FILTER DESIGN FOR THE DETECTION OF COMPACT SOURCES EMBEDDED IN NON-STATIONARY NOISE PLUS A DETERMINISTIC BACKGROUND

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ABSTRACT

Detection of a compact signal with a given profile mixed with a deterministic background and non-stationary noise is an issue of great interest in many scientific areas. In this paper, we propose a methodology based on filter design, that allows both the removal of the background and the estimation of the signal amplitude. Under certain conditions this filter is a wavelet with several vanishing moments. In particular, we apply this technique to the detection and estimation of extragalactic point sources in microwave maps. We show with numerical simulations that the filter is very effective at source detection/estimation. We want to remark that this method can be also applied to the analysis of signals and images in other fields.

1. INTRODUCTION

Detection of deterministic signals embedded in non-stationary noise is a relevant problem in many scientific areas. A further complication can be the contribution of a deterministic background, which manifests itself on large scales, masking the presence of more compact signals. This situation is common in Astrophysics, for instance emission and absorption lines appear superposed to continuum emission in optical spectra of quasars [1]. Another example is given in microwave astronomy: typical maps of the cosmic microwave background (CMB) on the sky include the emission from extragalactic point sources and clusters of galaxies superposed on foreground emission due to our own Galaxy [2]. Moreover, non-stationary noise produced by the detectors contributes to the data.

The standard approach to this problem has been to fit the continuous deterministic emission to a baseline [1, 2] and remove such a contribution, applying then a matched filter [3, 4] or a wavelet [5, 6] to amplify the compact signal. This method has some drawbacks: the parameters of the baseline function are always obtained with statistical errors, a goodness of fit assumes implicitly that the noise is Gaussian and, in general, the equations for the parameters can be complicated.

In this paper we propose a different methodology that allows the removal of the continuous background without any specific fitting. This is an elegant procedure which is based on the design of a filter that gives the maximum amplification and removes the undesired background. Such a filter is obtained by solving a system of linear equations i.e. the inversion of a matrix constructed from the constraints and the non-stationary noise dispersion.

First of all, we will establish a general framework for designing the suitable filter. We shall in particular consider a background given by the superposition of a constant plus a gradient plus a curvature. This specific case can be applied to the detection of extragalactic point sources in CMB maps, an issue of great importance in the field of Cosmology [2].

In Section 2, we develop our methodology and design the filter in a general case and also in the specific case of a constant plus a gradient plus a curvature. In this last case, the filter is a wavelet with vanishing moments up to the second order [7, 8]. In Section 3, we carry out two-dimensional numerical simulations consisting of extragalactic point sources, non-stationary noise and a second order polynomial, defining the background. For the simulations, we take the characteristics of one of the channels, 70 GHz, of the ESA Planck satellite [9, 10]. In this section we also show the results. Finally, we draw the conclusions in Section 4.

2. METHODOLOGY

Let us assume an N-dimensional data set $d(\vec{x})$ written as a signal $s(\vec{x})$ corrupted by non-stationary noise $n(\vec{x})$

$$d(\vec{x}) = s(\vec{x}) + n(\vec{x}), \tag{1}$$

where the deterministic signal is given by two terms: a compact source of known profile $\tau(\vec{x})$ and unknown amplitude A and a second term, $b(\vec{x})$, that represents a background that can be decomposed in a known local basis $f^t(\vec{x}) = (f_1(\vec{x}), ..., f_r(\vec{x}))$ as

$$s(\vec{x}) = A\tau(\vec{x}) + b(\vec{x}), \quad b(\vec{x}) = \lambda^t f(\vec{x}), \tag{2}$$

λ being the coefficients of such a decomposition.

Let us assume that the noise properties can be characterized by a non-stationary white noise of dispersion $\sigma(\vec{x})$ and mean value zero

$$\langle n(\vec{x})\rangle = 0, \quad \langle n(\vec{x})n(\vec{x'})\rangle = S_N \sigma^2(\vec{x})\delta^N(\vec{x} - \vec{x'}), \tag{3}$$

S_N being the N-dimensional volume of the image.

We will design a filter $\psi(\vec{x})$ that will operate on the image in linear form

$$d_F(\vec{x}) = \int d\vec{y}\, d(\vec{y})\psi(\vec{x} - \vec{y}). \tag{4}$$

We will impose certain conditions on such a filter: i) we want to recover the amplitude of the source at the center of the source, ii) to have a minimum variance of the filtered field and iii) to erase the background. The first and second conditions give an unbiased estimator with maximum efficiency for the amplitude and the maximum amplification. The third condition allows us to eliminate the background and avoids making a fit to a baseline.

Therefore, let us assume that the source is at the origin, the first condition is equivalent to

$$\int d\vec{x}\,\tau(\vec{x})\psi(\vec{x}) = 1, \tag{5}$$

whereas the third condition is equivalent to the constraints

$$\int d\vec{x}\,f(\vec{x})\psi(\vec{x}) = 0. \tag{6}$$

On the other hand the variance of the filtered field is

$$\sigma_F^2(\vec{0}) = S_N \int d\vec{x}\,\sigma^2(\vec{x})\psi^2(\vec{x}). \tag{7}$$

Therefore, the filter design is obtained through the functional minimization of such a variance as a constrained problem with the Lagrangian

$$L = S_N \int d\vec{x}\,\sigma^2(\vec{x})\psi^2(\vec{x}) - 2\mu^t\left(\int d\vec{x}\,g(\vec{x})\psi(\vec{x}) - \beta\right), \tag{8}$$

where $g(\vec{x})$, β and the multiplier μ are the following $r+1$ vectors

$$g^t(\vec{x}) \equiv (\tau(\vec{x}), f_1(\vec{x}), ..., f_r(\vec{x})), \tag{9}$$

$$\beta^t \equiv (1,0,0,...0), \quad \mu^t \equiv (\mu_1, ..., \mu_{r+1}). \tag{10}$$

Then, it can be proven that the filter is given by

$$\psi(\vec{x}) = \sigma^{-2}(\vec{x})g^t M^{-1}\beta, \quad M \equiv \int d\vec{x}\,\sigma^{-2}(\vec{x})gg^t. \tag{11}$$

We remark that M is a $(r+1)\times(r+1)$ matrix and M^{-1} is its inverse matrix. The variance of the filtered field is now

$$\sigma_F^2(\vec{0}) = \beta^t M^{-1}\beta = M_{11}^{-1}. \tag{12}$$

As an application of this scheme, we shall consider that the background can be characterized by a constant plus a gradient. In such a case

$$g^t(\vec{x}) \equiv (\tau(\vec{x}), 1, x^1, x^2) \tag{13}$$

and the explicit solution for the filter is

$$\psi(\vec{x}) = \frac{C}{\sigma^2(\vec{x})}\langle \tau(\vec{x}) + \frac{Q}{P}[-1 + \sum_{i,j=1,2} h_{ij}^{-1} l^i (x^j - \frac{P^j}{P})]\rangle, \tag{14}$$

$$l^i \equiv \frac{P^i}{P} - \frac{Q^i}{Q}, \quad h^{ij} \equiv \frac{P^{ij}}{P} - \frac{P^i P^j}{P^2}, \tag{15}$$

$$C \equiv [R - \frac{Q^2}{P}(1 + \sum_{i,j=1,2} h_{ij}^{-1} l^i l^j)]^{-1}, \quad P \equiv \int \frac{d\vec{x}}{\sigma^2}, \tag{16}$$

$$P^i \equiv \int \frac{d\vec{x}}{\sigma^2} x^i, P^{ij} \equiv \int \frac{d\vec{x}}{\sigma^2} x^i x^j, Q \equiv \int \frac{d\vec{x}}{\sigma^2}\tau, \tag{17}$$

$$Q^i \equiv \int \frac{d\vec{x}}{\sigma^2}\tau x^i, R \equiv \int \frac{d\vec{x}}{\sigma^2}\tau^2. \tag{18}$$

The variance of the filtered field is given by $\sigma^2(\vec{0}) = S_N C$. We remark that in this particular case the inverse matrix of h_{ij} is only required.

Similarly, one can also include a background with curvature. In such case

$$g^t(\vec{x}) \equiv (\tau(\vec{x}), 1, x^1, x^2, (x^1)^2, (x^2)^2, x^1x^2) \tag{19}$$

and explicit analytical formulas can be obtained through the inversion of two matrices 2×2 and 3×3.

3. NUMERICAL SIMULATIONS AND RESULTS

The formalism developed above can be applied to source detection in a background with non-stationary noise plus a gradient and curvature. One particularly interesting case is source detection in astrophysical signals and images (CMB maps, optical spectra, infrared and radio images,...). We have simulated two-dimensional images with the following components: 1) a point source filtered with a Gaussian beam, placed at random in the image, 2) non-stationary noise, 3) a function $f(x^1, x^2)$ incorporating gradient and curvature

$$f \equiv a_{11}(x^1)^2 + a_{12}x^1x^2 + a_{22}(x^2)^2 + a_1x^1 + a_2x^2 + a_0 \tag{20}$$

The images have 64×64 pixels with a pixel size of $3'$ and are filtered with a Gaussian-shaped beam whose full width half maximum (FWHM) is $14'$. These conditions correspond to the characteristics of the 70 GHz channel of the Planck satellite, which will be launched in 2008 and will measure with unprecedented accuracy the anisotropies of the CMB. The non-stationary noise added to the images also fulfils the specifications of the 70 GHz Planck channel. We carry out simulations with two types of noise: one corresponding to the high noise zones (those in which the instruments take less data) and another for the low noise zones. In the former case the mean rms deviation of the noise is $\sigma = 3.5\times 10^{-5}$ (the observations are given in terms of $\frac{\Delta T}{T}$, i.e. the relative CMB temperature anisotropy) and the standard deviation of this rms deviation, which changes from pixel to pixel, is 8×10^{-6}; in the latter case the values are respectively $\sigma = 2.2\times 10^{-5}$ and 3.2×10^{-6}.

We consider Gaussian-filtered point sources with an amplitude $A = 1\sigma, 2\sigma, 3\sigma, 4\sigma$, so that we can study the performance of our filter for different signal-to-noise ratios. Finally, we add the function given in (20) and fix different values for the parameters, from 0.1σ to 1000σ, so that we explore the contribution of higher and lower gradients and curvatures. We design our filter according to (11), so that all the first and second moments vanish, what means that the results should be independent of the gradient and curvature. We will show that this is the case.

We carry out twenty simulations for each combination of specific values of the noise, amplitude of the source, gradient and curvature. We filter the data with the designed filter and analyze the filtered maps searching for the source. We detect the pixel of maximum temperature in the map and consider this pixel as the source position, we measure the temperature of this pixel and estimate the source amplitude as this value. We also calculate the amplification, defined as

$$\Delta = \frac{A_F/\sigma_F}{A/\sigma}, \tag{21}$$

Noise	A	error	Δ
low	**2**σ	0.12(0.06)	3.10(0.35)
	3σ	0.08(0.05)	2.75(0.19)
	4σ	0.06(0.03)	2.44(0.12)
high	**2**σ	0.11(0.05)	3.15(0.28)
	3σ	0.07(0.04)	2.80(0.14)
	4σ	0.05(0.03)	2.47(0.08)

Table 1: Relative errors in the source amplitude and amplification for low and high noise. The results are the same for different gradients and curvatures and change with the source amplitude, expressed in the second column in terms of the average noise r.m.s.. The relative error and the amplification are written in the third and fourth column respectively, we write the average of 20 simulations and its r.m.s. deviation (in parentheses)

i.e. the ratio between the signal to noise in the filtered and unfiltered maps (A_F is the amplitude of the source after applying the designed filter and σ_F is the r.m.s deviation of the filtered map).

The results are the same for the different values of gradient and curvature as expected from the characteristics of the filter. We prove, indeed, that the designed filter eliminates the contribution of $f(x^1,x^2)$. In Table 1, we show the average results for the different levels of the noise and source amplitude. We write the relative error in the estimated amplitude and the amplification. The error in the position of the source is at most of one pixel, but for the case $A = 1\sigma$, where in general the source is not detected, since even the filtered maps are dominated by the noise. We do not write the results for this case, since the detection is not possible. As shown in Table 1, the error in the estimated amplitude is just of a few percents and the amplification is high, allowing the detection of the source, assuming for instance a 5σ threshold.

We also show in Figure 1 four images corresponding to: 1) a Gaussian filtered source, 2) the source plus non-stationary noise, 3) the source plus non-stationary noise and the deterministic background (gradient and curvature) and 4) the image obtained after filtering the previous one with our designed filter. It is clear that we can recover the source which had been masked by the noise and the deterministic background.

4. CONCLUSIONS

In this paper, we have addressed the problem of designing a filter, able to recover the position and amplitude of a compact source embedded in non-stationary noise plus a deterministic background. This situation is common in Astrophysics: for instance, emission and absorption lines in quasars or images of the cosmic microwave background.

In section 1, we define our problem in precise mathematical terms, assuming that our data are composed by a compact source of known profile plus noise and a given deterministic background. We will design a suitable filter which satisfies the following conditions: i) we want to recover the amplitude of the source at its center ii) to have a minimum variance of the filtered field and iii) to erase the background. The first and second conditions give an unbiased estimator with maximum efficiency for the amplitude and the maximum amplification.

In order to find the filter, we have considered the appropriate Lagrangian, (8), for the constrained optimization problem in the general N-dimensional case. We have to minimize the variance (7), under conditions (5) and (6), which mean the unbiased estimation of the source amplitude and the removal of the background. The filter is easily obtained, (11), in the general case. In the particular case of a background given by a constant plus a gradient, we obtain explicitly the analytical expression for the filter, eqs 14-18.

In section 2 we apply our filter to a problem of great interest in Astrophysics: the detection of compact sources in CMB two-dimensional images. We carry out simulations of 64×64 pixels including one point source, filtered with a Gaussian beam, non-stationary noise and a deterministic background given by (20), i.e. composed by a constant, gradient and curvature. This background could be an approximation to Galactic components which contaminate CMB maps.

The simulations have the characteristics of the 70 GHz channel of the ESA Planck satellite, to be launched in 2008. The noise corresponds to the high and low noise zones of the survey. We include point sources with different signal to noise ratios (SNR=1-4) and backgrounds with different levels (coefficients from 0.1σ to 1000σ, with σ the average noise r.m.s.). We filter the maps with the filter given by (11), where we have imposed as constraints the vanishing of all the moments till the second order.

We check that the background is always erased by the filter and that point sources are detected in their original position and with very low relative errors (a few percents) in the amplitude, provided that the signal to noise ratio is equal or higher than two. The amplification is high, allowing the source detection.

The main results can be seen in Table 1 and Figure 1 shows how a source masked in a contaminated map is clearly seen in the filtered map.

We have proven the good performance of the filter; a more realistic approach should involve the simulation of several sources and the detection of local maxima in the filtered map which had to be compared with the source location in the original maps [5]. The contribution of the CMB and other Galactic foregrounds should also be taken into account. However, we want to point out that we only tried to show the effectiveness of the new filter in a simple scenario and its ability to amplify the signal and remove the moments. In a further work, we will analyze the possibility of removing the Galactic foregrounds in CMB maps by using this type of filter.

Acknowledgments

We acknowledge partial financial support from the Spanish Ministry of Education (MEC) under project AYA2007-

68058-C03-02. J. L. S. , on sabbatical leave, acknowledges financial support from the Spanish Ministry of Education and from the joint CNR-CSIC research project 2006-IT-0037. J. L. S. and F. A. thank the C.N.R. ISTI in Pisa for their hospitality during the sabbatical and a short stay, respectively, when part of this work was done.

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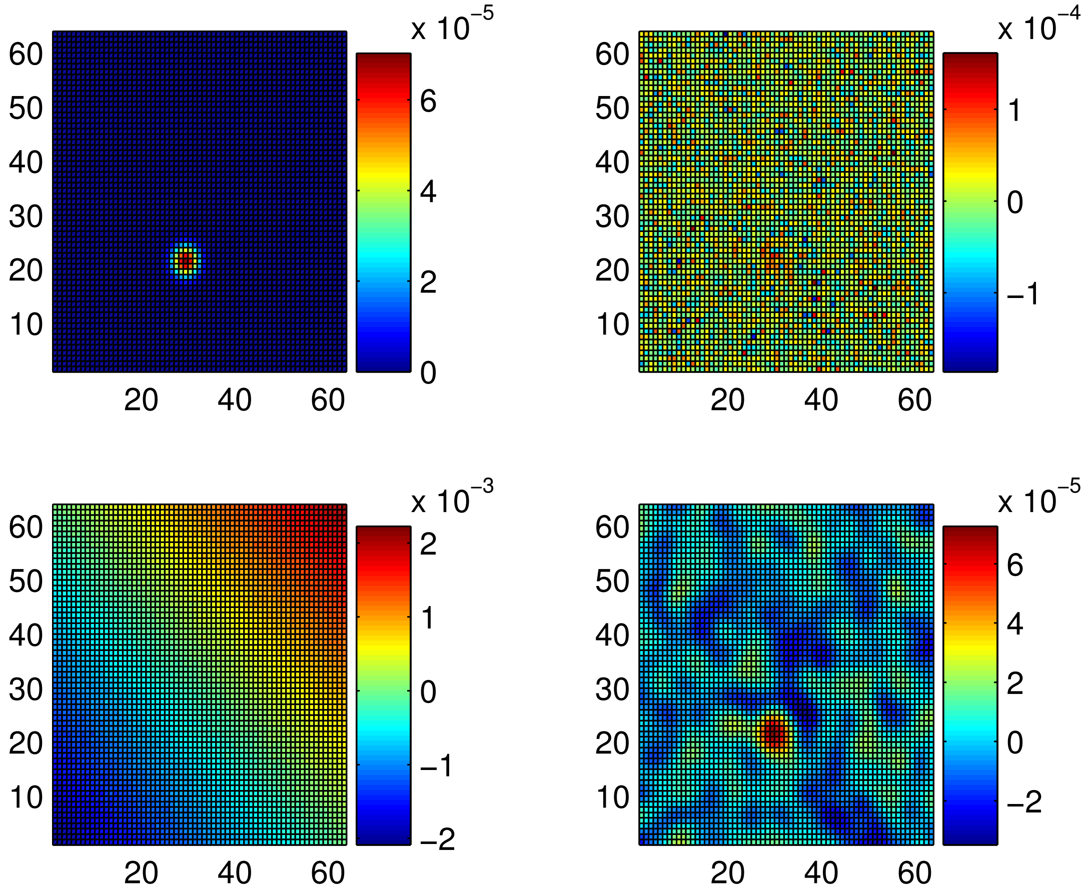

Figure 1: Top left: Image of a source filtered with a Gaussian beam, the amplitude of the source is 2σ with σ the average noise r.m.s. Top right: Image of the source plus non-stationary noise. The noise has been simulated with the characteristics of the high noise zone of the 70 GHz Planck channel. Bottom left: The previous image plus a deterministic background including gradient and curvature, the coefficients of eq.(20) are 1000 times the average noise r.m.s . Bottom right: The previous image after being filtered with the filter, eq. (11), designed in this paper.