16th European Signal Processing Conference (EUSIPCO 2008), Lausanne, Switzerland, August 25-29, 2008, copyright by EURASIP

DESIGN OF IIR FILTERS USING NEW THREE ALLPASS FILTERS STRUCTURE

Alfonso Fernandez-Vazquez,[1,2] Gordana Jovanovic-Dolecek,[2] and Xue Mei Xie[1]

[1]School of Electronic Engineering; Xidian University
Xi'an, 710071, P. R. China
[2]Department of Electronics; Instituto Nacional de Astrofisica, Optica, y Electronica; INAOE
Puebla, 72000, Mexico
afernan@inaoep.mx, gordana@inaoep.mx, xmxie@mail.xidian.edu.cn

ABSTRACT

We present a new structure for the IIR filter design composed of three allpass filters, i.e., one real-valued and two complex-valued allpass filters. In particular, applying the inverse discrete Fourier transform to the allpass filters, we get three real doubly complementary IIR filters. We also provide the procedure to design a lowpass filter with a desired characteristic using a complex-valued allpass filter with pre-determined characteristics. The proposed structure is used for three different lowpass filter designs, that is, eigenfilter, equiripple, and maximally flat filter design. Closed form equations for the computation of the maximally flat filter coefficients are also provided. The design techniques are illustrated by means of examples.

1. INTRODUCTION

Traditional design of lowpass IIR filters is based on the design of analog filters and a transformation from the analog domain to the digital domain. This design is usually applied for Butterworth, Chebyshev, and elliptic filters [1]. Furthermore, it is well known that the resulting IIR filters can be implemented using a parallel connection of two allpass filters [2, 3]. Since the IIR filter implementation is structurally passive [1], the passband magnitude response has low sensitivity to the filter quantization and a low noise level [2, 3]. Moreover, using the allpass structure, we can obtain a complementary highpass filter. This means that the lowpass and highpass transfer functions satisfy the *doubly complementary* property. To be specific, they are allpass complementary as well as power complementary [2, 3].

Consequently, different direct design methods for IIR filters, based on two allpass filters, have been proposed in the literature [4–7]. A direct design of IIR filters, which is based on complex allpass filters, is introduced in [4]. The technique applies Remez-like algorithm to design the corresponding allpass filters. A another approach is given in [5] where the technique is reduced to the design of one real-valued allpass filter, which can be unstable; however, the desired IIR filter is always stable. In this method, Remez algorithm is used to compute the allpass filter coefficients. In a similar way, the method given in [6] designs IIR filters with flat magnitude responses. A direct design of Butterworth filters based on real- and complex-valued allpass filters is discussed in [7].

An interesting method to design M-band lowpass IIR filters based on M real-valued allpass filters is given in [8]. However, the main disadvantage of this method is that for $M > 2$ there are frequency regions where the stop attenuation cannot be controlled, i.e., the regions in the vicinity of $3\pi/M$, $5\pi/M$, $7\pi/M$, etc. The authors called those regions *don't care bands* because the final application is the design of multistage decimators and interpolators [9]. Additionally the passband edge frequency ω_p is restricted to $0 < \omega_p < \pi/M$.

In this paper we address the magnitude approximation of real-valued lowpass filters based on a new parallel connection of three allpass filters, that is, the proposed structure is composed by one real- and two complex-valued allpass filters. By applying 3-point inverse discrete Fourier transform (IDFT) to the designed allpass filters, we can obtain three transfer functions, which satisfy the doubly complementary property. Moreover, the IIR filter implementation is structurally passive. The design problem of lowpass filter is reduced further to design one complex-valued allpass filter with desired characteristics. We present three different designs to approximate the desired phase response of the allpass filters, i.e., eigenfilter design, equiripple design, and maximally flat design. Examples are provided in order to illustrate the proposed lowpass filter design.

The rest of the paper is organized as follows. The proposed IIR filter based on real- and complex-valued allpass filters is introduced in Section 2. Section 3 presents the design of allpass filters with desired phase responses and complex coefficients based on eigenfilter, equiripple, and maximally flat approaches. Design examples are also given in Section 3.

This work was supported by CONACyT Mexico under projet number 49640.

2. PROPOSED IIR FILTER STRUCTURE

This section introduces the proposed structure to design real-valued and stable IIR filters. Additionally, some properties of the proposed filters are given.

Suppose that the lowpass IIR filter can be expressed by $H(z) = [A_0(z)+A_1(z)+A_2(z)]/3$, where $A_0(z)$, $A_1(z)$, and $A_2(z)$ are stable allpass filters. If the coefficients of the allpass filters are real, then it can be shown that the minimum value of $|H(e^{j\omega})|$ at $\omega = \pi$ is $1/3$. So we consider the case where two allpass filters, e.g., $A_1(z)$ and $A_2(z)$, have complex coefficients. Due that the desired IIR filter $H(z)$ should have real coefficients, the allpass filters must be related as $A_2(z) = \widetilde{A}_1(z^{-1})$, where $\widetilde{A}_1(z)$ is the paraconjugate of $A_1(z)$, that is, it is obtained by conjugating the filter coefficients and by replacing z by z^{-1}. In this way, the proposed IIR filter is given by

$$H(z) = \frac{1}{3}\left[A_0(z)+A_1(z)+\widetilde{A}_1(z^{-1})\right]. \quad (1)$$

The allpass filters $A_0(z)$ and $A_1(z)$ must be stable in order that the filter $H(z)$ be stable.

2.1 Doubly complementary property

Now, we show how the proposed filter $H(z)$ along with two other filters satisfy the doubly complementary property.

By applying 3-point IDFT to the allpass filters $A_0(z)$, $A_1(z)$, and $\widetilde{A}_1(z^{-1})$, respectively, we can obtain three real-valued filters, namely $H(z)$, $G(z)$, and $F(z)$, which are shown in Fig. 1. The filters $F(z)$ and $G(z)$ have highpass and bandpass characteristics, respectively.

It can be shown that the resulting transfer functions satisfy the power complementary property, that is,

$$|F(e^{j\omega})|^2+|G(e^{j\omega})|^2+|H(e^{j\omega})|^2 = 1. \quad (2)$$

Moreover, if we apply 3-point discrete Fourier transform (DFT) to

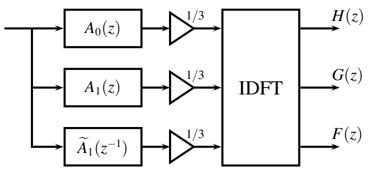

Figure 1: Filters $H(z)$, $G(z)$, and $F(z)$ obtained by applying 3-point IDFT to $A_0(z)$, $A_1(z)$, and $\widetilde{A}_1(z^{-1})$.

$H(z)$, $G(z)$, and $F(z)$, we arrive at

$$A_0(z) = H(z) + G(z) + F(z), \tag{3}$$

$$A_1(z) = H(z) + \mathrm{e}^{\mathrm{j}2\pi/3} G(z) + \mathrm{e}^{-\mathrm{j}2\pi/3} F(z). \tag{4}$$

Consequently, from (2)–(4), we can say that the transfer functions $H(z)$, $G(z)$, and $F(z)$ satisfy the doubly complementary property.

2.2 Structural passivity

Some efficient implementation structures for the stable allpass filters $A_0(z)$ and $A_1(z)$ can be found in [1]. Using those structures, each multiplier coefficient $|m_i|$ in the structure is in the range $[0,1]$. Furthermore, from (2), we see that the magnitude response of $H(z)$ is bounded by the unity, i.e., $|H(\mathrm{e}^{\mathrm{j}\omega})| \leq 1$.

Therefore, the implementation is structurally passive. This means that the magnitude response has low sensitivity to the filter quantization.

2.3 Auxiliary complex-valued allpass filter

In this subsection we show that the problem of designing lowpass and stable IIR filter is reduced to designing a complex-valued allpass filter with desired characteristics.

At first, notice that (1) can be rewritten as

$$H(z) = \frac{A_0(z)}{3}\left[1 + A(z) + \widetilde{A}(z^{-1})\right], \tag{5}$$

where $A(z) = A_1(z)/A_0(z)$ is a complex-valued allpass filter, which can have poles outside the unit circle due to the zeros of $A_0(z)$.

As a consequence, the problem of designing IIR filters is reduced to designing an auxiliary allpass filter with complex coefficients and desired characteristics. The design of such kind of filters is discussed in Section 3.

In the following, some characteristics of $A(z)$ are described.

From (5), the corresponding magnitude response of $H(z)$ is

$$|H(\mathrm{e}^{\mathrm{j}\omega})| = \frac{1}{3}\left|1 + \mathrm{e}^{\mathrm{j}\phi_A(\omega)} + \mathrm{e}^{-\mathrm{j}\phi_A(-\omega)}\right|, \tag{6}$$

where $\phi_A(\omega)$ is the phase response of $A(z)$.

In order to have the values of $|H(\mathrm{e}^{\mathrm{j}\omega})|$ in the passband and stopband be 1 and 0, respectively (ideal case), the condition $\phi_A(\omega) = \phi_A(-\omega)$ must be satisfied, that is, the phase response is an even function of ω.

Using this property, it follows that $A(z) = A(z^{-1})$. Consequently, the magnitude response $|H(\mathrm{e}^{\mathrm{j}\omega})|$ becomes

$$|H(\mathrm{e}^{\mathrm{j}\omega})| = \frac{1}{3}\left|1 + 2\cos\left(\phi_A(\omega)\right)\right|, \tag{7}$$

and the ideal phase response $\phi_A(\omega)$ is expressed as

$$\phi_A(\omega) = \begin{cases} 0, & \omega \leq \omega_\mathrm{p}; \\ \pm\frac{2\pi}{3}, & \omega_\mathrm{s} \leq \omega \leq \pi. \end{cases} \tag{8}$$

Here, we consider a complex-valued allpass filter of order N given by [10],

$$\begin{aligned} A(z) &= \frac{a_N^* + a_{N-1}^* z^{-1} + \cdots + a_0^* z^{-N}}{a_0 + a_1 z^{-1} + \cdots + a_N z^{-N}} \\ &= \alpha \frac{b_N^* + b_{N-1}^* z^{-1} + \cdots + z^{-N}}{1 + b_1 z^{-1} + \cdots + b_N z^{-N}}, \end{aligned} \tag{9}$$

where $\alpha = a_0^*/a_0$, $b_n = a_n/a_0$, and a_n, $n = 0,\ldots,N$, are complex coefficients, i.e., $a_n = a_{\mathrm{R}n} + \mathrm{j}a_{\mathrm{I}n}$, where $a_{\mathrm{R}n}$ and $a_{\mathrm{I}n}$ are the real and imaginary part of a_n.

In order to achieve the condition $A(z) = A(z^{-1})$, the corresponding filter coefficients a_n, $n = 0,\ldots,N$, need to satisfy $a_n = a_{N-n}$, i.e., they must be a symmetric sequence. Generally, there are two cases that should be considered: N odd and N even. However, one can verify that N odd implies at least one pole of $A(z)$ is on the unit circle. As a consequence, in our design, we only consider the case where N is even.

In summary, the allpass filter $A(z)$ possesses the following properties:

1. The order N must be even.
2. The filter coefficients a_n, $n = 0,\ldots,N$, are a symmetric sequence, i.e., $a_n = a_{N-n}$, which implies that the resulting phase response $\phi_A(\omega)$ is an even function.
3. The ideal phase response is given by (8).

Finally, we wish to find the allpass filters $A_0(z)$ and $A_1(z)$. First note that $B(z)$, the z-transform of b_n, can be rewritten as $B(z) = z^{-N/2} B_0(z^{-1}) B_0(z)/\beta$, where $B_0(z)$ is a polynomial with all zeros inside the unit circle, i.e., $B_0(z) = 1 + b_{0,1}z^{-1} + \cdots + b_{0,N/2} z^{-N/2}$, and $\beta = b_{0,N/2}$.

Accordingly, the corresponding allpass filters are expressed as,

$$A_0(z) = z^{-N}\frac{B_0(z^{-1})\widetilde{B}_0(z)}{\widetilde{B}_0(z^{-1})B_0(z)}, \quad A_1(z) = z^{-N}\alpha\frac{\beta}{\beta^*}\frac{\widetilde{B}_0^2(z)}{B_0^2(z)}. \tag{10}$$

3. DESIGN OF COMPLEX-VALUED ALLPASS FILTERS

This section focuses on the design of allpass filters $A(z)$ using three different optimality criteria, that is, eigenfilter, equiriple, and maximally flat.

3.1 Eigenfilter method

The eigenfilter method to design allpass filters involves the computation of an eigenvector of a real, symmetric and definite positive matrix. In our case, such matrix is partitioned into four $(N+1)\times(N+1)$ centrosymmetric submatrices.

The equation, which approximates the ideal phase response of $\phi_A(\omega)$, is a linear combination of the real and imaginary parts of a_n, $n = 0,\ldots,N$ [11], i.e.,

$$\mathbf{D}^\mathrm{T}\mathbf{a} \approx 0, \tag{11}$$

where $\mathbf{D}^\mathrm{T}$ denotes matrix transposition of $\mathbf{D}$ and

$$\mathbf{a} = [a_{\mathrm{R}0}\, a_{\mathrm{R}1} \cdots a_{\mathrm{R}N}\, a_{\mathrm{I}0}\, a_{\mathrm{I}1} \cdots a_{\mathrm{I}N}]^\mathrm{T}, \tag{12}$$

$$\mathbf{D} = [\sin(\Phi_0) \cdots \sin(\Phi_N)\; -\cos(\Phi_0) \cdots -\cos(\Phi_N)]^\mathrm{T}, \tag{13}$$

where $\Phi_k = (k - N/2)\omega - \phi_A(\omega)/2$.

In [11], the authors consider the minimizing of the mean-squared error for (11) as

$$E = \mathbf{a}^\mathrm{T}\int_{\Re} W\mathbf{D}\mathbf{D}^\mathrm{T}\,\mathrm{d}\omega\,\mathbf{a} = \mathbf{a}^\mathrm{T}\mathbf{R}\mathbf{a}, \tag{14}$$

where $\mathbf{a}^\mathrm{T}\mathbf{a} = 1$, W is a weighting constant, $\Re$ is the region $(-\pi,-\omega_\mathrm{s}]\bigcup[-\omega_\mathrm{p},\omega_\mathrm{p}]\bigcup[\omega_\mathrm{s},\pi]$ and $\mathbf{R}$ is a real, symmetric and positive definite matrix of size $2(N+1)\times 2(N+1)$. Using the

Rayleigh's principle, the minimum value for E equals $\lambda_{\min}$, where $\lambda_{\min}$ is the smallest eigenvalue of $\mathbf{R}$.

Using (14), we can rewrite the matrix $\mathbf{R}$ as follows

$$\mathbf{R} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{C} \end{bmatrix}, \tag{15}$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are centrosymmetric matrices of size $(N+1)\times(N+1)$, i.e., they satisfy $\mathbf{JMJ} = \mathbf{M}$ [12–14], where $\mathbf{J}$ is the antidiagonal matrix. It is well known that if the eigenvectors of a centrosymmetric matrix are linearly independent, then they are symmetric or antisymmetric [12–14]. This means that the coefficients a_n can satisfy the Property 2 in Section 2.3.

Considering the weighting constant W in the passband and stopband as unity and W_s, respectively, the elements for each matrix are given in a closed form equations as,

$$\begin{aligned}[\mathbf{A}]_{n,m} = {} & \omega_\mathrm{p}\Big(\operatorname{sinc}((n-m)\omega_\mathrm{p}) - \operatorname{sinc}((n+m-N)\omega_\mathrm{p})\Big) \\ & + W_s\cdot(\pi-\omega_\mathrm{s})\Big((-1)^{n-m}\operatorname{sinc}((n-m)(\pi-\omega_\mathrm{s})) \\ & + \frac{1}{2}(-1)^{n+m-N}\operatorname{sinc}((n+m-N)(\pi-\omega_\mathrm{s}))\Big),\end{aligned} \tag{16}$$

$$[\mathbf{B}]_{n,m} = \frac{\sqrt{3}W_s}{2}(-1)^{n+m-N}(\pi-\omega_\mathrm{s})\operatorname{sinc}((n+m-N)(\pi-\omega_\mathrm{s})), \tag{17}$$

$$\begin{aligned}[\mathbf{C}]_{n,m} = {} & \omega_\mathrm{p}\Big(\operatorname{sinc}((n-m)\omega_\mathrm{p}) + \operatorname{sinc}((n+m-N)\omega_\mathrm{p})\Big) \\ & + W_s\cdot(\pi-\omega_\mathrm{s})\Big((-1)^{n-m}\operatorname{sinc}((n-m)(\pi-\omega_\mathrm{s})) \\ & - \frac{1}{2}(-1)^{n+m-N}\operatorname{sinc}((n+m-N)(\pi-\omega_\mathrm{s}))\Big),\end{aligned} \tag{18}$$

where $n,m = 0,\ldots,N$.

Example 1. We wish to design a lowpass IIR filter based on the parallel connection of three allpass filters with $N=8$, $\omega_\mathrm{p} = 0.4\pi$, and $\omega_\mathrm{s} = 0.5\pi$. Note that this filter cannot be designed using a 3-band IIR filters described in [8] because ω_p is not in $[0,\pi/3]$. As we pointed out before, the design of lowpass filter is reduced to designing a complex-valued allpass filter. By computing the smallest eigenvalue of $\mathbf{R}$ with $W_s = 60.9$, we obtain the filter coefficients a_n, $n = 0,\ldots,N$, from the corresponding eigenvector (see (12)). The resulting magnitude response of the designed filter is shown in Figs. 2(a) and 2(b) in solid line.

3.2 Equiripple method

Now we consider the design of optimal allpass filters in the Chebyshev sense. In [15], it is demonstrated that an equiripple phase error has an optimal solution in the Chebyshev sense. Furthermore, the authors propose an efficient iterative eigenfilter design method.

The problem is to find the smallest, positive, and real eigenvalue λ and the corresponding eigenvector $\mathbf{a}$ from

$$\mathbf{Pa} = \lambda\mathbf{Qa}, \tag{19}$$

where the vector $\mathbf{a}$ contains the filter coefficients a_n (see (12)) and the matrices $\mathbf{P}$ and $\mathbf{Q}$ are given by [15]

$$[\mathbf{P}]_{n,m} = \begin{cases} W_e\sin\Theta_m(\omega_n), & 0\le m\le N; \\ W_e\cos\Theta_{m-N-1}(\omega_n), & N<m\le 2N+1; \end{cases} \tag{20}$$

$$[\mathbf{Q}]_{n,m} = \begin{cases} (-1)^{n+l}\cos\Theta_m(\omega_n), & 0\le m\le N; \\ (-1)^{n+l+1}\sin\Theta_{m-N-1}(\omega_n), & N<m\le 2N+1; \end{cases} \tag{21}$$

where $l = 0$ or 1 to guarantee $\lambda > 0$, W_e is a weighted constant, and

$$\Theta_m(\omega_n) = \begin{cases} \frac{m-N}{2}\omega_n, & |\omega_n|\le\omega_\mathrm{p}; \\ \frac{m-N}{2}\omega_n - \frac{\pi}{3}, & \omega_\mathrm{s}\le|\omega_n|\le\pi; \end{cases} \tag{22}$$

where ω_n, $n = 0,\ldots,2(N+1)$, are the extremal frequencies.

To obtain equiripple phase error and the optimal Chebyshev solution as well, we use the iterative procedure introduced in [15].

Example 2. We use the same design parameters as in Example 1. Additionally, we select $W_e = 0.052$. This ensures that the stopband attenuation is 60 dB. To get the desired property $a_n = a_{N-n}$, the number of extremal frequencies in both passband and stopband must be the same, i.e., $N+1$. Figures 2(a) and 2(b) illustrate the magnitude characteristic of the lowpass filter in dash-dotted line.

3.3 Maximally flat method

In the following, we describe the maximally flat design. In [10], a method to design complex-valued allpass filters with flat group delay response at any desired set of frequency points is proposed. In our case we select three points, i.e., the points $\omega = 0$, $\omega = -\omega_\mathrm{r}$, and $\omega = \omega_\mathrm{r}$. Notice that we can get a Butterworth-like filter if $\omega_\mathrm{r} = \pi$.

Applying the method [10] with the number of null derivatives of the group delay at $\omega = 0$, $\omega = -\omega_\mathrm{r}$, and $\omega = \omega_\mathrm{r}$ equal $N-2$, $N/2-2$, and $N/2-2$, respectively (the reason is that this combination gives the filter coefficients with the property $b_n = b_{N-n}$), we find that the desired filter coefficients are expressed as

$$b_n = (-1)^n\left[\binom{N}{n} - \frac{4e^{j\phi_\alpha/2}}{\sqrt{3}}\binom{N/2}{n}c_{N,n}(\omega_\mathrm{r})\cos(\phi_\alpha/2+\pi/6)\right], \tag{23}$$

where $n = 0,\ldots,N/2$, ϕ_α is the phase value of α, and the function $c_{N,n}(\omega_\mathrm{r})$ for different values of N is given in Table 1. Moreover, we have $c_{N,0}(\omega_\mathrm{r}) = 0$.

The allpass filters described here are maximally flat in the following sense. Given the allpass filter order N, the phase and group delay values at $\omega = 0$, $\omega = -\omega_\mathrm{r}$, and $\omega = \omega_\mathrm{r}$, the coefficients are chosen so that as many derivatives of the group delay as possible vanish at $\omega = 0$, $\omega = -\omega_\mathrm{r}$, and $\omega = \omega_\mathrm{r}$.

We now discuss the problem of obtaining the values ϕ_α. To do so we consider the passband edge frequency ω_p and the attenuation in dB at this frequency point A_p. From (7) we define

$$\phi_\mathrm{p} = \cos^{-1}\left(\frac{3\cdot 10^{-A_\mathrm{p}/20}-1}{2}\right), \tag{24}$$

which gives the desired phase $\phi_A(\omega)$ at ω_p.

In order to find the value of ϕ_α, we solve [10]

$$\begin{aligned}&\sum_{n=0}^{N}\sin\left((n-N/2)\omega_\mathrm{p} + (\phi_\alpha-\phi_\mathrm{p})/2\right)b_{\mathrm{R}n} \\ &\qquad - \sum_{n=0}^{N}\cos\left((n-N/2)\omega_\mathrm{p} + (\phi_\alpha-\phi_\mathrm{p})/2\right)b_{\mathrm{I}n} = 0.\end{aligned} \tag{25}$$

From (25), it follows that

$$\phi_\alpha(\omega_\mathrm{p}, A_\mathrm{p}, \omega_\mathrm{r}) = 2\cdot\angle\left\{R_\mathrm{p}A'_\mathrm{p} + 1 + j\sqrt{3}(R_\mathrm{p}+1)\right\}, \tag{26}$$

where $\angle\{x\}$ stands for the angle of x, and

$$R_\mathrm{p} = \frac{-2^{N-1}\sin^N\left(\frac{\omega_\mathrm{p}}{2}\right)}{c_{N,N/2}(\omega_\mathrm{r}) + 2C_N(\omega_\mathrm{r},\omega_\mathrm{p})}, \quad A'_\mathrm{p} = \sqrt{\frac{1+3\cdot 10^{-A_\mathrm{p}/20}}{1-10^{-A_\mathrm{p}/20}}}, \tag{27}$$

where

$$C_N(\omega_\mathrm{r},\omega_\mathrm{p}) = \sum_{n=1}^{N/2-1}(-1)^{N/2+n}\binom{N/2}{n}c_{N,n}(\omega_\mathrm{r})\cos\left((N/2-n)\omega_\mathrm{p}\right). \tag{28}$$

In a similar way, we can define the values $\phi_\alpha(\omega_\mathrm{s}, A_\mathrm{s}, \omega_\mathrm{r})$ and $\phi_\alpha(\pi, A_\mathrm{s}, \omega_\mathrm{r})$, where A_s is the attenuation at both $\omega = \omega_\mathrm{s}$ and $\omega = \pi$.

N	n	$c_{N,n}(\omega_\mathrm{r})$
2	1	$1-\cos(\omega_\mathrm{r})$
4	1	$1-\cos(\omega_\mathrm{r})$
	2	$1-\cos(2\omega_\mathrm{r})$
6	1	$1-\cos(\omega_\mathrm{r})$
	2	$1-\cos(2\omega_\mathrm{r})$
	3	$10-9\cos(\omega_\mathrm{r})-\cos(3\omega_\mathrm{r})$
8	1	$1-\cos(\omega_\mathrm{r})$
	2	$1-\cos(2\omega_\mathrm{r})$
	3	$7-6\cos(\omega_\mathrm{r})-\cos(3\omega_\mathrm{r})$
	4	$17-16\cos(2\omega_\mathrm{r})-\cos(4\omega_\mathrm{r})$
10	1	$1-\cos(\omega_\mathrm{r})$
	2	$1-\cos(2\omega_\mathrm{r})$
	3	$6-5\cos(\omega_\mathrm{r})-\cos(3\omega_\mathrm{r})$
	4	$11-10\cos(2\omega_\mathrm{r})-\cos(4\omega_\mathrm{r})$
	5	$126-100\cos(\omega_\mathrm{r})-25\cos(3\omega_\mathrm{r})-\cos(5\omega_\mathrm{r})$
12	1	$1-\cos(\omega_\mathrm{r})$
	2	$1-\cos(2\omega_\mathrm{r})$
	3	$11/2-9/2\cos(\omega_\mathrm{r})-\cos(3\omega_\mathrm{r})$
	4	$9-8\cos(2\omega_\mathrm{r})-\cos(4\omega_\mathrm{r})$
	5	$66-50\cos(\omega_\mathrm{r})-15\cos(3\omega_\mathrm{r})-\cos(5\omega_\mathrm{r})$
	6	$262-225\cos(2\omega_\mathrm{r})-36\cos(4\omega_\mathrm{r})-\cos(6\omega_\mathrm{r})$
14	1	$1-\cos(\omega_\mathrm{r})$
	2	$1-\cos(2\omega_\mathrm{r})$
	3	$26/5-21/5\cos(\omega_\mathrm{r})-\cos(3\omega_\mathrm{r})$
	4	$8-7\cos(2\omega_\mathrm{r})-\cos(4\omega_\mathrm{r})$
	5	$143/3-35\cos(\omega_\mathrm{r})-35/3\cos(3\omega_\mathrm{r})-\cos(5\omega_\mathrm{r})$
	6	$127-105\cos(2\omega_\mathrm{r})-21\cos(4\omega_\mathrm{r})-\cos(6\omega_\mathrm{r})$
	7	$1761-1225\cos(\omega_\mathrm{r})-441\cos(3\omega_\mathrm{r})-49\cos(5\omega_\mathrm{r})-\cos(7\omega_\mathrm{r})$
16	1	$1-\cos(\omega_\mathrm{r})$
	2	$1-\cos(2\omega_\mathrm{r})$
	3	$5-4\cos(\omega_\mathrm{r})-\cos(3\omega_\mathrm{r})$
	4	$37/5-32/5\cos(2\omega_\mathrm{r})-\cos(4\omega_\mathrm{r})$
	5	$39-28\cos(\omega_\mathrm{r})-10\cos(3\omega_\mathrm{r})-\cos(5\omega_\mathrm{r})$
	6	$87-70\cos(2\omega_\mathrm{r})-16\cos(4\omega_\mathrm{r})-\cos(6\omega_\mathrm{r})$
	7	$715-490\cos(\omega_\mathrm{r})-196\cos(3\omega_\mathrm{r})-28\cos(5\omega_\mathrm{r})-\cos(7\omega_\mathrm{r})$
	8	$3985-3136\cos(2\omega_\mathrm{r})-784\cos(4\omega_\mathrm{r})-64\cos(6\omega_\mathrm{r})-\cos(8\omega_\mathrm{r})$

Table 1: Function $c_{N,n}(\omega_\mathrm{r})$ for different values of N.

In order to find the value ω_r, we solve $\phi_\alpha(\omega_\mathrm{p}, A_\mathrm{p}, \omega_\mathrm{r}) = \phi_\alpha(\pi, A_\mathrm{s}, \omega_\mathrm{r})$. Similarly, we can estimate the order of the allpass filter N by solving $\phi_\alpha(\omega_\mathrm{p}, A_\mathrm{p}, \omega_\mathrm{r}) = \phi_\alpha(\omega_\mathrm{s}, A_\mathrm{s}, \omega_\mathrm{r})$.

Example 3. We consider the design parameters as in Examples 1 and 2, with $A_\mathrm{p} = 3.42\,\mathrm{dB}$ and $\omega_\mathrm{r} = 0.788687\pi$. The resulting magnitude response is shown in Figs. 2(a) and 2(b) in dashed line.

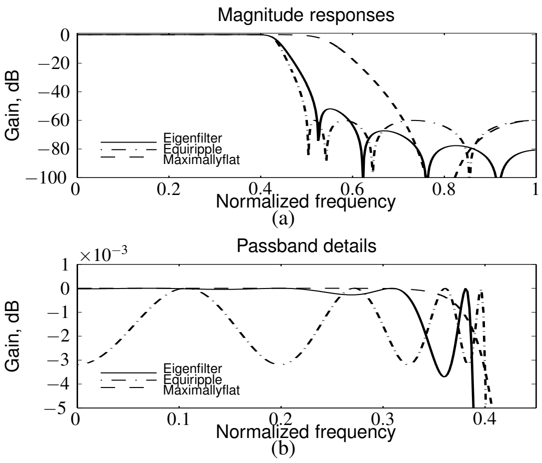

Figure 2: Magnitude responses of the designed filters.

4. CONCLUDING REMARKS

A novel structure for the design of real-valued, causal, and stable IIR filters is presented. It is composed of one real-valued and two complex-valued allpass filters. We show that by applying 3-point IDFT to the allpass filters, a set of three IIR filters that are doubly complementary can be obtained. This means that the resulting lowpass filter has low sensitivity to the filter quantization in the passband region. We also show how the problem of designing a lowpass filter is reduced to designing a complex-valued allpass filter with desired characteristics. Additionally, three different approaches to design lowpass filters suitable for the proposed structure are presented, that is, eigenfilter, equiripple, and maximally flat approaches. Provided design examples illustrate the techniques.

REFERENCES

[1] S. K. Mitra, *Digital Signal Processing: A computer based approach*, 3rd ed. Mc Graw Hill, 2006.

[2] P. P. Vaidyanathan, S. K. Mitra, and Y. Neuvo, "A new approach to the realization of low sensitivity IIR digital filters," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 34, no. 2, pp. 350–361, April 1986.

[3] P. P. Vaidyanathan, P. A. Regalia, and S. K. Mitra, "Design of doubly complementary IIR digital filters using a single complex allpass filter, with multirate applications," *IEEE Trans. Circuits Syst.*, vol. 34, no. 4, pp. 378–389, April 1987.

[4] M. Ikehara, M. Funaishi, and H. Kuroda, "Design of complex allpass networks using Reméz algorithm," *IEEE Trans. Circuits Syst. II*, vol. 39, no. 8, pp. 549–556, August 1992.

[5] M. Lang, "Allpass filter design and applications," *IEEE Trans. Signal Process.*, vol. 46, no. 9, pp. 2505–2514, September 1998.

[6] I. W. Selesnick, "Low-pass filter realizable as all-pass sums: Design via a new flat delay filter," *IEEE Trans. Circuits Syst. II*, vol. 46, no. 1, pp. 40–50, January 1999.

[7] A. Fernandez-Vazquez and G. Jovanovic-Dolecek, "A new method for the design of IIR filters with flat magnitude response," *IEEE Trans. Circuits Syst. I*, vol. 53, no. 8, pp. 1761–1771, August 2006.

[8] M. Renfords and T. Saramäki, "Recursive *N*th-band digital filters—part I: Design and properties," *IEEE Trans. Circuits Syst.*, vol. 34, no. 1, pp. 24–39, January 1987.

[9] ——, "Recursive *N*th-band digital filters—part II: Design of multistage decimators and interpolators," *IEEE Trans. Circuits Syst.*, vol. 34, no. 1, pp. 40–51, January 1987.

[10] A. Fernandez-Vazquez and G. Jovanovic-Dolecek, "Design of complex allpass filters," in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing (ICASSP'04)*, vol. II, Montreal, Canada, May 2004, pp. 393–396.

[11] S.-C. Pei and J.-J. Shyu, "Eigenfilter design of 1-D and 2-D IIR digital all-pass filters," *IEEE Trans. Signal Process.*, vol. 42, no. 4, pp. 966–968, April 1994.

[12] J. Makhoul, "On the eigenvectors of symmetric Toeplitz matrices," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 29, no. 4, pp. 868–872, August 1981.

[13] L. Datta and S. D. Morgera, "Comments and corrections on "on the eigenvectors of symmetric Toeplitz matrices"," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 32, no. 2, pp. 440–441, April 1984.

[14] A. L. Andrew, "Further comments on "on the eigenvectors of symmetric Toeplitz matrices"," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 33, no. 4, p. 1013, August 1985.

[15] X. Zhang and H. Iwakura, "Design of IIR digital allpass filters based on eigenvalue problem," *IEEE Trans. Signal Process.*, vol. 47, no. 2, pp. 554–559, February 1999.