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MODELING OF MULTICOMPONENT AM-FM SIGNALS USING FB EXPANSION AND LINEAR TVAR PROCESS

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ABSTRACT

In this paper, we propose a novel method of modeling multicomponent amplitude and frequency modulated (AM-FM) signals using linear time varying autoregressive (TVAR) process. In the proposed method, the Fourier-Bessel (FB) expansion decomposes the multicomponent AM-FM signal into a number of single component signals and a 2-order TVAR process is used to model each single component signal. The estimation of the TVAR parameters of a multicomponent signal requires the inversion of a large correlation matrix, whereas the proposed method requires the inversion of a number of relatively small correlation matrices with better numerical stability properties. In presence of noise, it is demonstrated that our method improves the estimation of the time varying poles. Simulation results are presented for two-component damped AM sinusoidal signal and two-component damped FM sinusoidal signal.

1. INTRODUCTION

In most system identification and estimation problems it is often assumed that the signal is stationary. This assumption ensures that the underlying statistics and the model parameters that characterize the system and the signal that it generates are not dependent on time. However, this assumption is often inaccurate for many physical signals encountered in speech processing, biological and physiological signal analysis, and seismology because such signals may have time varying amplitude and frequency. Signals having time varying amplitude and frequency are represented by the amplitude and frequency modulated (AM-FM) signals [1-3].

Spectral analysis of nonstationary signals, with high frequency-resolution is obtained by using the time varying autoregressive (TVAR) process [4, 5]. In the modeling of nonstationary signals by a TVAR process, the AR process parameters are spanned by a basis-set of time functions, thus allowing the time varying parameters to be represented by a set of constant coefficients of the basis functions. In effect, this allows us to model a nonstationary signal by a set of constant coefficients which can be estimated by a standard technique [4]. The goodness of fit achieved using this technique, known as basis function expansion, however is dependent on the subspace spanned by the chosen set of time functions as bases. With the assumption of slow moving poles of the process, the generated signal can be modeled using a fixed basis-set of time functions and the corresponding set of constant coefficients [5-7].

Modeling by a TVAR process is a general approach, and all other nonstationary models (AM, FM, AM-FM, and more) can be shown to be special cases of the general approach [7]. A real AM-FM signal can be modeled using a 2-order TVAR process, and a signal composed of M real components will require a 2M-order TVAR process [7]. In the modeling by a TVAR process, the estimation of the TVAR parameters require the inversion of an autocorrelation matrix of size $[2M(q+1)\times 2M(q+1)]$, where q is the required number of basis functions to represent each TVAR parameter.

In the basis function expansion, two issues need to be resolved. First, a general class of basis functions is to be chosen, which can suitably capture the time variation, and then, the significant number of basis functions need to be selected. Several classes of functions have been proposed in the literature including the polynomial, wavelet and prolate spheroidal functions. However, no uniform rule exists to indicate which class should be adopted. The approach of choosing the significant number of basis functions (order selection) is based on trial and error [7]. Moreover, the expansion of the TVAR parameters into the basis sequences substantially increases the number of model parameters (coefficients) that is to be estimated. For all the above mentioned problems, the method of analysis through modeling by a TVAR process has not become popular for multicomponent signals.

It is therefore suggested that in such cases of modeling by a TVAR process it would be wiser to perform a component separation before doing estimation of the process parameters. This enables us to decompose a complex estimation problem into a set of subproblems, each much simpler and more favorable from a numerical-computation point of view.

In this paper, we develop a method of signal analysis based on the Fourier-Bessel (FB) expansion and the linear TVAR process. The multicomponent AM-FM signal has been expanded into the FB series. Each component has non-overlapping cluster of FB coefficients. The components can be reconstructed directly from the real FB coefficients of the separated clusters. Then, we model each reconstructed component by a 2- order TVAR process. In this way it requires the inversion of a number of relatively small correlation matrices with better numerical stability properties.

2. AM-FM SIGNAL SEPARATION USING FB EXPANSION

The zero-order FB series expansion of a discrete-time signal $x(n)$ considered over some arbitrary interval $(0,\ N)$ is expressed as

$$x(n) = \sum_{m=1}^{M} C_m J_0\left(\lambda_m n/N\right) \quad (1)$$

where $J_0\left(.\right)$ are the zero-order Bessel functions [8]. The coefficients C_m are computed by using the relation

$$C_m = \frac{2\int_0^N nx(n)J_0\left(\lambda_m n/N\right)dn}{N^2[J_1(\lambda_m)]^2} \quad (2)$$

where $J_1(.)$ are the first-order Bessel functions, and $\{\lambda_m;\ m=1,...,\ M\}$ are the ascending order positive roots of $J_0(\lambda)=0$. Note that in the integration of (2) the integer time index n is treated symbolically as continuous variable.

It has been demonstrated in [9] that the order and range of non-zero coefficients of the FB series expansion of a test signal are changed as the center frequency and the bandwidth of the signal are varied. In particular, it is shown that the range widens with larger bandwidth and the order increases with higher center frequency.

It has been shown that there is a one-to-one correspondence between the frequency content of a signal and the order of the FB expansion where the coefficient attains peak magnitude [10]. It should be noted that the FB series coefficients C_m are unique for a given signal, similarly as the Fourier series coefficients are unique for a given signal. However, unlike the sinusoidal basis functions in the Fourier series, the Bessel functions decay over time. This feature of the Bessel functions make the FB series expansion suitable for nonstationary signals [10, 11].

In the present study, we assume that the AM-FM signals are well separated in the frequency domain, and the signals will be associated with various distinct non-overlapping clusters of the FB coefficients. Therefore, each signal of the multicomponent AM-FM signal can be reconstructed separately by identifying and separating the corresponding FB coefficients.

3. ESTIMATION OF TVAR PARAMETERS OF MONOCOMPONENT SIGNALS

First, we briefly review the TVAR process. For details of the TVAR process, please refer to Grenier [5].

A discrete time TVAR process $r(n)$ of order p is expressed as

$$r(n) = -\sum_{i=1}^{p} a_i(n)r(n-i) + w(n) \quad (3)$$

where $w(n)$ is a stationary white noise process with zero mean and variance σ^2, and the TVAR parameters $\{a_i(n),\ i=1,2,...,p\}$ are modeled as linear combinations of a set of basis time functions $\{u_k(n),\ k=0,1,...,q\}$;

$$a_i(n) = \sum_{k=0}^{q} a_{ik} u_k(n) \quad (4)$$

where $\{u_k(n),\ k=0,1,...,q\}$ can be any appropriate set of basis functions and a_{ik} are the corresponding set of coefficients. If $\{u_k(n)\}$ are chosen as power of time, then $\{a_i(n)\}$ are polynomial function of time. If $u_k(n)$ are trigonometric functions, then (4) is a finite order Fourier series expansion. The k^{th} Fourier basis is defined as

$$u_k(n) = \begin{cases} \cos\left(\frac{\pi(n-1)k}{N-1}\right) & \text{for even values of } k \\ \sin\left(\frac{\pi(n-1)k}{N-1}\right) & \text{for odd values of } k \end{cases} \quad (5)$$

where $1 \le n \le N$. The k^{th} Chebyshev basis is defined as

$$u_k(n) = \cos\left(k\cos^{-1}(l-1)\right) \quad (6)$$

where $l = \frac{2(n-1)}{N-1}$ and $1 \le n \le N$.

In any case, the time varying AR process is described completely by the set of coefficients $\{a_{ik},\ i=1,2,...,p;\ k=0,1,...,q;\ \sigma^2\}$

The estimation of $\{a_{ik}\}$ aims at minimizing the total squared prediction error in predicting the sequence $r(n)$:

$$J = \sum_n \left| r(n) + \sum_{i=1}^{p}\sum_{k=0}^{q} a_{ik}u_k(n)r(n-i)\right|^2 \quad (7)$$

If we define the generalized covariance function as

$$C_{kl}(i,j) = \frac{1}{N-p}\sum_{n=p}^{N-1} u_k(n)u_l(n)r(n-i)r(n-j) \quad (8)$$

then the solution $\{a_{ik},\ i=1,2,...,p;\ k=0,1,...,q\}$ that minimizes (7) can be solved for from the generalized covariance equations:

$$\sum_{i=1}^{p}\sum_{k=0}^{q} a_{ik}C_{kl}(i,j) = -C_{0l}(0,j);\ 1 \le j \le p,\ 0 \le l \le q \quad (9)$$

This is a system of $p(q+1)$ linear equations, from which the coefficients are solved for.

The time varying poles from the TVAR parameters can be estimated by calculating the roots z_i of the prediction error filter (PEF) polynomial given by

$$z^p + a_1(n)z^{p-1} + a_2(n)z^{p-2} + ... + a_p(n) = 0 \quad (10)$$

The TVAR process for each component-signal $r_i(n)$ reconstructed from the FB coefficients is represented by the following equation

$$r_i(n) = -b_{i1}(n)r_i(n-1) - b_{i2}(n)r_i(n-2) \quad (11)$$

for $n=1,2,...,N$; where $p=2$ is the order of the TVAR process. The time varying parameters $b_{im}(n)$ are modeled by a set of basis functions $u_k(n)$,

$$b_{im}(n) = \sum_{k=0}^{q} b_{imk}u_k(n),\ m=1,2 \quad (12)$$

where b_{imk} are the corresponding set of coefficients. Note that the TVAR parameters for the component signals can be determined from (9) and (12).

Since the reconstructed component signals are modeled as AM-FM signals, the i^{th} component-signal is given by

$$r_i(n) = A_i(n)\cos(\Phi_i(n)) \tag{13}$$

The relations between the TVAR process parameters of a component signal and the amplitude and frequency modulation functions of the signal are given by [12]

$$b_{i1}(n) = -\frac{A_i(n)}{A_i(n-1)}\left(\frac{\sin\left(d\Phi_i(n) + d\Phi_i(n-1)\right)}{\sin\left(d\Phi_i(n-1)\right)}\right)$$

$$b_{i2}(n) = \frac{A_i(n)}{A_i(n-2)}\left(\frac{\sin\left(d\Phi_i(n)\right)}{\sin\left(d\Phi_i(n-1)\right)}\right) \tag{14}$$

where $d\Phi_i(n) = \Phi_i(n) - \Phi_i(n-1)$. For a given AM-FM signal, the TVAR parameters $b_{i1}(n)$, $b_{i2}(n)$ can be computed using the amplitude envelope $A_i(n)$ and instantaneous phase $\Phi_i(n)$. The time varying poles are computed using the characteristic equation of the TVAR process (10).

4. PROPOSED ALGORITHM

Our procedure based on the FB expansion and the TVAR process for modeling a multicomponent AM-FM signal requires the following steps:

Step 1: Component separation

Calculate the FB coefficients C_m for a given signal using (2). Every component of the multicomponent AM-FM signal has non-overlapping cluster of FB coefficients. Since coefficients are real; each component is directly reconstructed by using (1) from coefficient versus order plot.

Step 2: Modeling of component signals by TVAR process

Choose the basis functions $\{u_k(n), k = 0, 1, ..., q\}$ with appropriate q, and then solve (9) for b_{imk}. Construct the TVAR coefficients $b_{im}(n)$ using (12).

Step 3: Estimation of the time varying poles of the composite signal

Using (10), calculate the time varying poles of the component signals. Movements of the poles of the component signals over time provide the pole loci of the composite signal. The set of basis functions and the orders p and q should be selected using a priori knowledge of the signal.

5. SIMULATION RESULTS

In the simulation study, we have considered a two-component damped AM sinusoidal signal and a two-component damped FM sinusoidal signal.

Example 1: Two-Component Damped AM Sinusoidal Signal

The two-component damped AM sinusoidal signal given by

$$x(n) = (0.998)^n\left[1 + 0.8\cos(\pi n/128)\right]\cos(\pi n/6)$$
$$+5(0.992)^n[1 + 0.6\cos(\pi n/64)]\cos(\pi n/2)$$

is used for simulation. The two-component signal is expanded into the FB series. Each of the two reconstructed

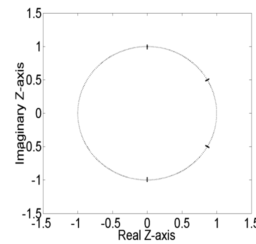

Figure 1: Pole loci of two-component damped AM signal (Actual)

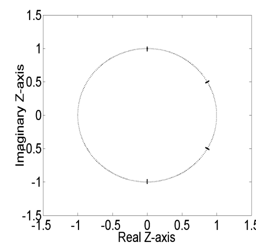

Figure 2: Pole loci of two-component damped AM signal (Proposed method)

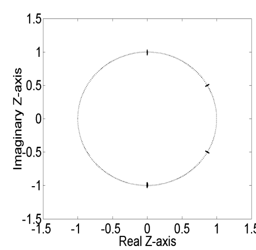

Figure 3: Pole loci of two-component damped AM signal (Grenier's method)

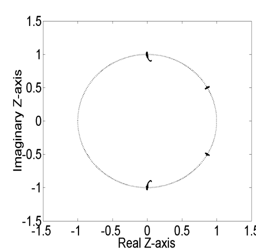

Figure 4: Pole loci of two-component damped AM signal at SNR=40 dB (Proposed method)

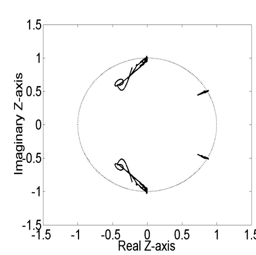

Figure 5: Pole loci of two-component damped AM signal at SNR=40 dB (Grenier's method)

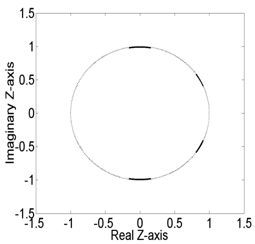

Figure 6: Pole loci of two-component damped FM signal (Actual)

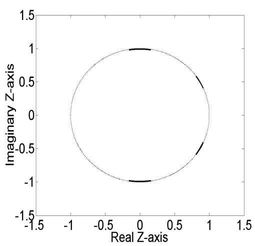

Figure 7: Pole loci of two-component damped FM signal (Proposed method)

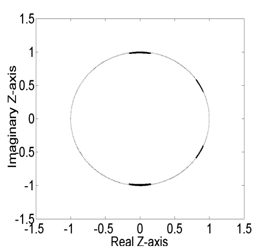

Figure 8: Pole loci of two-component damped FM signal (Grenier's method)

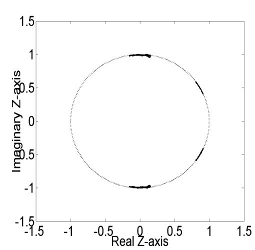

Figure 9: Pole loci of two-component damped FM signal at SNR=30 dB (Proposed method)

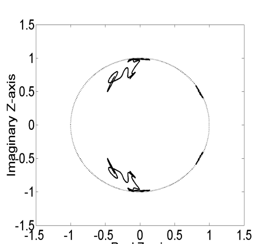

Figure 10: Pole loci of two-component damped FM signal at SNR=30 dB (Grenier's method)

AM signals from the clusters of non-overlapped FB coefficients, is modeled by a 2-order TVAR process. While using the Fourier and Chebyschev bases, the sets of 33 and 28 basis functions respectively are found to give best results. The actual time varying poles using the TVAR process with Chebyschev basis functions are shown in Figure 1, whereas the estimated time varying poles are shown in the Figure 2. The estimated time varying poles using the Grenier's method [5] are shown in Figure 3.

The additive white Gaussian noise is used for simulation of noisy signal condition. A total of 100 realizations are implemented with independent noise sequences at each signal-to-noise ratio (SNR) level. When noise is added to the signal, the loci of the estimated time varying poles at SNR 40 dB using the proposed method and the Grenier's method are shown in Figures 4 and 5 respectively. It is clear from the simulation that our proposed method improves the accuracy of estimation of the time varying poles under noisy signal condition. The mean square error (MSE) measures of the estimated poles using the proposed method and the Grenier's method are shown in Tables 1, 2.

Example 2: Two-Component Damped FM Sinusoidal Signal

The two-component damped FM sinusoidal signal given by

$$x(n) = (0.998)^n \cos\left[\pi n/6 + 0.2\pi/240 \sum_{j=1}^{n} cos(\pi j/140)\right]$$

$$+5(0.992)^n \cos\left[\pi n/2 + 0.1\pi/120 \sum_{j=1}^{n} \cos(\pi j/120)\right]$$

is used for simulation. The signal is expanded into the FB series. Each of the two reconstructed FM signal from the clusters of non-overlapped FB coefficients is modeled by a 2-order TVAR process. While using the Fourier and Chebyschev bases, the sets of 33 and 28 basis functions respectively are found to give best results. The actual time-varying poles using the TVAR process with Chebyschev basis functions are shown in Figure 6, whereas the estimated time varying poles are shown in Figure 7. The estimated time varying poles using the Grenier's method are shown in Figure 8. In the presence of additive white Gaussian noise, the loci of the time varying poles at SNR 30 dB using the proposed method and the Grenier's method are shown in Figures 9 and 10 respectively. It is clear from the simulation that our proposed method improves the accuracy of estimation of the time varying poles of FM signals as well. The performance of the proposed method and the Grenier's method are compared in Tables 3, 4.

6. CONCLUSION

A new method for parametric modeling of multicomponent AM-FM signals is presented in this paper. The method decomposes a multicomponent AM-FM signal into a number of single component signals, and each component is modeled by using a second order TVAR process. In this way, the estimation of the model parameters of a multicomponent signal is done accurately in presence of noise.

The proposed method is less computational-intensive compared to the Grenier's method, because the method requires inversion of relatively small matrices and needs a two order characteristic equation to be solved for computing the time varying poles.

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Table 1: MSE measures in the estimation of the time-varying poles: First component of the two-component damped AM sinusoidal signal

SNR (dB)	Grenier's Method	Proposed Method
Infinity	1.0133×10^{-6}	2.5344×10^{-5}
60	1.6397×10^{-6}	5.0104×10^{-5}
50	4.1655×10^{-5}	6.6067×10^{-5}
40	2.0123×10^{-3}	6.6471×10^{-5}
30	1.1301×10^{-1}	6.8000×10^{-5}
20	3.3155×10^{-1}	1.2042×10^{-4}
10	5.6434×10^{-1}	1.3974×10^{-4}
5	6.1321×10^{-1}	1.0101×10^{-3}

Table 2: MSE measures in the estimation of the time-varying poles: Second component of the two-component damped AM sinusoidal signal

SNR (dB)	Grenier's Method	Proposed Method
Infinity	5.1089×10^{-6}	2.3308×10^{-5}
60	5.4801×10^{-6}	3.6702×10^{-5}
50	4.8163×10^{-5}	2.3724×10^{-4}
40	1.2130×10^{-3}	2.4346×10^{-4}
30	0.9461×10^{-1}	4.0894×10^{-4}
20	3.2710×10^{-1}	1.1129×10^{-4}
10	5.3971×10^{-1}	2.4378×10^{-3}
5	5.9619×10^{-1}	2.1313×10^{-3}

Table 3: MSE measures in the estimation of the time-varying poles: First component of the two-component damped FM sinusoidal signal

SNR (dB)	Grenier's Method	Proposed Method
Infinity	2.9813×10^{-6}	3.5142×10^{-5}
60	3.9205×10^{-6}	6.1122×10^{-5}
50	4.2633×10^{-6}	6.1413×10^{-5}
40	6.2649×10^{-5}	6.3873×10^{-5}
30	2.8123×10^{-3}	7.1923×10^{-5}
20	4.1036×10^{-3}	2.5406×10^{-4}
10	5.2339×10^{-1}	1.6231×10^{-3}
5	5.9199×10^{-1}	6.3502×10^{-3}

Table 4: MSE measures in the estimation of the time-varying poles: Second component of the two-component damped FM sinusoidal signal

SNR (dB)	Grenier's Method	Proposed Method
Infinity	3.9923×10^{-5}	5.0019×10^{-4}
60	4.3931×10^{-5}	5.0075×10^{-4}
50	4.5911×10^{-5}	5.0408×10^{-4}
40	1.2219×10^{-4}	5.0680×10^{-4}
30	9.1834×10^{-3}	5.5731×10^{-4}
20	1.2138×10^{-1}	1.2327×10^{-3}
10	6.6695×10^{-1}	3.2564×10^{-3}
5	7.5576×10^{-1}	5.0013×10^{-3}