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A UNIFIED FULL-RATE STBC DESIGN WITH REDUCED-COMPLEXITY COHERENT AND DIFFERENTIAL ML DECODING

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***Abstract*— We propose a new full-rate space-time block code (STBC) for two transmit antennas which can be designed to achieve maximum diversity or maximum capacity while enjoying optimized coding gain and reduced-complexity maximum-likelihood (ML) decoding. The maximum transmit diversity (MTD) construction provides a diversity order of $2N_r$ for any number of receive antennas N_r at the cost of channel capacity loss. The maximum channel capacity (MCC) construction preserves the mutual information between the transmit and the received vectors while sacrificing diversity. Both constructions enjoy low-complexity ML decoding proportional to the square of the signal constellation size making them attractive alternatives to existing full-diversity full-rate STBCs in [6], [3] which have high ML decoding complexity proportional to the fourth order of the signal constellation size. Furthermore, we design a differential transmission scheme for our proposed STBC, derive the exact ML differential decoding rule, and compare its performance with competitive schemes. Finally, we investigate transceiver design and performance of our proposed STBC over frequency-selective channels.**

I. INTRODUCTION

V-BLAST, proposed in [7], is a well-known multi-input multi-output (MIMO) system which operates at very high spectral efficiency with low encoding and decoding complexities. However, it can not exploit the maximum diversity available in a MIMO channel and, therefore, can suffer appreciable performance loss. Orthogonal STBCs (OSTBC) [1] [2], on the other hand, are primarily designed to capture full transmit diversity of the MIMO channel while keeping the decoding complexity linear in the number of transmit antennas. Due to their orthogonal structure constraint, OSTBC suffers from rate loss. Moreover, with the exception of the Alamouti STBC with 1 receive antenna [1], OSTBC schemes do not preserve the mutual information between the received and the transmit signal vectors due to the induced space-time correlation on the channel matrix. These observations motivated researches to design several STBCs for $N_t = 2$ which not only achieve the capacity of the underlying MIMO channel but also ensure maximum diversity, thanks to their special algebraic structure. A number-theoretic STBC construction, called $B_{2,\phi}$, was proposed in [3] and proved to be a full-diversity capacity-achieving STBC. For more than one receive antenna, the performance of $B_{2,\phi}$ was shown to be superior to the Alamouti STBC at the same rate. The Golden code in [6] was shown to achieve the optimum diversity-multiplexing gain tradeoff [8]. However, the main drawback is its exponentially-growing ML decoding complexity as a function of the number of transmit antennas and constellation size. We derive a new STBC design in this paper for $N_t = 2$ transmit antennas and $T = 2$ time slots which enjoys low-complexity ML decoding (quadratic in the constellation size). [1]

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II. SYSTEM MODEL AND DESIGN CRITERIA

Consider a vector of 4 information symbols $\boldsymbol{s} = [s_1\ s_2\ s_3\ s_4]^t$ where $[.]^t$ denotes the matrix transpose and the information symbols $s_j, j = 1, \cdots, 4$ are carved from a q-QAM constellation and transmitted from $N_t = 2$ transmit antennas during $T = 2$ symbol periods. Define an $N_t \times T$ matrix $\boldsymbol{u}(\boldsymbol{s})$ as the STBC codeword associated with the vector $\boldsymbol{s}$ with the entries $u_{m\nu}$ which are transmitted simultaneously from N_t transmit antennas during each symbol period $\nu = 1, \cdots, T$. The received signal matrix of size $(N_r \times T)$ is given by

$$\boldsymbol{y} = \sqrt{\frac{\rho}{N_t}}\boldsymbol{H}\boldsymbol{u} + \boldsymbol{w} \tag{1}$$

where $\boldsymbol{H} \in \mathbb{C}^{N_r \times N_t}$ denotes the channel matrix with entries h_{ri} representing the fading coefficients associated with the ith transmit and the rth receive antenna. The channel coefficients are samples of an independent and identically distributed (i.i.d.) complex Gaussian random process with zero mean and variance 0.5 per real dimension. The channel coefficients are assumed quasi-static flat-fading i.e. fixed during one STBC transmission of T symbol periods. The noise matrix $\boldsymbol{w} \in \mathbb{C}^{N_r \times T}$ has entries $w_{r\nu}$ which are drawn from a white Gaussian distribution $\mathcal{CN}(0, \sigma^2)$. The received signal matrix $\boldsymbol{y} \in \mathbb{C}^{N_r \times T}$ is generated by stacking signal samples from the N_r receive antennas at time slots $1, \cdots, T$. The normalization factor $\sqrt{\frac{\rho}{N_t}}$ in (1) ensures that ρ is the SNR at each receive antenna since $\mathsf{E}[tr\{\boldsymbol{u}\boldsymbol{u}^*\}] = N_t$ where $\mathsf{E}[\cdot]$ and $tr\{\cdot\}$ are the expectation and trace operations, respectively. Since it takes T symbol periods to transmit a vector of size p, the transmission rate is defined as

$$\mathcal{R}_s = \frac{p}{T} \qquad \text{symbols per channel use (pcu)} \tag{2}$$

[1] During the review process of this paper the reviewers brought to our attention recent code constructions which were independently developed in [9], [11] and [10] and their unified treatment in [12]. As we will show in Section VII (c.f. Fig.4), these codes achieve almost the same performance as our proposed code at similar or higher decoding complexity. In addition we develop its non-coherent (differential) encoding/decoding scheme, and study its performance in frequency-selective channels. All of these issues were not considered in [9], [11], [10] and [12]

Note that an STBC is said to be full-rate if [8] $\mathcal{R}_s = N_t$ symbols pcu.

III. PROPOSED 2×2 STBC DESIGN

A. Maximum Transmit Diversity (MTD) Construction

Defining

$$\boldsymbol{u} \triangleq \text{diag}\left(\boldsymbol{R}\left[\begin{array}{cc} \mathcal{V}_1' & \mathcal{V}_2' \end{array}\right]\right) + \text{diag}\left(\boldsymbol{R}\left[\begin{array}{cc} \mathcal{V}_1 & \mathcal{V}_2 \end{array}\right] J\right) J \quad (3)$$

where the diag$(\cdot)$ operator constructs a diagonal matrix by setting the off-diagonal elements to zero and

$$J = \left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right] \qquad \boldsymbol{R} \triangleq \left[\begin{array}{cc} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{array}\right] \quad (4)$$

where $\mathcal{V}_i'$ for $i = 1, 2$ represents the ith column of one Alamouti code. Similarly, $\mathcal{V}_j, j = 1, 2$ is the jth column of another Alamouti code. Multiplying matrices in (3) and collecting terms we get

$$\boldsymbol{u} = \left[\begin{array}{cc} s_1'\alpha_1 - s_2'^*\beta_1 & s_3^*\beta_1 + s_4\alpha_1 \\ s_3\alpha_2 - s_4^*\beta_2 & s_1'^*\beta_2 + s_2'\alpha_2 \end{array}\right] \quad (5)$$

The matrix $\boldsymbol{R}$ is designed to maximize the coding gain and guarantee full diversity of the code in (5) as follows

$$\alpha_1 = \sin(\theta_1); \beta_1 = \cos(\theta_1); \alpha_2 = \sin(\theta_2); \beta_2 = \cos(\theta_2) \quad (6)$$

where

$$\{\theta_1^{\text{opt}}, \theta_2^{\text{opt}}\} = \{\arctan(2), \arctan(\frac{1}{2})\} \quad (7)$$

The proof is given in [4].

B. Maximum Channel Capacity (MCC) Construction

It can be shown that the eigenvalue distribution of the equivalent channel matrix for the MTD STBC in (5) is not the same as that of the original channel matrix due to the induced space-time correlation, hence this design is not a capacity-achieving STBC. Therefore, as we increase N_r or the constellation size, the MTD construction loses coding gain. It is possible, however, to re-design the code to preserve the mutual information between the transmit and received vectors. Let $\boldsymbol{P}$ be a 2×2 transformation matrix. Introducing $\boldsymbol{R}' \triangleq \left[\begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array}\right] = \boldsymbol{PR}$, where $\boldsymbol{R}$ is defined in (4), our goal is to design $\boldsymbol{P}$ such that the STBC

$$\begin{aligned} \boldsymbol{v} &\triangleq \text{diag}\left(\boldsymbol{R}'\left[\begin{array}{cc} \mathcal{V}_1' & \mathcal{V}_2' \end{array}\right]\right) + \text{diag}\left(\boldsymbol{R}'\left[\begin{array}{cc} \mathcal{V}_1 & \mathcal{V}_2 \end{array}\right] J\right) J \\ &= \left[\begin{array}{cc} s_1'a_1 - s_2'^*b_1 & s_3^*b_1 + s_4a_1 \\ s_3a_2 - s_4^*b_2 & s_1'^*b_2 + s_2'a_2 \end{array}\right] \end{aligned} \quad (8)$$

is information lossless. This is achieved by $\boldsymbol{R}' = \left[\begin{array}{cc} \alpha_1 & \beta_1 \\ \beta_2 & \alpha_2 \end{array}\right]$ where $\alpha_1, \beta_1, \alpha_2, \beta_2$ are the MTD design parameters. The proof is given in [5].

IV. DECODING

Since the MTD and MCC constructions have the same algebraic structure (i.e. linear combination of two Alamouti STBC), the same decoding algorithm can be used for both constructions. To decode the STBC schemes in [3], [6], a size-q^4 exhaustive ML search has to be performed over all 4 information symbols transmitted in each codeword. Therefore, real-time implementation of these schemes is challenging in practical systems, especially for large signal constellations. Any suboptimal receiver such as zero-forcing, minimum mean square error (MMSE) or ordered successive interference cancellation (OSIC), will degrade their performance compared to full ML decoding. Hence, the ultimate goal of these STBC schemes which is to achieve the optimum diversity-multiplexing trade-off would not be accomplished due to this performance loss.

Our proposed hybrid maximum likelihood interference cancellation (HMLIC) algorithm operates as follows. The decoding process starts by performing conditional ML decoding only on symbols s_2' and s_3 and for each of their q^2 possible choices, their interfering effect is canceled from the original received signal vector. Due to the special algebraic structure of the MTD and MCC codes, the resulting equivalent channel matrix for symbols s_1' and s_4 is *orthogonal*; hence, simple matched-filtering is ML-optimal for joint decoding of s_1' and s_4. Finally, for each of the q^2 candidate codewords, we evaluate the metric $\|\boldsymbol{\mathcal{Y}} - \sqrt{\frac{\rho}{N_t}}\boldsymbol{Hu}\|^2$ and choose the codeword that minimizes it. Therefore, the ML decoding complexity is reduced from a size-q^4 search to a size-q^2 search plus q^2 matched filters. For detailed mathematical description of the decoding process the reader is referred to [4].

V. DIFFERENTIAL TRANSMISSION

Channel estimation in a fast time-varying communication environment is computationally expensive and can result in substantial data rate loss due to training overhead. Moreover, considering the increased number of unknown channel coefficients in a MIMO system, it is sometimes desirable to eliminate channel estimation at the receiver at the cost of some performance loss. In this section, we investigate differential transmission for our proposed code and derive the exact ML differential decoding metric.

A. Differential Encoder

Assuming differential transmission of M MTD [2]codewords and denoting the kth transmitted codeword by $\boldsymbol{X}(k)$ for $0 \le k \le M-1$. The differential scheme is initialized by transmitting $\boldsymbol{X}(0) = \boldsymbol{I_2}$, where $\boldsymbol{I_2}$ is the 2×2 identity matrix, and proceeds as follows

$$\boldsymbol{B}(k) = \boldsymbol{X}(k-1)\boldsymbol{u}(k); \qquad \boldsymbol{X}(k) = \frac{\boldsymbol{B}(k)}{\sqrt{e(k)}} \quad (9)$$

The total transmitted energy from all antennas is constrained to be a constant independent of N_t; i.e.

$$\mathsf{E}\left[\sum_{r=1}^{N_r}\sum_{m=1}^{N_t}|x_{m,r}(k)|^2\right] = T \quad (10)$$

where $\boldsymbol{X}(k-1)$ and $\boldsymbol{X}(k)$ are the transmitted codewords at times $(k-1)$ and k, respectively, and $e(k)$ is the energy normalizer i.e. $e(k) = \frac{tr\{\boldsymbol{B}(k)\boldsymbol{B}^H(k)\}}{T}$.

[2]The same approach can be followed for the MCC code as well.

Keeping the average transmitted energy constant is critical here in the differential encoder since the MTD code is not orthogonal and successive multiplication of the codewords may cause its energy to blow up or diminish. The corresponding received signal blocks over codeword transmissions k and $k-1$ are

$$\begin{aligned}[\boldsymbol{Y}(k-1) \quad \boldsymbol{Y}(k)] &= [\boldsymbol{H}(k-1)\boldsymbol{X}(k-1) \quad \boldsymbol{H}(k)\boldsymbol{X}(k)] \\ &+ [\boldsymbol{W}(k-1) \quad \boldsymbol{W}(k)]\end{aligned} \tag{11}$$

Substituting (9) into (11) and applying the quasi-static channel assumption $\boldsymbol{H}(k) = \boldsymbol{H}(k-1)$, we can write

$$\boldsymbol{Y}(k) = \frac{\boldsymbol{Y}(k-1)\boldsymbol{u}(k)}{\sqrt{e(k)}} + \tilde{\boldsymbol{W}}(k) \tag{12}$$

where the equivalent noise matrix seen by the differential decoder is given by

$$\tilde{\boldsymbol{W}}(k) = \boldsymbol{W}(k) - \frac{\boldsymbol{W}(k-1)\boldsymbol{u}(k)}{\sqrt{e(k)}} \tag{13}$$

which is a colored noise with the variance of

$$\begin{aligned}\sigma^2_{\tilde{\boldsymbol{W}}} &= \mathsf{E}\left[tr\left\{\tilde{\boldsymbol{W}}(k)\tilde{\boldsymbol{W}}^H(k)\right\}\right] \\ &= \sigma^2\left(1 + \frac{1}{e(k)}\mathsf{E}\left[tr\{\boldsymbol{u}(k)\boldsymbol{u}^H(k)\}\right]\right)\end{aligned} \tag{14}$$

B. *Differential Decoding*

Due to the non-orthogonality of the MTD code, its ML differential decoding can not be performed using simple linear processing as in OSTBC and an exhaustive search is needed in general whose complexity increases exponentially with the constellation size and the number of transmit antennas. Starting from (11), we can derive the ML decoding rule for our non-orthogonal differential MTD as follows. Define

$$\begin{aligned}\boldsymbol{Y_E} &\triangleq [\boldsymbol{Y}(k) \quad \boldsymbol{Y}(k-1)] \\ &= \boldsymbol{H}(k)\left[\boldsymbol{X}(k) \quad \boldsymbol{X}(k-1)\right] + [\boldsymbol{W}(k) \quad \boldsymbol{W}(k-1)] \\ &= \boldsymbol{H}(k)\boldsymbol{X}(k-1)\left[\boldsymbol{u}^{'}(k) \quad \boldsymbol{I_2}\right] + [\boldsymbol{W}(k) \quad \boldsymbol{W}(k-1)] \\ &\triangleq \boldsymbol{H_E}\boldsymbol{G} + \boldsymbol{W_E}\end{aligned} \tag{15}$$

where $\boldsymbol{H_E} \triangleq \boldsymbol{H}(k)\boldsymbol{X}(k-1)$ and $\boldsymbol{u}^{'}(k) = \frac{\boldsymbol{u}(k)}{\sqrt{e(k)}}$. Since $\boldsymbol{W_E} = [\boldsymbol{W}(k) \quad \boldsymbol{W}(k-1)]$ is AWGN and *independent* of $\boldsymbol{G}$, we can write the exact ML decoding metric as follows

$$J_{ML} = \arg\min \|\boldsymbol{Y_E} - \boldsymbol{H_E}\boldsymbol{G}\|^2 \tag{16}$$

which can be equivalently written as

$$J_{ML} = \min\left\{\|\boldsymbol{Y}(k) - \boldsymbol{H_E}\boldsymbol{u}^{'}(k)\|^2 + \|\boldsymbol{Y}(k-1) - \boldsymbol{H_E}\|^2\right\} \tag{17}$$

We can eliminate the dependence of J_{ML} on $\boldsymbol{H_E}$ by differentiating J_{ML} with respect to $\boldsymbol{H_E}$ to find the choice of $\boldsymbol{H_E}$ which minimizes J_{ML}. Using the identity on the derivative of a quadratic function of a matrix with respect to that matrix we get

$$\begin{aligned}\boldsymbol{0} &= \frac{\partial J_{ML}}{\partial \boldsymbol{H_E}} \\ &= -(\boldsymbol{Y}(k)\boldsymbol{u}^{'H}(k) + \boldsymbol{Y}(k-1)) + \boldsymbol{H_E}(\boldsymbol{I_2} + \boldsymbol{u}^{'}(k)\boldsymbol{u}^{'H}(k))\end{aligned} \tag{18}$$

Therefore, we can solve for the optimum choice of $\boldsymbol{H_E}$ from (18) to be

$$\boldsymbol{H_E} = (\boldsymbol{Y}(k)\boldsymbol{u}^{'H}(k) + \boldsymbol{Y}(k-1))(\boldsymbol{I_2} + \boldsymbol{u}^{'}(k)\boldsymbol{u}^{'H}(k))^{-1} \tag{19}$$

Substituting back for $\boldsymbol{H_E}$ from (19) in (17), we get

$$\begin{aligned}J_{ML} &= \arg\min \|\boldsymbol{Y}(k) - \left[\boldsymbol{Y}(k)\boldsymbol{u}^{'H}(k) + \boldsymbol{Y}(k-1)\right] \times \\ &\left[\boldsymbol{I_2} + \boldsymbol{u}^{'}(k)\boldsymbol{u}^{'H}(k)\right]^{-1}\boldsymbol{u}^{'}(k)\|^2 \\ &+ \|\boldsymbol{Y}(k-1) - \left[\boldsymbol{Y}(k)\boldsymbol{u}^{'H}(k) + \boldsymbol{Y}(k-1)\right] \times \\ &\left[\boldsymbol{I_2} + \boldsymbol{u}^{'}(k)\boldsymbol{u}^{'H}(k)\right]^{-1}\|^2\end{aligned} \tag{20}$$

This metric is very complex to implement. Using (12), the following *approximate* ML decoder can be derived by minimizing the following suboptimal[3] metric

$$J_{ML}^{\text{approx}} = \arg\min \left\|\boldsymbol{Y}(k) - \frac{\boldsymbol{Y}(k-1)\boldsymbol{u}(k)}{\sqrt{e(k)}}\right\|^2 \tag{21}$$

The performances of the metrics J_{ML}^{approx} and J_{ML} will be compared in Section VII for both the MTD construction and the Golden code. Note that J_{ML} collapses to J_{ML}^{approx} for orthogonal STBC[4]. Even more important are the two observations that the differential MTD code outperforms the differential Golden code and that the performance gap between differential and coherent decoding is less for the MTD code than for the Golden code.

VI. Frequency-Selective Channels

Assume that the maximum delay spread between the $N_r \times N_t$ channel paths is L and define $h_{ri}(l)$ as the lth channel tap between the ith transmit antenna and the rth receive antenna where $r \in [1, N_r], i \in [1, N_t], l \in [0, L]$. We generate 4 information blocks each of length N and combine them in the time domain so that after DFT processing at the receiver, we get the MTD code structure in the frequency domain. To make the channel matrices circulant, a cyclic prefix of length L is appended at the end of each information block in the time domain. Denoting the nth symbol of the kth transmitted block over the ith transmit antenna by $u_i^k(n)$ we can write

$$\begin{aligned}u_1^k(n) &= \alpha_1 s_1^k(n) - \beta_1 s_2^{*k}([N-n]_N) \\ u_2^k(n) &= \alpha_2 s_3^k(n) - \beta_2 s_4^{*k}([N-n]_N)\end{aligned} \tag{22}$$

where $[\cdot]_N$ is the modulo-N operation. Equation (22) can be represented in vector notation as follows

$$\boldsymbol{u_1^k} = \alpha_1\boldsymbol{s_1^k} - \beta_1\overline{\boldsymbol{s_2^k}}; \qquad \boldsymbol{u_2^k} = \alpha_2\boldsymbol{s_3^k} - \beta_2\overline{\boldsymbol{s_4^k}} \tag{23}$$

where $\overline{[\cdot]}$ stands for the complex-conjugate operation over a vector of length N. The received block from the rth receive antenna during the kth block transmission in the time domain is given by

$$\boldsymbol{y_r^k} = \boldsymbol{H_{c_{1r}}}\left(\alpha_1\boldsymbol{s_1^k} - \beta_1\overline{\boldsymbol{s_2^k}}\right) + \boldsymbol{H_{c_{2r}}}\left(\alpha_2\boldsymbol{s_3^k} - \beta_2\overline{\boldsymbol{s_4^k}}\right) + \boldsymbol{w_r^k}$$

where $\boldsymbol{H_{c_{ir}}}$ is the circulant channel matrix from the ith transmit antenna to the rth receive antenna. Assuming quasi-static

[3] This metric is suboptimal since the equivalent noise term $\tilde{\boldsymbol{W}}^k$ in (13) is not white and is dependent on $\boldsymbol{u}^{'}(k)$.

[4] This can be easily verified by substituting $\boldsymbol{u}^{'}(k)\boldsymbol{u}^{'H}(k) = \boldsymbol{I_2}$ in (17).

channels, the channel taps remain constant over two consecutive block transmissions. Performing the same block processing for the $(k+1)$th block transmission, the $(k+1)$th received block in the time domain is given by

$$\begin{aligned} \boldsymbol{y}_r^{k+1} &= \boldsymbol{H}_{c_{1r}}\left(\beta_1\overline{\boldsymbol{s}_3^{k+1}}+\alpha_1\boldsymbol{s}_4^{k+1}\right) \\ &+ \boldsymbol{H}_{c_{2r}}\left(\alpha_2\boldsymbol{s}_2^{k+1}+\beta_2\overline{\boldsymbol{s}_1^{k+1}}\right)+\boldsymbol{w}_r^{k+1} \end{aligned}$$

Since all channel matrices are circulant, they can be diagonalized using the DFT orthonormal matrix $\boldsymbol{F}$ whose (p,q) element is given by $\boldsymbol{F}(p,q)=\frac{1}{\sqrt{N}}\exp(-j\frac{2\pi pq}{N})$ for $0\le p,q\le N-1$. At the receiver, the time domain blocks are transformed to the frequency domain by first removing the cyclic prefix and then multiplying by $\boldsymbol{F}$ as follows

$$\begin{aligned} \boldsymbol{Y}_r^k \triangleq \boldsymbol{F}\boldsymbol{y}_r^k &= \boldsymbol{F}\boldsymbol{F}^H\boldsymbol{D}_{r1}\boldsymbol{F}\left(\alpha_1\boldsymbol{s}_1^k-\beta_1\overline{\boldsymbol{s}_2^k}\right) \\ &+\boldsymbol{F}\boldsymbol{F}^H\boldsymbol{D}_{r2}\boldsymbol{F}\left(\alpha_2\boldsymbol{s}_3^k-\beta_2\overline{\boldsymbol{s}_4^k}\right)+\boldsymbol{F}\boldsymbol{w}_r^k \\ &=\boldsymbol{D}_{r1}\left(\alpha_1\boldsymbol{S}_1^k-\beta_1\boldsymbol{S}_2^{*k}\right) \\ &+\boldsymbol{D}_{r2}\left(\alpha_2\boldsymbol{S}_3^k-\beta_2\boldsymbol{S}_4^{*k}\right)+\boldsymbol{W}_r^k \end{aligned} \tag{24}$$

where $\boldsymbol{S}_i^k=\boldsymbol{F}\boldsymbol{s}_i^k$, $i=1,\cdots,4$ are the transmitted blocks in the frequency domain while $\boldsymbol{D}_{r1}$ and $\boldsymbol{D}_{r2}$ are $N\times N$ diagonal matrices with (n,n) elements given by

$$\boldsymbol{D}_{ri}(n,n)=\frac{1}{\sqrt{N}}\sum_{l=0}^{L}h_{r,i}(l)e^{-j\frac{2\pi nl}{N}} \tag{25}$$

Following the same procedure, one can derive the received block during the $(k+1)$ block transmission to be

$$\begin{aligned} \boldsymbol{Y}_r^{k+1} &= \boldsymbol{D}_{r1}\left(\beta_1\boldsymbol{S}_3^{*k+1}+\alpha_1\boldsymbol{S}_4^{k+1}\right) \\ &+\boldsymbol{D}_{r2}\left(\alpha_2\boldsymbol{S}_2^{k+1}+\beta_2\boldsymbol{S}_1^{*k+1}\right)+\boldsymbol{W}_r^{k+1} \end{aligned} \tag{26}$$

Taking the element-wise complex conjugation of $\boldsymbol{Y}_r^{k+1}$ and collecting all the received blocks from the N_r receive antennas, we get the system of equations in the frequency domain. Note that performing HMLIC on this system of equations is not possible since equalization is done in the frequency-domain where the lattice structure (due to the signal constellation) has been destroyed by the application of the DFT matrix. Instead, we consider the MMSE estimate of the nth element of each of 4 received blocks in the frequency-domain which is given by

$$\hat{\boldsymbol{S}}^k(n)=\left(\boldsymbol{D}^H(n,n)\boldsymbol{D}(n,n)+\frac{N_t}{\rho}\boldsymbol{I}_4\right)^{-1}\boldsymbol{D}^H(n,n)\boldsymbol{Y}^k(n) \tag{27}$$

where $\hat{\boldsymbol{S}}^k(n)=[\boldsymbol{S}_1^k(n)\ \ \boldsymbol{S}_2^{*k}(n)\ \ \boldsymbol{S}_3^k(n)\ \ \boldsymbol{S}_4^{*k}(n)]^t$ is a 4×1 vector of the MMSE estimates of the nth symbols from each of the 4 information blocks. Similarly, $\boldsymbol{D}(n,n)$ is a $2N_r\times 4$ matrix constructed by choosing the nth diagonal element of $\boldsymbol{D}_{ri}$ for $1\le r\le N_r$ and $i=1,2$. In addition, $\boldsymbol{Y}^k(n)=[\boldsymbol{Y}_1^k(n)\ \ \boldsymbol{Y}_1^{*k+1}(n)\cdots\boldsymbol{Y}_{N_r}^k(n)\ \ \boldsymbol{Y}_{N_r}^{*k+1}(n)]^t$ is the vector of the nth elements from each of the 4 received blocks. Therefore, MMSE-based SC-FDE is performed over the nth elements of the 4 information blocks in the frequency domain. Stacking all symbol estimates in vectors, they can be transformed back to the time domain followed by a slicer to decode the information blocks.

VII. Simulation Results

The BER performances of the MTD and MCC constructions are compared under different transmission scenarios in Fig.1. It can be seen from this figure that when the number of receive antennas and the spectral efficiency are small, the MTD construction achieves better performance since achieving full diversity is more critical than full capacity under these scenarios. However, as we increase the number of receive antennas, the capacity of the underlying MIMO channel will increase and hence preserving the mutual information between the transmit and the received signal vectors becomes more crucial. The same trend also holds as we increase the signal constellation size and the MCC construction becomes preferable in this case as well. Next, we compare in Fig.2 the BER performance of the J_{ML} and J_{ML}^{approx} differential decoder metrics for the MTD and Golden codes with $N_r=1,2$ and QPSK constellation. Thanks to the special algebraic structure of the MTD code, the performance gap between the two metrics is not as high as that of the Golden code for $N_r=1$. Furthermore, it is clear that the differential MTD code outperforms the differential Golden code by about 2.2dB at high SNR for $N_r=1$.

The BER performance of the MTD code is investigated for the frequency-selective channel scenario with $L=3,5,7$ and a block size of $N=64$. As it can be seen from Fig.3, the MMSE-based SC-FDE is clearly capable of capturing multi-path diversity and outperforms V-BLAST at a spectral efficiency of 4 bits pcu and $N_r=2$.

Fig.4 shows a performance comparison of MTD with those of [9], [11][5], and [10] with $N_r=2$ and spectral efficiency of 4 bits pcu. At the same figure, MCC is compared with the schemes in [9], [10] and [11] and in the presence of $N_r=5$ receive antennas and spectral efficiency of 4 bits pcu. As it can be seen from this figure, if the right construction i.e. MTD or MCC is used in the right transmission environment, then the BER performances of these schemes are almost the same.

VIII. Conclusion

We designed a closed-form rate-2 space-time block code for two transmit antennas through a judicious application of rotation and linear combination operations on two parallel Alamouti codes. We presented two different constructions of the proposed code design, related through a simple transformation, where one construction maximizes the diversity gain while the other one guarantees the information lossless property. We show how to use our proposed code in differential transmission scheme and in frequency-selective channels.

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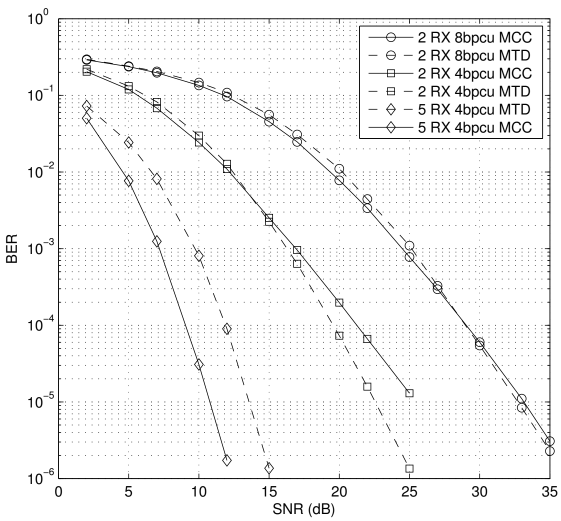

Fig. 1. Comparison of MTD and MCC constructions with different number of receive antennas and different spectral efficiencies.

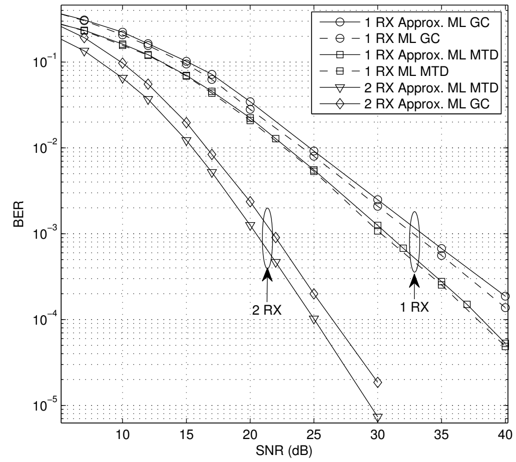

Fig. 2. BER comparison between J_{ML} and J_{ML}^{approx} assuming differential transmission for the MTD and Golden codes with QPSK constellation and 1 a[illegible]

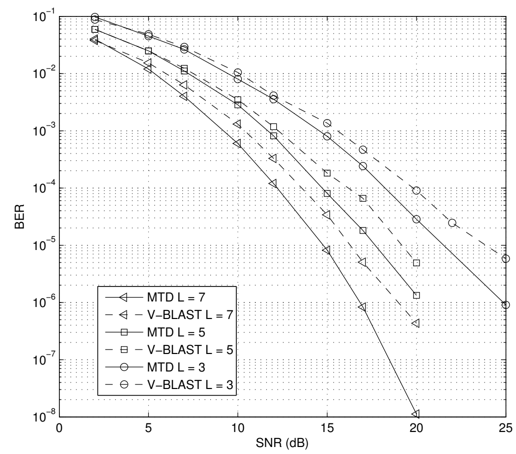

Fig. 3. BER performance of the MMSE SC-FDE receiver with the MTD code compared with V-BLAST for different channel delay spread $L = 3, 5, 7$ at the rate of 4 bits pcu and $N_r = 2$.

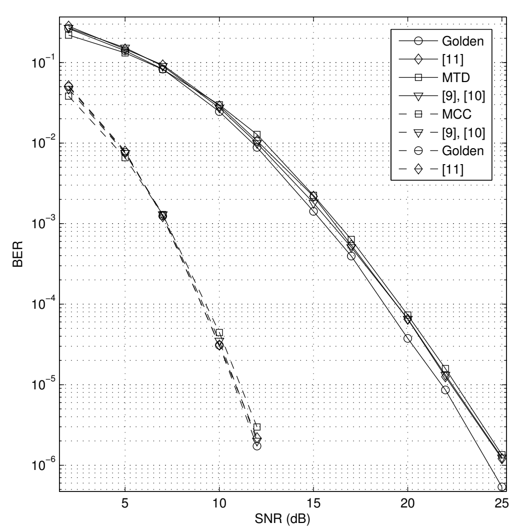

Fig. 4. Comparison between MTD and MCC with the codes in [9], [10] and [11] and . The dashed curves are for comparison of MCC with others using 5 receive antennas. The solid curves are for comparison of MTD with others using 2 receive antennas. Spectral efficiency is 4 bits pcu.