike the CM technique, the KM estimate depends on the ratio $\|\mathbf{H}\|^2/\sigma^2$, which is proportional to the instantaneous signal to noise ratio ($\mathrm{SNR}_{\mathbf{H}}$), through the weights given by (6). However, this value can be easily estimated by the direct analysis of the signal and noise subspaces of $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}$.
- In the case of uncorrelated sources with the same power ($\lambda_k = \lambda, \forall k$), both approaches are identical ($\mathbf{\Phi}^{\mathrm{KM}} \propto \mathbf{\Phi}^{\mathrm{CM}}$), and equivalent to the relaxed blind ML decoder [14], which amounts to minimizing

$$\arg\min_{\hat{\mathbf{H}}, \hat{\mathbf{s}}[n]} \sum_{n=0}^{N-1} \left\| \tilde{\mathbf{y}}[n] - \tilde{\mathbf{W}}(\hat{\mathbf{H}})\hat{\mathbf{s}}[n] \right\|^2. \tag{8}$$

- In some practical cases, such as the Alamouti code and most of the multiple-input single-output (MISO) systems, the relaxed blind ML decoder is affected by a set of indeterminacies which preclude the unambiguous blind channel recovery from second-order statistics (see [11, 12] for a review of the identifiability conditions). In order to solve this problem, in [14] the authors have proposed a linear precoding of the information symbols $\mathbf{s}[n]$ and a weighted version of the relaxed blind ML decoder, whose solution is given by the principal eigenvector of

$$\mathbf{\Phi} = \sum_{k=1}^{M'} \rho_k \tilde{\mathbf{D}}_k^T \hat{\mathbf{R}}_{\tilde{\mathbf{y}}} \tilde{\mathbf{D}}_k, \tag{9}$$

  where the weights $\rho_1 \geq \rho_2 \geq \ldots \geq \rho_{M'}$ are free parameters to be selected by the user. Therefore, the CM and KM matrices (5) and (7) can be viewed as particular cases of (9), which provide two different criteria for the selection of the weights and shed some light into the technique proposed in [14].
- Taking (6) into account, the KM technique can be easily interpreted:
  - In the high noise regime ($\mathrm{SNR}_{\mathbf{H}} \to 0$), $\rho_k^{\mathrm{KM}} = \lambda_k$ and the KM criterion is equivalent to the CM, i.e., both techniques try to extract the channel state information from the prior knowledge of $\mathbf{\Lambda}_{\mathbf{s}}$.
  - In the low noise regime ($\mathrm{SNR}_{\mathbf{H}} \to \infty$), the KM criterion is equivalent to the relaxed blind ML decoder ($\rho_k^{\mathrm{KM}} = \sigma^2/(2\|\mathbf{H}\|^2)$), i.e., the channel state information is no longer extracted from the information about the correlation matrices (which is not exact due to the finite number of observations), but from the congruence between the observations and the data model (see eq. (8)).

5. SIMULATION RESULTS

In this section the performance of the proposed techniques is illustrated by means of some numerical examples. In all the cases the information symbols belong to a QPSK constellation, and they are transmitted with unit power by channel use, which implies an instantaneous signal to noise ratio $\mathrm{SNR}_{\mathbf{H}} = \frac{\|\mathbf{H}\|^2}{n_T n_R \sigma^2}$. The MIMO channel follows a Rayleigh distribution, i.e., each element of $\mathbf{H}$ is a complex Gaussian random variable with zero mean and unit variance. Therefore, the average SNR is defined as $\mathrm{SNR} = 1/\sigma^2$.

The information symbols have been encoded with the complex OSTBC $\mathbf{Z}_3$ proposed in [15, Chapter 3], whose parameters are $n_T = L = 8$ and $M = 4$ ($M' = 8$ and $R = 1/2$), and which is defined by the following transmission matrix

$$\mathbf{S}[n] = \begin{bmatrix} \mathbf{S}_1[n] & \mathbf{S}_2[n] \\ \mathbf{S}_3[n] & -\mathbf{S}_1^H[n] \end{bmatrix},$$

where, omitting the temporal index $[n]$,

$$\mathbf{S}_1 = \frac{1}{2}\begin{bmatrix} s_1+js_2 & s_1+js_2 & s_1+js_2 & s_1+js_2 \\ s_1+js_2 & -s_1+js_2 & s_1+js_2 & -s_1+js_2 \\ s_1+js_2 & s_1+js_2 & -s_1+js_2 & -s_1+js_2 \\ s_1+js_2 & -s_1+js_2 & -s_1+js_2 & s_1+js_2 \end{bmatrix},$$

$$\mathbf{S}_2 = \begin{bmatrix} s_3+js_4 & 0 & s_7+js_6 & s_5+js_8 \\ 0 & s_3+js_4 & -s_5+js_8 & s_7-js_6 \\ -s_7+js_6 & s_5+js_8 & s_3-js_4 & 0 \\ -s_5+js_8 & -s_7-js_6 & 0 & s_3-js_4 \end{bmatrix},$$

$$\mathbf{S}_3 = \begin{bmatrix} s_3-js_8 & s_5-js_6 & s_7-js_4 & 0 \\ -s_5-js_6 & s_3+js_8 & 0 & s_7-js_4 \\ -s_7-js_4 & 0 & s_3+js_8 & -s_5+js_6 \\ 0 & -s_7-js_4 & s_5+js_6 & s_3-js_8 \end{bmatrix}.$$

This code was designed to provide better peak to average power ratio (PAR) than that of the conventional designs for $n_T = 8$ [9]. For this code, it has been proven in [15, Chapter 4] that, in order to optimize the bit error rate (BER) under QPSK constellations and Rayleigh channels, the source correlation matrix should be

$$\mathbf{\Lambda}_{\mathbf{s}} = \mathrm{diag}([3,3,1,1,1,1,1,1])/12,$$

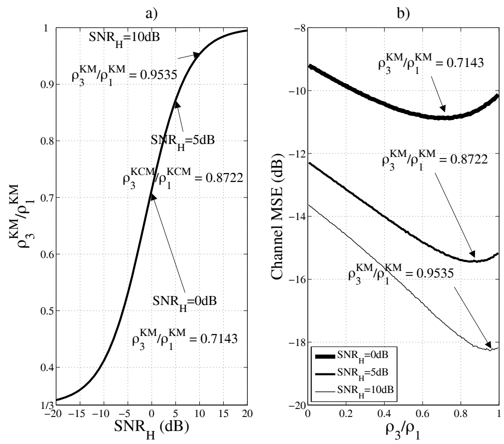

Figure 1: Analysis of the KM criterion. a) Evolution of the ratio $\rho_3^{\text{KM}}/\rho_1^{\text{KM}}$ with the instantaneous SNR. b) MSE in the channel estimate for different ratios ρ_3/ρ_1, $N = 50$.

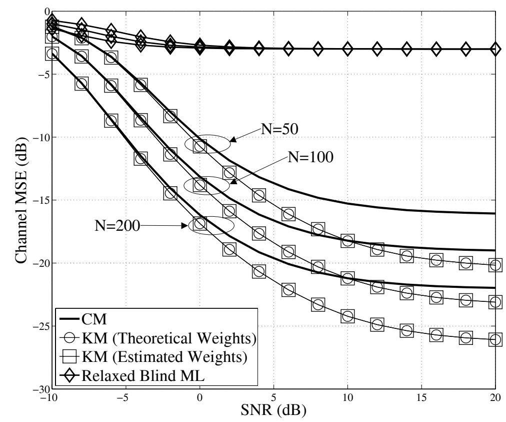

Figure 2: Performance of the proposed techniques in the non-identifiable case ($n_R = 1$). Channel MSE vs. SNR.

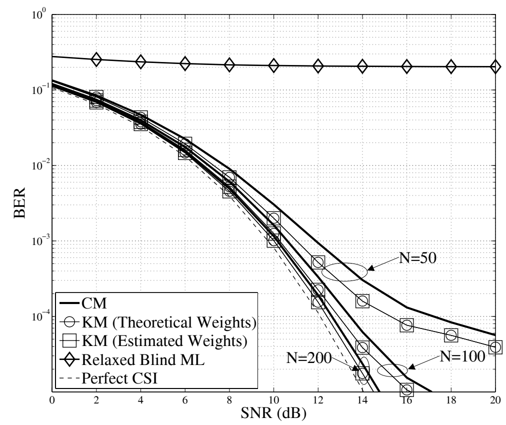

Figure 3: Performance of the proposed techniques in the non-identifiable case ($n_R = 1$). BER vs. SNR.

i.e., the energy of the first complex symbol is three times higher than that of the three remaining symbols. As follows from the CM and KM techniques, this suggests the use of only two different weights, i.e., in all the cases (CM, KM, and the technique in [14]) we select

$$\rho_1 = \rho_2 \qquad \text{and} \qquad \rho_3 = \ldots = \rho_8.$$

Finally, we must note that the results in [11, 12] ensure the blind identifiability of the channel by means of the relaxed blind ML receiver in the case $n_R > 1$. However, when $n_R = 1$ the channel can not be unambiguously extracted without exploiting the correlation properties of the sources.

5.1 First Example: Evolution of the KM Weights

In the first example the performance of the KM criterion is analyzed in the case $n_R = 1$. Fig. 1.a shows the evolution of the ratio $\rho_3^{\text{KM}}/\rho_1^{\text{KM}}$ with the instantaneous SNR ($\text{SNR}_{\mathbf{H}}$). As can be seen, this value ranges from $1/3$ (low $\text{SNR}_{\mathbf{H}}$), which is equivalent to the CM approach, to 1 (high $\text{SNR}_{\mathbf{H}}$), which matches the relaxed blind ML decoder. Additionally, Fig. 1.b represents the mean square error (MSE) in the channel estimate as a function of the ratio ρ_3/ρ_1, where we can see that the minimum MSE is obtained with the value $\rho_3^{\text{KM}}/\rho_1^{\text{KM}}$ provided by the KM criterion.

5.2 Second Example: Non-Identifiable Case ($n_R = 1$)

In the second example, the CM and KM techniques are evaluated in the case $n_R = 1$. Fig. 2 shows the MSE in the channel estimate versus the average SNR for different numbers N of available OSTBC blocks at the receiver side. As can be seen, the relaxed blind ML decoder in [14] is not able to recover the channel due to the indeterminacy problems pointed out in [11, 12, 14], whereas the CM and KM techniques are affected by a noise floor due to the finite number of observations. However, unlike the CM criterion, which is solely based on the previous knowledge of the source correlation matrix, the KM approach also exploits the congruence of the data model, which translates into a lower noise floor. Furthermore, we must note that the performance of the KM technique is practically identical in the cases of exact and estimated (from the instantaneous SNR) weights ρ^{KM}. Finally, Fig. 3 shows the BER after decoding, where we can see that the gap between the CM and KM criteria increases with the SNR and decreases with the number of available blocks at the receiver. Analogously to the previous case, this can be seen as a direct consequence of the additional information provided by the data model, which is more significative when the estimates of the correlation matrices are inaccurate (low N), or when the data model is very reliable (high SNR).

5.3 Third Example: Identifiable Case ($n_R = 2$)

In the final example, the previous experiment has been repeated for $n_R = 2$. In this case, the channel can be unambiguously recovered by means of the relaxed blind ML decoder, which is equivalent to the KM approach in the high SNR regime. As can be seen in Figs. 4 and 5, this translates into the fact that, unlike the CM approach, the KM technique is not affected by the noise floor, i.e, in the absence of noise it exactly recovers the channel within a finite number of observations.

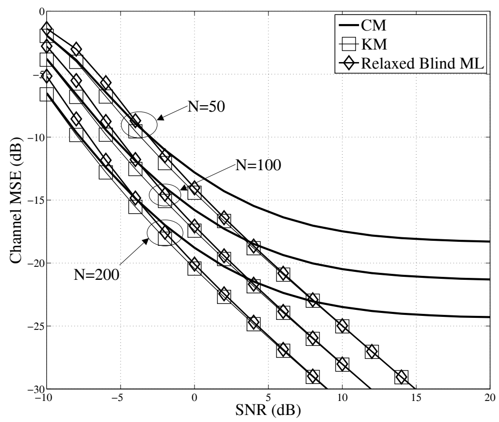

Figure 4: Performance of the proposed techniques in the identifiable case ($n_R = 2$). Channel MSE vs. SNR.

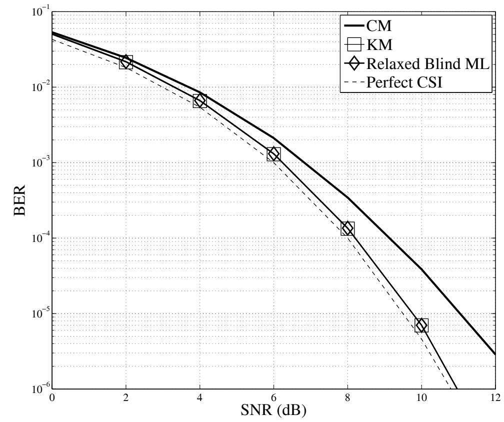

Figure 5: Performance of the proposed techniques in the identifiable case ($n_R = 2$). BER vs. SNR for $N = 50$.

6. CONCLUSIONS

In this paper the correlation (CM) and Kullback matching (KM) criteria have been applied to the problem of blind channel estimation under orthogonal space-time block coded (OSTBC) transmissions. Both techniques are based on the knowledge of the correlation properties of the sources and, due to the special structure of the codes, their solutions can be obtained in closed form. For both criteria, the channel estimation problem reduces to an eigenvalue problem, which is formed from the correlation matrix of the observations modified by the code matrices and a set of weights. The performance of the KM technique, whose weights depend on the SNR, is in general better than that of the CM approach (fixed weights). Finally, in the limiting cases of zero and infinite noise variance, the KM method is equivalent to the CM and the relaxed blind ML decoder, respectively.

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