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ESPRITWED-UG AND AV-ESPRITWED : TWO NEW LINEAR SUBSPACE ALGORITHMS FOR TIME DELAY ESTIMATION

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ABSTRACT

Two improvements of the “linear ESPRIT” algorithm in [1] are proposed and applied to the time delay estimation (TDE) from radar data within the microwave range. At first, “linear ESPRIT” is adapted to the TDE. Then, two new linear subspace algorithms, namely ESPRITWED-UG (ESPRIT Without EigenDecomposition) and AV-ESPRITWED (AVerage ESPRIT Without EigenDecomposition) are proposed. Contrary to [1], both algorithms take the whole bandwidth into account. Computer tests enable to assess the performances of the algorithms on the measurements of the layer thickness of civil engineering materials from GPR data. The new methods show improved noise robustness and smaller standard deviation in comparison with linear ESPRIT [1] and SWEDE [2].

1. INTRODUCTION

Among the subspace algorithms, ESPRIT affords a direct parameter estimation with both a lower computational load and high resolution capability [3]. In the nineties, a new family of subspace algorithms, called “linear” subspace algorithm, i.e. BEWE, PM, OPM, SWEDE [4]-[8], has emerged. They provide high resolution capability without any eigen decomposition requirement. By combining the advantages of both families, [9] and [1] have further reduced the computer burden of ESPRIT with the help of PM and SWEDE algorithms, respectively.

When applied to time delay estimation (TDE), the latter idea can lead to different algorithms according to the data arrangements in the processing. For example, a first solution has been proposed in [10] in context of any known noise, which is based on the processing of data from no-overlapping frequency bandwidths.

Contrary to [10], in this paper, the noise covariance matrix is unknown and assumed to be block-diagonal as in [2]. Under these conditions, the performances of the method proposed in [10] are not optimal since only reduced frequency bandwidth is used.

This paper focuses on the “linear ESPRIT” method [1], which processes the data from overlapping frequency bandwidths as the ESPRIT algorithm conventionnally does. This paper proposes to adapt the “linear ESPRIT” method in order to measure small thicknesses of stratified materials by processing Ground Penetrating Radar (GPR) data in microwave range.

In practice, the GPR measurements can provide a very large number of data. Thus, a quick algorithm is necessary. The enhanced resolution of subspace algorithms is also required because new pavement techniques allow to estimate reduced layer thickness between 1 and 3 cm. In comparison, the performances of a classical band-limited GPR radar are currently limited to measure layer thickness larger than 5 cm [10, 11].

In this paper, two improvements of the “linear ESPRIT” method are proposed, namely ESPRITWED-UG and AV-ESPRITWED. As opposed to [1], both algorithms take the whole bandwidth into account. The second algorithm can be viewed as the generalization of the first one.

The new methods show simultaneously an improved noise robustness and smaller standard deviation on the estimated thickness in comparison with both the ”linear ESPRIT” [1] and SWEDE [2] algorithms.

2. RADAR DATA MODELING

We consider the backscattered signal from an horizontal stratified medium with $K-1$ layers. Each layer of the medium is characterized by its thickness and its dielectric permittivity. In far field conditions, i.e. plane waves, the received signal is composed of the K backscattered echoes from each interface. For non-dispersive medium, each echo is a simple time-shifted replica of the emitted radar pulse $e(t)$. At vertical incidence, the amplitudes s_k depend on the dielectric contrast between layers through the Fresnel coefficients at nadir [11]. An additive noise $n(t)$ is assumed to represent the measurement uncertainties. Then the received signal can be written as:

$$r(t) = \sum_{k=1}^{K} s_k e(t-T_k) + n(t) \tag{1}$$

To perform the TDE with spectral analysis techniques, the received time signal is transformed to the frequency domain [12]. It is then written as a linear combination of cisoids modulated by the radar pulse as follows:

$$\tilde{r}(f) = \sum_{k=1}^{K} s_k \tilde{e}(f) e^{-2j\pi f T_k} + \tilde{n}(f) \tag{2}$$

where the $\tilde{\ }$ symbol represents the associated Fourier transform of the time function. For N discrete frequencies (f_n) within the bandwidth B, the received signal, called observation vector $\mathbf{r}$, can be written in the following matrix form :

$$\mathbf{r} = \mathbf{\Lambda A s} + \mathbf{n} \tag{3}$$

with the following notations :

- $\mathbf{r} = [\tilde{r}(f_1)\ \tilde{r}(f_2)\ \cdots\ \tilde{r}(f_N)]^T$ is the data vector which may represent either the Fourier transform of the GPR signal or the measurements from a step frequency radar;
- $\mathbf{\Lambda} = diag(\tilde{e}(f_1), \tilde{e}(f_2), ..., \tilde{e}(f_N))$ is a diagonal matrix whose diagonal elements are the amplitudes of the Fourier tranform of the radar pulse
- $\mathbf{A} = [\mathbf{a}(T_1)\ \mathbf{a}(T_2)\ \ldots\ \mathbf{a}(T_K)]$ is called mode matrix whose columns are defined as below;
- $\mathbf{a}(T_k) = \left[e^{-2j\pi f_1 T_k}\ e^{-2j\pi f_2 T_k}\ \ldots\ e^{-2j\pi f_N T_k}\right]^T$ is called either mode or steering vector; $\mathbf{\Lambda a}(T_k)$ represents the parameterized modeling of the steering vector;
- $\mathbf{s} = [s_1\ s_2\ \cdots\ s_K]^T$ is the source vector composed of the echo amplitudes s_k;
- $\mathbf{n} = [\tilde{n}(f_1)\ \tilde{n}(f_2)\ \cdots\ \tilde{n}(f_N)]^T$ is the noise vector in which each element is a white Gaussian noise with zero mean and variance σ^2;
- $f_n = f_1 + (n-1)\Delta f$ are the equispaced frequency samples, where f_1 is the beginning of the bandwidth and Δf is the frequency difference between samples.

Let $\mathbf{\Gamma_S}$ and $\sigma^2\mathbf{\Sigma}$ be the covariance matrices of the source vector and the noise vector respectively, the covariance matrix of the data vector (3) can be written as :

$$\mathbf{\Gamma} = \mathbf{\Lambda A \Gamma_S A}^H \mathbf{\Lambda}^H + \sigma^2 \mathbf{\Sigma} \tag{4}$$

3. THE “LINEAR ESPRIT” ALGORITHM FOR TDE

This method has been proposed in [1] to reduce the computational burden of the ESPRIT method. The principle of the ESPRIT algorithm has been combined with that of the SWEDE method. In comparison with the conventional ESPRIT algorithm, “linear ESPRIT” does not require the eigendecomposition of the data covariance matrix. Besides, in comparison with SWEDE, “linear ESPRIT” does not require the pseudo-spectrum calculation, it allows a direct estimation of time delays.

This method has been initially formulated for array signal processing [1]. For this algorithm, the property of invariance between the submatrices of the covariance matrix (eq. 4) is exploited. In this section, this method is adapted for time delay estimation (TDE).

As the first step of this adaptation, the observation vector is whitened by the pulse. Thus, the new observation vector, $\mathbf{y}$, becomes:

$$\mathbf{y} = \mathbf{\Lambda}^{-1}\mathbf{r} = \mathbf{As} + \mathbf{\Lambda}^{-1}\mathbf{n} \tag{5}$$

and the associated covariance matrix is :

$$\mathbf{\Gamma_y} = \mathbf{A\Gamma_S A}^H + \sigma^2 \mathbf{\Lambda}^{-1}\mathbf{\Sigma}\mathbf{\Lambda}^{-H} \tag{6}$$

The noise covariance matrix $\sigma^2\mathbf{\Xi} = \sigma^2\mathbf{\Lambda}^{-1}\mathbf{\Sigma}\mathbf{\Lambda}^{-H}$ is assumed to be block diagonal as in [2].

Then, the algorithm is based on the following partitioning of the mode matrix $\mathbf{A}$ into three frequency subbands $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$, each of them being of size (K,K), (K,K) and $(N-2K,K)$ respectively:

$$\mathbf{A}^H = \left[\mathbf{A}_1^H\ \mathbf{A}_2^H\ \mathbf{A}_3^H\right] \tag{7}$$

To make the source identification possible, the condition $N-2K \geq 1$ is assumed. In addition to that, it is assumed that the matrix $\mathbf{A}$ is of full rank, therefore the submatrices $\mathbf{A}_1$ and $\mathbf{A}_2$ are not singular. Considering the covariance matrix $\mathbf{\Gamma_y}$ whose partitioning is fitted to that of $\mathbf{A}$ as:

$$\mathbf{\Gamma_y} = \begin{bmatrix} \mathbf{\Gamma_y}_{11} & \mathbf{\Gamma_y}_{12} & \mathbf{\Gamma_y}_{13} \\ \mathbf{\Gamma_y}_{21} & \mathbf{\Gamma_y}_{22} & \mathbf{\Gamma_y}_{23} \\ \mathbf{\Gamma_y}_{31} & \mathbf{\Gamma_y}_{32} & \mathbf{\Gamma_y}_{33} \end{bmatrix} \tag{8}$$

where $\mathbf{\Gamma_y}_{ij}$ are the block elements of the covariance matrix $\mathbf{\Gamma}_y$ and are defined as:

$$\mathbf{\Gamma_y}_{ij} = \mathbf{A}_i \mathbf{\Gamma_s} \mathbf{A}_j^H + \sigma^2 \mathbf{\Xi}_{ij} = \mathbf{\Gamma}_{\mathbf{y}ji}^H \tag{9}$$

with $\sigma^2\mathbf{\Xi}_{ij} = \sigma^2\mathbf{\Lambda}_i^{-1}\mathbf{\Sigma}_{ij}\mathbf{\Lambda}_j^{-H}$ a block-diagonal noise covariance matrix. This method proposes to use the partitioning of the ESPRIT method on the $\mathbf{U}$ matrix defined as:

$$\mathbf{U} = \begin{pmatrix} \mathbf{\Gamma_y}_{21} \\ \mathbf{\Gamma_y}_{31} \end{pmatrix} = \begin{pmatrix} \mathbf{U}_1 \\ - \end{pmatrix} = \begin{pmatrix} - \\ \mathbf{U}_2 \end{pmatrix} \tag{10}$$

with $\mathbf{\Gamma_y}_{21}$ and $\mathbf{\Gamma_y}_{31}$ the two off-diagonal blocks of the covariance matrix of size (K,K) and $(N-2K,K)$ respectively. The two sub-matrices $\mathbf{U}_1$ and $\mathbf{U}_2$ are defined as the $N-K-1$ upper lines and the $N-K-1$ lower lines of $\mathbf{U}$ respectively. This method is theoretically not disturbed by the presence of noise as long as the noise covariance matrix is block diagonal, i.e. $\mathbf{\Sigma}_{ij} = 0$ for $i \neq j$. Then, this algorithm can handle a greater variety of noise models than other subspace algorithms can do.

Next, let us define an $(N-K,K)$ matrix $\mathbf{A}_4$ including the $N-K$ lowest lines of $\mathbf{A}$ as:

$$\mathbf{A}_4 = \begin{pmatrix} \mathbf{A}_2 \\ \mathbf{A}_3 \end{pmatrix} = \begin{pmatrix} - \\ \mathbf{A}_6 \end{pmatrix} = \begin{pmatrix} \mathbf{A}_5 \\ - \end{pmatrix} \tag{11}$$

with $\mathbf{A}_5$ and $\mathbf{A}_6$ two sub-matrices defined as the $N-K-1$ upper lines and the $N-K-1$ lower lines of $\mathbf{A}_4$ respectively. The two sub-matrices $\mathbf{A}_5$ and $\mathbf{A}_6$ are related to each other by a diagonal matrice $\mathbf{\Phi}$ of size (K,K), whose elements are related to the time delays to be estimated:

$$\mathbf{A}_6 = \mathbf{A}_5\mathbf{\Phi} \tag{12}$$

$$\mathbf{\Phi} = diag\left(e^{-2i\pi\Delta f T_1}, ..., e^{-2i\pi\Delta f T_K}\right) \tag{13}$$

diag means the diagonal matrix whose diagonal elements are those in brackets. Theoretically, time delays can be estimated directly for the diagonal elements of the matrix $\mathbf{\Phi}$. However, the matrix $\mathbf{\Phi}$ can not be estimated directly from the data. In the following, we show how to retrieve the phase shifts of $\mathbf{\Phi}$ from the block elements of the data covariance matrix $\mathbf{\Gamma_y}$. It can be shown that there exists a non singular matrix $\mathbf{Z}$ satisfying the following linear relation between the two block elements of the matrix $\mathbf{U}$:

$$\mathbf{U}_1 = \mathbf{U}_2\mathbf{Z} \tag{14}$$

From equations (10) and (11), we get $\mathbf{U}_1 = \mathbf{A}_5\mathbf{\Gamma_s}\mathbf{A}_1^H$ and $\mathbf{U}_2 = \mathbf{A}_6\mathbf{\Gamma_s}\mathbf{A}_1^H$. Using the latter relations and equation (12), we obtain:

$$\mathbf{\Phi}^{-1} = \mathbf{\Gamma_s}\mathbf{A}_1^H\mathbf{Z}\left(\mathbf{\Gamma_s}\mathbf{A}_1^H\right)^{-1} \tag{15}$$

This last formula means that the two matrices $\mathbf{Z}$ and $\mathbf{\Phi}^{-1}$ are similar, and consequently share the same eigenvalues. Then the search for the eigenvalues of $\mathbf{Z}$ leads to the time delay associated with each echo. The time delay is estimated by the angles of the eigenvalues of $\mathbf{Z}$. Finally, the thickness is estimated from the estimated time delays and from the known dielectric constant. The matrix $\mathbf{Z}$ is given from eq. (14) by the following least square solution (LS):

$$\mathbf{Z} = \left(\mathbf{U}_2^H\mathbf{U}_2\right)^{-1}\mathbf{U}_2^H\mathbf{U}_1 \tag{16}$$

4. ESPRITWED-UG

In the above method, the number of lines of the matrix $\mathbf{U}$ depends on the number of echoes K, i.e. it is equal to $N-K$. Some improvement can be expected if the whole frequency bandwidth is taken into account in the method. Therefore, we propose a new algorithm called ESPRITWED-UG which works with a new definition of the $\mathbf{U}$ matrix, for which the number of lines is equal to N. Let us define the matrix $\mathbf{U}_G$ of size (N,K) as:

$$\mathbf{U}_G = \begin{bmatrix} \mathbf{\Gamma}_{\mathbf{A}11} \\ \mathbf{\Gamma}_{\mathbf{y}21} \\ \mathbf{\Gamma}_{\mathbf{y}31} \end{bmatrix} = \mathbf{A}\mathbf{\Gamma}_{\mathbf{s}}\mathbf{A}_1^H \tag{17}$$

where $\mathbf{\Gamma}_{\mathbf{A}11}$ represents the diagonal sub-matrix of the covariance matrix of the data vector without noise, i.e. $\mathbf{A}_1\mathbf{\Gamma}_{\mathbf{S}}\mathbf{A}_1^H$. It is defined as in [2] from the block elements of the covariance matrix $\mathbf{\Gamma}_y$, according to:

$$\mathbf{\Gamma}_{\mathbf{A}11} = \mathbf{\Gamma}_{\mathbf{y}12}\left(\mathbf{\Gamma}_{\mathbf{y}23}\mathbf{\Gamma}_{\mathbf{y}32}\right)^{-1}\mathbf{\Gamma}_{\mathbf{y}23}\mathbf{\Gamma}_{\mathbf{y}31}. \tag{18}$$

Then, the property of rotation invariance can be applied on the matrix $\mathbf{U}_G$, by introducing the following partitioning of $\mathbf{U}_G$:

$$\mathbf{U}_G = \begin{pmatrix} \mathbf{U}_1 \\ - \end{pmatrix} = \begin{pmatrix} - \\ \mathbf{U}_2 \end{pmatrix} \tag{19}$$

where $\mathbf{U}_1$ and $\mathbf{U}_2$ are the two sub-matrices defined as the $N-1$ upper lines and the $N-1$ lower lines of $\mathbf{U}_G$ respectively. The relations (14) and (15) remain valid and can be used to retrieve the time delays from the angles of the eigenvalues of $\mathbf{Z}$. The matrix $\mathbf{Z}$ is estimated according to (16).

5. AV-ESPRITWED : AVERAGE ESPRITWED

In this section, the mode matrix of data in (7) is partitioned into an increasing number of blocks M. Moreover, in comparison with the two last sections, the matrix $\mathbf{U}$ (on which the invariance property of ESPRIT is valid), is calculated as the average over the matrix columns of the modified covariance matrix of data (whose noise contribution is cancelled). Therefore, the new algorithm which is proposed in this section may be viewed as the general case of the latter one, when $M>3$.
AV-ESPRITWED method uses the following matrix which is divided into M^2 block matrices as follows:

$$\mathbf{\Gamma}_{\mathbf{x}} = \begin{bmatrix} \mathbf{\Gamma}_{\mathbf{A}11} & \mathbf{\Gamma}_{\mathbf{y}12} & \cdots & \mathbf{\Gamma}_{\mathbf{y}1M} \\ \mathbf{\Gamma}_{\mathbf{y}21} & \mathbf{\Gamma}_{\mathbf{A}22} & \cdots & \mathbf{\Gamma}_{\mathbf{y}2M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Gamma}_{\mathbf{y}M1} & \mathbf{\Gamma}_{\mathbf{y}M2} & \cdots & \mathbf{\Gamma}_{\mathbf{A}MM} \end{bmatrix} \tag{20}$$

with $\mathbf{\Gamma}_{\mathbf{x}ij} = \mathbf{\Gamma}_{\mathbf{y}ij} = \mathbf{A}_i\mathbf{\Gamma}_{\mathbf{S}}\mathbf{A}_j^H$ for $i \neq j$. $\mathbf{\Gamma}_{\mathbf{x}ii} = \mathbf{\Gamma}_{Aii}$ represents diagonal sub-matrix of $\mathbf{\Gamma}_{\mathbf{y}}$ without noise and is defined as $\mathbf{\Gamma}_{Aii} = \mathbf{A}_i\mathbf{\Gamma}_{\mathbf{S}}\mathbf{A}_i^H$. The mode matrix $\mathbf{A}$ is partitioned into M sub-matrices, as

$$\mathbf{A}^H = \begin{pmatrix} \mathbf{A}_1^H & \mathbf{A}_2^H & \dots & \mathbf{A}_M^H \end{pmatrix} \tag{21}$$

with $dim(\mathbf{A}_1) = \cdots = dim(\mathbf{A}_{M-1}) = (K,K)$, $dim(\mathbf{A}_M) = (K,(N-K(M-1)))$ and $N-K(M-1) \geq 1$.
Let us define $\mathbf{U}_i = \mathbf{A}\mathbf{\Gamma}_{\mathbf{s}}\mathbf{A}_i^H$, so $\mathbf{\Gamma}_{\mathbf{x}}$ becomes

$$\mathbf{\Gamma}_{\mathbf{x}} = (\mathbf{U}_1\ \mathbf{U}_2\ \cdots\ \mathbf{U}_M) \tag{22}$$

Next, introduce

$$\mathbf{U}_\Sigma = \frac{1}{M-1}\sum_{i=1}^{M-1}\mathbf{U}_i = \mathbf{A}\mathbf{\Gamma}_{\mathbf{s}}\mathbf{B} \tag{23}$$

with $\mathbf{B} = \frac{1}{M-1}\sum_{i=1}^{M-1}\mathbf{A}_i^H$.
Then , we can use the principle of ESPRITWED-UG by using the matrix $\mathbf{U}_\Sigma$ instead of the matrix $\mathbf{U}_G$. In this case, the matrix $\mathbf{Z}$ is obtained by the least square solution (LS):

$$\mathbf{Z} = \left(\mathbf{U}_{\Sigma2}^H\mathbf{U}_{\Sigma2}\right)^{-1}\mathbf{U}_{\Sigma2}^H\mathbf{U}_{\Sigma1} \tag{24}$$

with the two sub-matrices $\mathbf{U}_{\Sigma1}$ and $\mathbf{U}_{\Sigma2}$ defined as the $N-1$ upper lines and the $N-1$ lower lines of $\mathbf{U}_\Sigma$ respectively.
To use this method, we must estimate the diagonal sub-matrices without noise $\mathbf{\Gamma}_{Aii}$. For the case $M=3$, it can be shown that the two diagonal sub-matrices of size (K,K) are given by:

$$\mathbf{\Gamma}_{\mathbf{A}11} = \mathbf{\Gamma}_{\mathbf{y}12}\left(\mathbf{\Gamma}_{\mathbf{y}23}\mathbf{\Gamma}_{\mathbf{y}32}\right)^{-1}\mathbf{\Gamma}_{\mathbf{y}23}\mathbf{\Gamma}_{\mathbf{y}31} \tag{25}$$

$$\mathbf{\Gamma}_{\mathbf{A}22} = \mathbf{\Gamma}_{\mathbf{y}21}\left(\mathbf{\Gamma}_{\mathbf{y}13}\mathbf{\Gamma}_{\mathbf{y}31}\right)^{-1}\mathbf{\Gamma}_{\mathbf{y}13}\mathbf{\Gamma}_{\mathbf{y}32} \tag{26}$$

For any $M>3$, some recursive formulas can be found for calculating the diagonal sub-matrix without noise $\mathbf{\Gamma}_{\mathbf{A}(M-1)(M-1)}$ of size (K,K); it is given by:

$$\begin{aligned}\mathbf{\Gamma}_{A(M-1)(M-1)} &= \mathbf{\Gamma}_{\mathbf{y}(M-1)(M-2)} \times \\ &\left[\begin{pmatrix}\mathbf{\Gamma}_{\mathbf{y}(M-2)(M)} & \mathbf{\Gamma}_{\mathbf{y}(M-2)(1:M-3)}\end{pmatrix}\begin{pmatrix}\mathbf{\Gamma}_{\mathbf{y}(M)(M-2)} \\ \mathbf{\Gamma}_{\mathbf{y}(1:M-3)(M-2)}\end{pmatrix}\right]^{-1} \\ &\times \begin{pmatrix}\mathbf{\Gamma}_{\mathbf{y}(M-2)(M)} & \mathbf{\Gamma}_{\mathbf{y}(M-2)(1:M-3)}\end{pmatrix}\begin{pmatrix}\mathbf{\Gamma}_{\mathbf{y}(M)(M-1)} \\ \mathbf{\Gamma}_{\mathbf{y}(1:M-3)(M-1)}\end{pmatrix}\end{aligned}$$

The diagonal sub-matrix $\mathbf{\Gamma}_{\mathbf{A}(M-i-1)(M-i-1)}$ of size (K,K) can be estimated from the matrix $\mathbf{\Gamma}_{\mathbf{y}}$, which is divided into $(M-i)^2$ blocks, with i ranging between 1 and $M-3$.
Thereafter, AV-ESPRITWED with indexing $m = M-1$ means the use of the method AV-ESPRITWED with an average of m sub-matrices of size (N,K) each, i.e. the matrix $\mathbf{\Gamma}_{\mathbf{x}}$ is divided into $(m+1)^2$ sub-matrices.
Finally, the AV-ESPRITWED ($m = M-1$) algorithm is processed according to the following nine steps : *i)* Estimate the non-diagonal sub-matrices $\mathbf{\Gamma}_{\mathbf{x}ij}$ for $i \neq j$, with $i \leq m$ and $j \leq m$; *ii)* Estimate the diagonal sub-matrices without noise $\mathbf{\Gamma}_{\mathbf{A}ii}$, with $i \leq m$; *iii)* Form the matrices $\mathbf{U}_i$ from eqn. (22), with $i \leq m$; *iv)* Compute the matrix $\mathbf{U}_\Sigma$ from eqn. (23); *v)* Form the matrices $\mathbf{U}_{\Sigma1}$ and $\mathbf{U}_{\Sigma2}$ from the partitioning of the matrix $\mathbf{U}_\Sigma$; *vi)* Estimate the matrix $\mathbf{Z}$ by the least

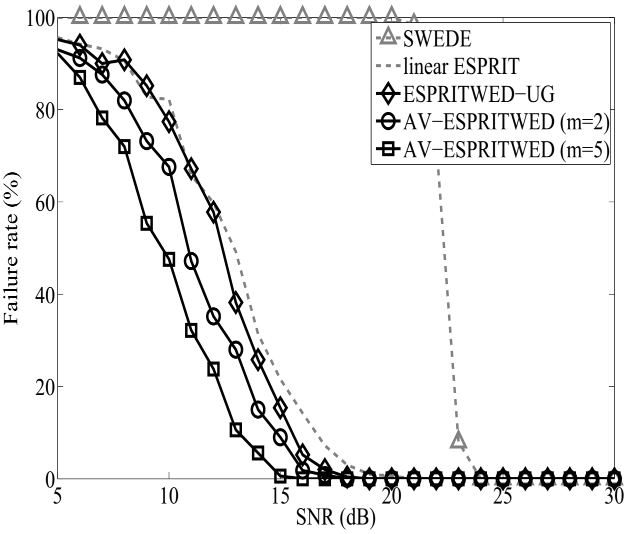

Figure 1: Failure rate vs. SNR for the four methods after 500 Monte Carlo runs; $B\Delta\tau$= 0.3.

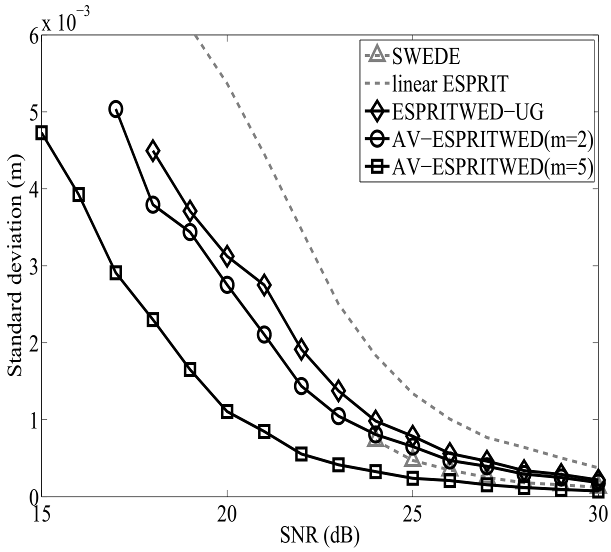

Figure 2: Standard deviation vs. SNR for the four methods after 500 Monte Carlo runs; $B\Delta\tau$= 0.3.

square solution given in eqn. (24); *vii)* Calculate the K eigenvalues of the matrix $\mathbf{Z}$; *viii)* Estimate the time delays from the arguments of the eigenvalues of $\mathbf{Z}$; *ix)* Compute the thickness from the estimated time delays and from the given dielectric constant.

6. COMPUTER TESTS

In this section, the performances of the proposed algorithms are compared with respect to those of both the algorithm presented in section 3 and the G-SWEDE algorithm by [2] which was adapted in [10] for TDE.

The simulated data obey to the model in eqn.(3). They represent two overlapping backscattered echoes ($K = 2$) from a one layer medium, which is 2 cm thick with 5 as dielectric constant. The radar pulse $e(t)$ is a ricker pulse and is defined as the second derivative of a gaussian pulse. The central frequency of the pulse is 1.5 GHz and the frequency bandwidth is 1 GHz. The data vector is made of 41 samples within a 1 GHz frequency bandwidth, B. It means that the simulations are carried out with a $B\Delta\tau$ product of 0.3. These simulations are performed with a Gaussian white noise.

To compare the performances of the algorithms at the best conditions, the two echoes are assumed to be uncorrelated. The signal to noise ratio (SNR) is defined as the power ratio between the strongest echo and the noise variance. The powers of the two sources are fixed to 0 and -6 dB respectively.

The performance is established with a Monte Carlo process which consists of 500 independent runs of the algorithms. For each run, the correlation matrix is estimated from $L = 200$ independent snapshots, from which the various algorithms perform an estimation of the thickness and a failure test. The rank of the signal subspace has been fixed to the expected number of echoes, i.e. 2. It can be also estimated with a detection criterion [13]. Then, the failure rate and the standard deviation on the measured thickness have been calculated to assess the overall performance. The detection from each run of the algorithms is considered as successful when the estimated time delays belong to the interval $[T_1 - \Delta\tau/2\ ,\ T_2 + \Delta\tau/2\ ]$, where T_1 and T_2 ($T_1 < T_2$) are the two true time delays and $\Delta\tau = T_2 - T_1$ is the corresponding differential time shift.

Figures (1) and (2) show the failure rate and the standard deviation on the estimated thickness with regards to SNR, for the four algorithms : SWEDE, linear ESPRIT, ESPRITWED-UG and AV-ESPRITWED. As expected, the performances improve with increasing SNR.

Among the tested algorithms, AV-ESPRITWED (m=5) shows the best noise robustness, followed by AV-ESPRTIWED (m=2), ESPRITWED-UG, Linear ESPRIT and SWEDE. The 1% failure rate is commonly used in the literature to define the detection threshold. The corresponding SNR thresholds are 15 dB, 16.8 dB, 17.6 dB, 19 dB, 24dB for AV-ESPRITWED (m=5), AV-ESPRITWED (m=2), ESPRITWED-UG, linear ESPRIT and SWEDE algorithms respectively. Figure (2) shows the standard deviation (std) variations on the estimated thickness with regards to SNR when the failure rate is below 1 %. As expected, the accuracy on the thickness improves with the increasing of SNR. AV-ESPRITWED (m=5) shows the best accuracy, followed by SWEDE, AV-ESPRTIWED (m=2), ESPRITWED-UG, and Linear ESPRIT. From these simulation results, AV-ESPRITWED (m=5) shows the best performance.

Table 1 shows the approximated number of complex multiplications in order to estimate the useful block elements of the covariance matrix, the matrix $\mathbf{Z}$, and to build the matrices $\mathbf{U}_G$ and $\mathbf{U}_\Sigma$ for linear ESPRIT, ESPRITWED-UG and AV-ESPRITWED algorithms. In comparaison with linear ESPRIT, the proposed methods have a computational load a little higher. Indeed, the proposed methods take into account all off-diagonal blocks of the covariance matrix and the diagonal sub-matrices of the covariance matrix without noise.

Algorithms	Linear ESPRIT	ESPRITWED UG	AV-ESPRIT WED(m=M-1)
Correlation sub-matrices	$K(N-K)L$	$K(2N-3K)L$	$K(2N-3K)L$
Build of matrix $\mathbf{U}_i$	-	$i=G$ $2NK^2$	$i=\Sigma$ $2mNK^2$
Estimation of $\mathbf{Z}$	$2NK^2$	$2NK^2$	$2NK^2$

Table 1: Order of necessary complex multiplications for linear ESPRIT, ESPRITWED-UG and AV-ESPRITWED algorithms ($N >> K$).

7. CONCLUSION

In this paper, the linear ESPRIT method has been firstly adapted to solve the problem of time delay estimation for processing GPR data within the scope of thickness measurement.
Besides, two new linear subspace algorithms have been proposed, namely ESPRITWED-UG and AV-ESPRITWED, which allow to exploit the whole frequency bandwidth. They afford high resolution capability with reduced computational burden.
The performances of ESPRITWED-UG and AV-ESPRITWED have been compared with those of both the linear ESPRIT and the conventional SWEDE algorithms. Simulation results show the improvement in estimation performance provided by exploiting the whole frequency bandwidth of received signal. The AV-ESPRITWED algorithm has shown the best noise robustness and the smallest standard deviation for $m \geq 2$.

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