TURBO EQUALIZATION OF DOUBLY SELECTIVE CHANNELS

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ABSTRACT

In this paper we discuss turbo equalization for transmission over doubly selective channels. The maximum a-posteriori (MAP) algorithm is used for detection as well as for channel decoding. The detection/decoding constituents can exchange soft information in an iterative manner resulting in the so-called turbo-equalization. The time-varying multi-path fading channel is modeled using the basis expansion model (BEM). In this BEM, the time-varying channel is viewed as a bank of time-invariant FIR filters and the time-variation is captured by means of time-varying complex exponential basis functions. Therefore, the time-varying transition tables that characterize the time-varying channel can also follow the BEM. This allows for a lower complexity MAP detector with comparable complexity to the MAP detector for time-invariant channels.

1. INTRODUCTION

Soft-input soft-output (SISO) processing based decoding and/or detection has gained a lot of attention in the past years due to its superior performance over hard decision decoding and/or equalization. Turbo equalization inspired by turbo coding [1], is a form of SISO joint detection and channel decoding [2]. In this sense the inter-symbol interference (ISI) channel is considered as a convolutional code serially concatenated with the channel encoder. The two decoding constituents (the detector and the channel decoder) can exchange soft information in an iterative manner resulting in the so-called turbo equalization.

In this paper, the channel equalizer is based on the optimal symbol-by-symbol maximum a-posteriori (MAP) detector implemented using the BCJR algorithm [3]. To reduce complexity the MAP detector can be replaced by an ISI canceller (linear or decision feedback equalizer) [4, 5, 6]. In these works, the wireless channel is assumed to be time-invariant (frequency-selective). For slowly time-varying channels, similar algorithms can be devised and proved to provide acceptable performance. However, these algorithms have difficulty to track and equalize rapidly time-varying channels.

In this paper, we consider transmission over doubly selective channels, where the channel is selective in time as well as in frequency. The channel frequency selectivity arises from the inter-symbol interference as a result of the multi-path propagation, while the channel time selectivity arises from the Doppler shift and/or the carrier frequency offset between the transmitter and receiver. The underlying time-varying multi-path channel has a rather complex structure [7] which may indeed be prohibitive for developing channel equalization techniques. Therefore, the time-varying channel is modeled here using the basis expansion model (BEM), where the complex exponential version of the BEM is considered. Using the BEM allows for an efficient low complexity mechanism to update the time-varying branch metrics of the time-varying channel in the MAP algorithm. Hence a lower complexity MAP-based equalization algorithm can be devised with comparable complexity to the MAP detector for time-invariant channels. In this sense, the turbo-equalization principle can be applied for single carrier (SC) transmission as well as multi-carrier (MC) transmission over doubly selective channels. The MC system considered in this paper is implemented using orthogonal frequency-division multiplexing (OFDM).

This paper is organized as follows. In Section 2, the system model is introduced. The turbo equalizer for SC and OFDM transmission over doubly selective channels is introduced in Section 3. Our findings are confirmed by numerical results introduced in Section 4. Finally, conclusions are drawn in Section 5.

*N*otation: We use lower (upper) case bold face letters to denote time-domain (frequency-domain) vectors. Matrices are also denoted using bold face upper letters which should be differentiated from the frequency-domain vectors from the context. Superscript ^{H} is used to denote the Hermitian operation. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$, the $M \times N$ all zeros matrix by $0_{M\times N}$, and $\mathbf{e}_k$ as the length N unity vector with 1 at position k. Finally, $\text{diag}\{\mathbf{x}\}$ denotes the diagonal matrix with the vector $\mathbf{x}$ on the diagonal.

2. SYSTEM MODEL

We consider transmission over a doubly selective channel, with one transmit antenna and one receive antenna. The input bits $a_k \in \{0,1\}$ for $k = 0,\dots,K-1$ are first encoded using a recursive systematic convolutional (RSC) encoder with code rate r. The RSC encoder produces the coded bits $b_k \in \{0,1\}$ for $k = 0,\dots,(K/r)-1$. The coded bits are interleaved to protect them from burst errors first, and also to help in developing a turbo equalization scheme. The interleaved coded bits are denoted by c_k. The interleaved coded bits c_k are grouped in blocks of m-bits, and each block is then mapped to a symbol $x[n]$ drawn from a finite alphabet. The finite set is denoted as $\mathscr{X}$ with cardinality $|\mathscr{X}| = M = 2^m$. In this paper we assume block transmission, therefore the data symbols $x[n]$ are grouped in blocks of size N (given the above information $N = \frac{K}{mr}$), and transmitted at a rate of $1/T$ symbols per second over the time-varying channel. The data sequence can be thought of as time-domain QAM symbols for SC transmission, or a time-domain equivalent of QAM frequency-domain symbols for MC transmission. The discrete time base-band equivalent description of the received sequence at time index n is given by:

$$y[n] = \sum_{l=0}^{L} g[n;l]x[n-l] + v[n], \tag{1}$$

where $g[n;l]$ is the discrete time equivalent base-band representation of the time-varying frequency-selective channel taking into account the physical multi-path channel and the transmitter and receiver pulse shaping filters. The channel order $L = \lfloor \tau_{\max}/T \rfloor + 1$ with $\tau_{\max}$ is the channel maximum delay spread. Finally, $v[n]$ is the discrete time additive white noise (AWN).

We use the basis expansion model (BEM) [8, 9] to approximate the doubly selective channel $g[n;l]$, where the doubly selective channel is modeled as a time-varying FIR filter over a window of size N. For $n \in \{0,\dots,N-1\}$, each tap of the time-varying FIR

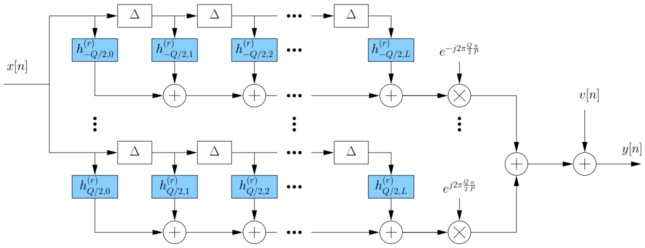

Figure 1: Input-output relationship using the BEM to approximate the doubly selective channel

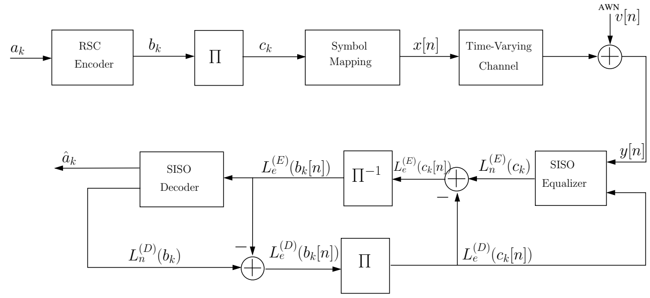

Figure 2: System model

filter is expressed as a superposition of complex exponential basis functions with frequencies on a discrete Fourier transform (DFT) grid, i.e.

$$h[n;l] = \sum_{q=-Q/2}^{Q/2} e^{j2\pi qn/P} h_{q,l}, \tag{2}$$

where Q is the number of time-varying basis functions satisfying $Q/(2PT) \geq f_{\max}$, with $f_{\max}$ is the channel maximum Doppler spread, and $P \geq N$ is the BEM resolution. $h_{q,l}$ is the coefficient of the qth basis function of the lth tap, which is kept invariant over a period of a transmission of a block of N symbols, but may change from block to block.

Substituting (2) in (1), we arrive at the following input-output relationship

$$y[n] = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} e^{j2\pi qn/P} h_{q,l} x[n-l] + v[n]. \tag{3}$$

The input-output relationship (3) is depicted in Figure 1. Defining $\mathbf{y} = [y[0], \ldots, y[N-1]]^T$, the received vector $\mathbf{y}$ can be written as

$$\mathbf{y} = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} h_{q,l} \mathbf{D}_q \mathbf{Z}_l \mathbf{x} + \mathbf{v}, \tag{4}$$

where $\mathbf{D}_q$ is a diagonal matrix with the qth basis function components on its diagonal $\mathbf{D}_q = \mathrm{diag}\{[1, e^{j2\pi q/P}, \ldots, e^{j2\pi q(N-1)/P}]^T\}$, and the $N \times (N+L)$ Toeplitz matrix $\mathbf{Z}_l$ is defined as $\mathbf{Z}_l = [0_{N\times(L-l)}, \mathbf{I}_N, 0_{N\times l}]$. The transmitted symbols vector $\mathbf{x}$ is defined as $\mathbf{x} = [x[-L], \ldots, x[N-1]]^T$, and finally the additive noise vector $\mathbf{v}$ is similarly defined as $\mathbf{y}$.

3. TURBO EQUALIZATION

In turbo equalization, the ISI channel can be viewed as a convolutional encoder serially concatenated to the channel convolutional encoder. The detector and channel decoder, if implemented by means of SISO algorithms, can then exchange soft information in an iterative manner resulting in the so-called turbo equalization scheme. The soft information produced by the detector is fed to the channel decoder as a-priori information and vice versa. The optimal symbol-by-symbol MAP algorithm is considered for detection as well as for channel decoding. An efficient implementation of the MAP algorithm based on the trellis diagram is the BCJR algorithm (sometimes referred to as the BCJR-MAP algorithm [3]). The MAP detector uses the channel model parameters and a-priori information provided by the channel decoder to maximize the a-posteriori probability (APP) $\Pr(x[n] = x|\mathbf{y})$ where $x \in \mathscr{X}$.

It is more convenient to work with log-likelihood-ratios (LLRs) rather than probabilities. The symbol LLR at time index n is defined as

$$L_n(x) = \log \frac{\Pr(x[n] = x|\mathbf{y})}{\Pr(x[n] = x_0|\mathbf{y})}, \tag{5}$$

where x_0 is some reference symbol. Therefore, the APP $\Pr(x[n] = x|\mathbf{y})$ is obtained from the symbol LLR as:

$$\Pr(x[n] = x|\mathbf{y}) = K_0 \exp[L_n(x)], \tag{6}$$

where $K_0 = \Pr(x[n] = x_0|\mathbf{y})$. Using Bayes' theorem, the LLR $L_n(x)$ can be written as

$$L_n(x) = \log \frac{\Pr(\mathbf{y}|x[n] = x)}{\Pr(\mathbf{y}|x[n] = x_0)} + \log \frac{\Pr(x[n] = x)}{\Pr(x[n] = x_0)}, \tag{7}$$

the first term in (7) is the extrinsic information, and the second term is the a-priori information, either initially known to the channel equalizer or provided by the channel decoder in later iterations.

For binary data $c \in \{0, 1\}$, the LLR can be obtained as

$$L(c) = \log \frac{\Pr(c = 1|\mathbf{y})}{\Pr(c = 0|\mathbf{y})}. \tag{8}$$

Here, the coded bits LLRs can be obtained from the data symbols LLRs as follows. The transmitted data symbols (QAM symbols) $x[n]$ are obtained by mapping m coded bits into a symbol $x \in \mathscr{X}$. Let the symbol $x[n]$ transmitted at time index n be mapped from the sequence of m coded bits $\{c_1[n], \ldots, c_m[n]\}$. Hence, the LLR of the ith coded bit of the transmitted symbol at time index n can be computed from the symbols LLRs as

$$L_n(c_i) = \log \frac{\sum_{x\in\mathscr{X}_i^1} \exp[L_n(x)]}{\sum_{x\in\mathscr{X}_i^0} \exp[L_n(x)]}, \tag{9}$$

where $\mathscr{X}_i^1$ ($\mathscr{X}_i^0$) is a subset of $\mathscr{X}$ containing symbols x mapped from the m-bits sequence $\{c_1[n], \ldots, c_m[n]\}$ with $c_i[n] = 1$ ($c_i[n] = 0$). Formula (9) is valid provided that the coded bits $c_i[n]$ are independent identically distributed (i.i.d) random variables, which is a valid assumption thanks to the interleaver used at the transmitter. The APPs that are necessary to compute the LLRs are obtained by applying the MAP algorithm for detection as well as for channel decoding. In the following we explain the MAP equalizer for the case of SC transmission as well as for MC transmission implemented using orthogonal frequency-division multiplexing (OFDM).

3.1 SC Transmission

For SC transmission the time-domain transmitted symbols $x[n]$ are drawn from a finite alphabet $\mathscr{X}$. Therefore, the MAP equalizer can be obtained by performing the BCJR algorithm where the forward coefficient $\alpha_n(s)$ and backward coefficient $\beta_n(s)$ at time index (recursion) n are obtained as:

$$\alpha_n(s) = \sum_{\forall s' \in \mathscr{S}} \alpha_{n-1}(s')\gamma_{n-1}(s', s) \tag{10}$$

and

$$\beta_n(s) = \sum_{\forall s' \in \mathscr{S}} \beta_{n+1}(s')\gamma_n(s, s'), \tag{11}$$

where $\mathscr{S}$ is the set of all possible states in the trellis describing the channel transitions. The initial conditions for the forward recursion coefficients are $\alpha_{-1}(s_0) = 1$ and $\alpha_{-1}(s \neq s_0) = 0$ assuming the initial state is known to be s_0. The initial conditions for the backward recursion coefficients are $\beta_N(s) = 1/|\mathscr{S}|\ \forall s \in \mathscr{S}$ assuming no trellis termination. If the trellis is terminated to some final state s_j, then the initial conditions for the backward recursion coefficients are $\beta_N(s_j) = 1$, and $\beta_N(s \neq s_j) = 0$. The term $\gamma_n(s_n, s_{n+1})$ is the transition probability $\Pr(s_n, y[n]|s_{n+1})$. Using Bayes' theorem, $\gamma_n(s_n, s_{n+1})$ can be written as

$$\gamma_n(s_n, s_{n+1}) = \Pr(s_{n+1}|s_n)\Pr(y[n]|s_n, s_{n+1}). \tag{12}$$

To compute the transition probability we have to compute the metric for every branch in the trellis describing the time-varying channel. To do so, we define $z^{(i,j)}[n]$ as the noiseless channel output symbol corresponding to the transition from state i to state j at time index (recursion) n. Also define the sequence $\mathscr{X}^{(i,j)}$ as the length $L+1$ sequence of symbols that characterizes the transition from state i to state j for $i, j = 0, \dots, M^L - 1$, $\mathscr{X}^{(i,j)} = \{x_0^{(i,j)}, \dots, x_L^{(i,j)}\}$. The noiseless channel output $z^{(i,j)}[n]$ can be written as

$$z^{(i,j)}[n] = \sum_{l=0}^{L} g[n;l] x_{L-l}^{(i,j)}. \tag{13}$$

Using the Euclidean distance, the channel branch metric for the transition from state i to state j at time index n is given by

$$r^{(i,j)}[n] = \left| y[n] - z^{(i,j)}[n] \right|^2. \tag{14}$$

For time-invariant channels, the outputs $z^{(i,j)}[n]$ are also time-invariant (independent of n), which means that to run the BCJR algorithm, one state diagram (transition table) has to be constructed and used for the equalization process. For time-varying channels, however, the output symbols $z^{(i,j)}[n]$ are also time-varying as they depend on the channel state information (CSI) at every time instant n. Therefore, to run the BCJR algorithm, the transition table has to be recomputed for each recursion. A more efficient procedure can be obtained by exploiting the BEM as will be clear next.

Using the BEM to approximate the doubly selective channel, we can write the noiseless channel output $z^{(i,j)}[n]$ in (13) as

$$\begin{aligned} z^{(i,j)}[n] &= \sum_{-Q/2}^{Q/2} \left(\sum_{l=0}^{L} h_{q,l} x_{L-l}^{(i,j)} \right) e^{j2\pi qn/P} \\ &= \sum_{q=-Q/2}^{Q/2} z_q^{(i,j)} e^{j2\pi qn/P}. \end{aligned} \tag{15}$$

Therefore, the branch metric for the transition from state i to state j at time index n is given by

$$r^{(i,j)}[n] = \left| y[n] - \sum_{q=-Q/2}^{Q/2} z_q^{(i,j)} e^{j2\pi qn/P} \right|^2. \tag{16}$$

Using the BEM as shown in (15), to run the BCJR algorithm, we have to construct $Q+1$ time-invariant transition tables. Each transition table corresponds to a branch of the BEM channel, and contains the elements $z_q^{(i,j)}$ $(i, j = 0, \dots, M^L - 1)$ for one particular q. The output of the transition from state i to state j at the qth branch $z_q^{(i,j)}$ is given by

$$z_q^{(i,j)} = \sum_{l=0}^{L} h_{q,l} x_{L-l}^{(i,j)}. \tag{17}$$

Assuming additive white Gaussian noise (AWGN) with zero-mean and variance σ_n^2, the transition probability from state i to state j, $\gamma_n(s_i, s_j)$ is obtained as

$$\gamma_n(s_i, s_j) = \tag{18}$$
$$\begin{cases} \Pr(x[n] = x^{(i,j)}) \frac{exp[-r^{(i,j)}/(2\sigma_n^2)]}{\sqrt{2\pi\sigma_n^2}} & s_i \rightarrow s_j \\ 0 & \text{otherwise} \end{cases}.$$

The LLR of the coded bits provided by the equalizer are then computed as

$$L_n^{(E)}(c_i) = \log \frac{\sum_{x \in \mathscr{X}_i^1} \alpha_n(s_i)\gamma_n(s_i, s_j)\beta_{n+1}(s_j)}{\sum_{x \in \mathscr{X}_i^0} \alpha_n(s_i)\gamma_n(s_i, s_j)\beta_{n+1}(s_j)}. \tag{19}$$

The SISO decoder follows the same principle, replacing the channel outputs by the convolutional encoder outputs. The a-priori information fed to the SISO decoder consists of the LLRs on coded bits provided by the SISO equalizer minus the a-priori information in the form of extrinsic information, i.e.

$$L_e^{(E)}(c_i[n]) = L_n^{(E)}(c_i) - L_e^{(D)}(c_i[n]), \tag{20}$$

where $L_n^{(E)}(c_i)$ is the LLR value on coded bits provided by the SISO equalizer, and $L_e^{(D)}(c_i[n])$ is the extrinsic information provided by the SISO decoder as a-priori information of the ith coded bit of the nth channel symbol. The SISO decoder produces soft information on uncoded bits as well as on coded bits. The LLRs on coded bits of the SISO decoder minus the input LLRs provides extrinsic information (similar to (20)) as a-priori information which is then fed back to the SISO equalizer as a-priori information. The SISO equalizer uses this a-priori information for another round of SISO equalization. The exchange of extrinsic information between the SISO equalizer and the SISO decoder is repeated in an iterative manner resulting in the so-called turbo equalization scheme. This iterative process results in a reduced bit-error-rate (BER). The turbo equalizer is depicted in Figure 2. For OFDM transmission the same principle may apply as follows.

3.2 OFDM Transmission

For OFDM transmission, the information-bearing symbols are parsed into blocks of N frequency-domain QAM symbols. Each block is then transformed to the time-domain by the inverse discrete Fourier transform (IDFT). A cyclic prefix (CP) of length $\nu \geq L$ is added to the head of each block. The time-domain blocks are then serially transmitted over the time-varying channel. Assuming S_k is the transmitted symbol on the kth sub-carrier of the OFDM block, $x[n]$ can be written as

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j2\pi(n-\nu)k/N}.$$

Note that this description includes the transmission of a CP of length ν. The received sequence after removing the CP, can be written as

$$\mathbf{y} = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} h_{q,l} \mathbf{D}_q \bar{\mathbf{Z}}_l \mathbf{F}^H \mathbf{S} + \mathbf{v}, \tag{21}$$

where $\mathbf{F}$ is the $N \times N$ unitary DFT matrix, $\bar{\mathbf{Z}}_l$ is an $N \times N$ circular matrix with $\mathbf{e}_l$ as its first column, and $\mathbf{S}$ is the frequency-domain transmitted vector $\mathbf{S} = [S_0, \ldots, S_{N-1}]^T$. The frequency-domain symbols S_k here are drawn from the finite input symbols alphabet[1] $\mathscr{X}$, i.e. $S_k \in \mathscr{X}$. Define $\mathbf{Y} = [Y_0, \ldots, Y_{N-1}]^T$ as the received vector in the frequency-domain, then $\mathbf{Y}$ can be written as

$$\mathbf{Y} = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} h_{q,l} \mathbf{F} \mathbf{D}_q \bar{\mathbf{Z}}_l \mathbf{F}^H \mathbf{S} + \mathbf{V}. \tag{22}$$

For $P = N$, i.e. when the BEM basis functions are taken on a DFT grid of size N, we can use the properties $\mathbf{D}_q = \mathbf{F}^H \bar{\mathbf{Z}}_q \mathbf{F}$ and $\bar{\mathbf{Z}}_l = \mathbf{F}^H \mathbf{D}_{-l} \mathbf{F}$. Hence, (22) can be written as

$$\mathbf{Y} = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} h_{q,l} \bar{\mathbf{Z}}_q \mathbf{D}_{-l} \mathbf{S} + \mathbf{V}. \tag{23}$$

Using the property $\bar{\mathbf{Z}}_q \mathbf{D}_{-l} = e^{j2\pi ql/N} \mathbf{D}_{-l} \bar{\mathbf{Z}}_q$, we obtain

$$\mathbf{Y} = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} \tilde{h}_{q,l} \mathbf{D}_{-l} \bar{\mathbf{Z}}_q \mathbf{S} + \mathbf{V}, \tag{24}$$

where $\tilde{h}_{q,l} = e^{j2\pi ql/N} h_{q,l}$. Note the similarity between (24) and (4). From (24), it is observed that when using a BEM resolution $P = N$, OFDM transmission corresponds to SC transmission over a doubly selective channel, where the doubly selective channel is now of order Q (instead of L for SC transmission), and the time-variation is captured by means of $L+1$ (instead of $Q+1$ for SC transmission) time-varying complex exponential basis functions, and the channel BEM coefficients are characterized by $\tilde{h}_{q,l}$. Hence, we can depict a figure similar to Figure 1, but with input S_k, and output Y_k to describe the input-output relationship in (24). Therefore, the SISO equalizer for OFDM transmission is obtained in a similar fashion to the one obtained for SC transmission. The branch metrics of the trellis are obtained as follows. Define the sequence $\mathscr{S}^{(i,j)}$ as the length $Q+1$ sequence of symbols that characterizes the transition from state i to state j, for $i, j = 0, \ldots, M^Q - 1$, $\mathscr{S}^{(i,j)} = \{S_0^{(i,j)}, \ldots, S_Q^{(i,j)}\}$. The noiseless channel output symbol $Z^{(i,j)}[k]$ of the transition from state i to state j at recursion k (recursion refers here to the sub-carrier index) can be written as

$$Z^{(i,j)}[k] = \sum_{l=0}^{L} Z_l^{(i,j)} e^{-j2\pi lk/N}, \tag{25}$$

where $Z_l^{(i,j)}$ is given by

$$Z_l^{(i,j)} = \sum_{q=-Q/2}^{Q/2} \tilde{h}_{q,l} S_{Q/2-q}^{(i,j)}. \tag{26}$$

Using the Euclidean distance, the branch metric for the transition from state i to state j at recursion k is given by

$$R^{(i,j)}[k] = \left| Y_k - Z^{(i,j)}[k] \right|^2. \tag{27}$$

The turbo equalization is then performed in the same manner as in the case of SC transmission, where the BCJR algorithm is used for the SISO equalizer as well as for the SISO decoder. The BCJR algorithm is implemented now in the frequency domain. The LLRs on the transmitted frequency-domain symbol on sub-carrier k is obtained as

$$L_k(S) = \log \frac{\Pr(S_k = S|\mathbf{Y})}{\Pr(S_k = S_{ref}|\mathbf{Y})}, \tag{28}$$

where S_{ref} is some reference symbol $S_{ref} \in \mathscr{X}$. The transition probability $\gamma_k(s_k, s_{k+1}) = \Pr(S_{k+1}|S_k)\Pr(Y_k, S_k, S_{k+1})$ is computed as in (12) replacing $y[n]$ by Y_k, and $r^{(i,j)}[n]$ by $R^{(i,j)}[k]$.

For a BEM resolution $P > N$, the ICI is unlimited and covers the whole OFDM block, which prevents the development of a practical SISO equalizer using the trellis based BCJR algorithm. However, the main part of the ICI comes from the neighboring Q sub-carriers, and so the above model and analysis may be used as a good approximation.

3.3 Complexity

Running the BCJR algorithm in the time-domain (SC transmission) and using perfect channel state information (CSI) requires $N(L+1)M^L$ multiply-add (MA) operations, in addition to M^L memory locations to store the transition tables (which are to be recomputed for each time instant (or recursion)). Running the BCJR algorithm in the time-domain and using the BEM requires $(L+1)(Q+1)M^L + N(Q+1)$ MA operations, in addition to $(Q+1)M^L$ memory locations to store the $Q+1$ transition tables that characterize the state diagrams. Note that these figures are updated/computed for every block of N samples.

Running the BCJR algorithm in the frequency-domain (OFDM) and using the BEM requires $(L+1)(Q+1)M^Q + N(L+1)$ MA operations, in addition to $(L+1)M^Q$ memory locations to store the $L+1$ transition tables that characterize the state diagrams.

4. NUMERICAL RESULTS

In this section we present some simulation results for the proposed equalization technique for SC as well as for OFDM transmission over doubly selective channels. In the simulations below, the input bits are encoded using a rate $r = 1/2$ RSC code $(23, 35)_8$. The encoded bits are interleaved using a random interleaver of size K/r (K is the number of uncoded bits in one block). The interleaved coded bits are then mapped to Quadrature Phase Shift Keying (QPSK) symbols $\mathscr{X} = \{\mp 1 \mp j\}/\sqrt{2}$, and then transmitted over the doubly selective channel. In all simulations, the channel taps are simulated as i.i.d random variables, correlated in time according to Jakes' model with correlation function $r_h[n] = J_0(2\pi n f_{max} T)$, where J_0 is the zeroth-order Bessel function of the first kind, $f_{\max} T = 0.001$ is the maximum normalized Doppler spread. The time-varying channels are normalized to have unity power. As the channel is time-varying, the channel taps over a block of N symbols are normalized as follows

$$g(t;\tau) = \frac{g_{ch}(t;\tau)}{\sqrt{\frac{1}{NT} \int\limits_{\tau=0}^{LT} \int\limits_{NT} |g_{ch}(t;\tau)|^2 \, dt d\tau}}.$$

The Jakes' Channel model was approximated by the BEM using least squares (LS) fitting. The BEM channel coefficients are assumed to be perfectly known at the receiver and used to equalize the underlying Jakes' channel model (taking into account the modeling error) and/or the exact BEM channel model as explained next. Turbo equalization with channel estimation is out of the scope of this paper and a topic of further investigation.

• *SC Transmission:* For SC transmission the doubly selective channel is assumed to be of order $L = 3$. A window of size $N = 800$ is considered. The BEM resolution is chosen to be $P = N$ and $P = 2N$. The number of basis functions in the BEM is $Q = 2$

[1] Note that we choose to define the frequency-domain input alphabet the same as the time-domain input alphabet for SC transmission $\mathscr{X}$.

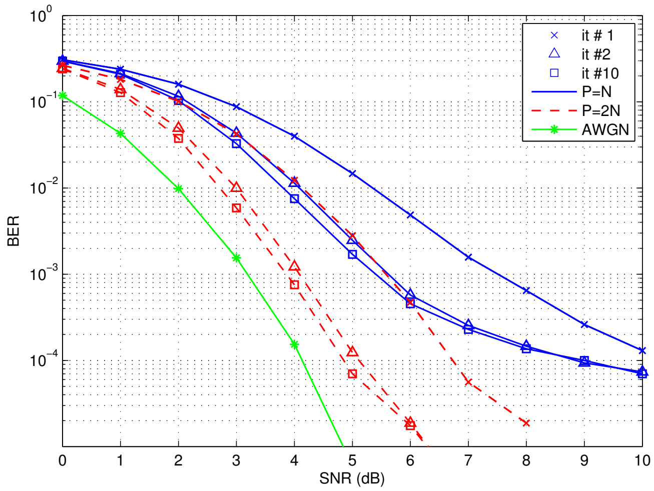

Figure 3: Turbo equalization of SC transmission over doubly selective channels

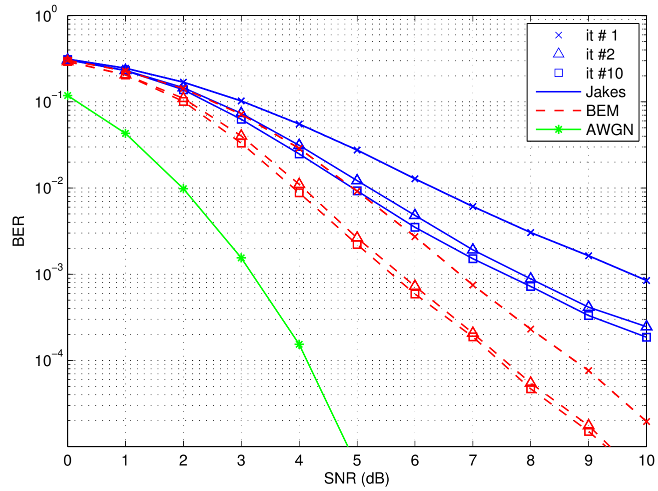

Figure 4: Turbo equalization of OFDM transmission over doubly selective channels

for $P = N$, and $Q = 4$ for $P = 2N$. The approximated BEM channel coefficients are used to equalize the underlying Jakes' channel model. The simulation results corresponding to this channel setup are shown in Figure 3. In addition to this setup, coded transmission over an additive white Gaussian (AWGN) channel is considered in the simulations. The performance of coded transmission over AWGN channel serves as a theoretical bound for the performance of the proposed turbo equalizers. In the simulations 10 iterations were performed. However, the difference between the third and the tenth iteration was very marginal, and most of the performance gain was obtained in iteration #2. As shown in Figure 3, the SNR gain for iteration #2 over iteration #1 is 2dB for $BER = 10^{-3}$ and $P = N$ and 1.8dB for $BER = 10^{-4}$ and $P = 2N$. If the BEM resolution is equal to the block size, i.e. $P = N$ the BER suffers from an error floor for high SNR values. This is due to the modeling error between the underlying real channel and the BEM. For $P = 2N$, the BER floor was not reached for the BER values shown in the figure. However, for the latter case, the SNR loss is 0.8dB at $BER = 10^{-4}$ and obtained in iteration #10 compared to the theoretical bound for coded transmission over AWGN channels.

• *OFDM Transmission:* For OFDM transmission the doubly selective channel is assumed to be of order $L = 6$. The window size (the number of sub-carriers in the OFDM system) is $N = 512$. The channel follows Jakes' model, and is modeled using the BEM with BEM resolution equal to the block size, i.e. $P = N$. This allows to limit the inter-carrier interference support to $Q = 2$ adjacent sub-carriers, and hence leads to a finite trellis for the MAP equalizer. The BEM channel coefficients are used to equalize the true Jakes' channel (with channel modeling error) as well as the approximated BEM channel (no channel modeling error). For the two channel setups, the performance of the proposed turbo equalizer is shown in Figure 4. In the simulations, 10 iterations were performed. However, the performance difference between iteration #3 and iteration#10 is marginal. As shown in the figure, the SNR loss is 3.2dB for the proposed turbo equalizer compared to the theoretical bound at $BER = 10^{-4}$ and when the channel follows exactly the BEM, and around 4.2dB when the channel follows Jakes' model at iteration #10 and $BER = 10^{-3}$. The BER curve for the case of Jakes' channel model tend to have an error floor (not shown in the figure) at $BER = 10^{-4}$. This is due to the modeling error between the Jakes' channel model and the BEM channel model.

5. CONCLUSIONS

In this paper we have discussed turbo-equalization for transmission over doubly selective channels. Single-carrier and multi-carrier transmission techniques have been considered. The doubly selective channel is modeled using the BEM. Using the BEM, it has been demonstrated that a simple time-invariant MAP detector can be utilized instead of the time-varying high complex MAP detector. The resulted MAP detector has a comparable complexity to the MAP detector for time-invariant channels.

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