

# SILVER SPACE–TIME TRELLIS-CODED MODULATION

Ezio Biglieri<sup>1</sup>, Yi Hong<sup>2</sup>, and Emanuele Viterbo<sup>3</sup>

<sup>1</sup>Universitat Pompeu Fabra, Barcelona, Spain, e.biglieri@ieee.org

<sup>2</sup>Institute of Advanced Telecom., University of Wales, Swansea, UK, y.hong@swansea.ac.uk

<sup>3</sup>DEIS-Università della Calabria, 89036 Rende (CS), Italy, viterbo@deis.unical.it

## ABSTRACT

Silver Code (SiLC) was originally discovered in [1–4] for  $2 \times 2$  multiple-input multiple-output (MIMO) transmission. It has non-vanishing minimum determinant  $1/7$ , slightly lower than Golden code, but is fast-decodable, i.e., it allows reduced-complexity maximum likelihood decoding [5–7]. In this paper, we present a multidimensional trellis-coded modulation scheme for MIMO systems [11] based on set partitioning of the Silver Code, named Silver Space-Time Trellis Coded Modulation (SST-TCM). This lattice set partitioning is designed specifically to increase the minimum determinant. The branches of the outer trellis code are labeled with these partitions. Viterbi algorithm is applied for trellis decoding, while the branch metrics are computed by using a sphere-decoding algorithm. It is shown that the proposed SST-TCM performs very closely to the Golden Space-Time Trellis Coded Modulation (GST-TCM) scheme, yet with a much reduced decoding complexity thanks to its fast-decoding property.

## 1. INTRODUCTION

High-speed wireless networks for multimedia traffic require high spectral efficiency schemes with low packet delay. Multiple-input multiple-output (MIMO) systems and algebraic space-time codes offer a good set of solutions to this challenging design problem. Wireless channels are commonly modeled as block fading, where it is assumed that the channel is fixed over the duration of a frame. For such channels, concatenated coding schemes are appropriate. Recently, a concatenated scheme, named Golden Space-Time Trellis Coded Modulation (GST-TCM), was proposed in [12, 20] for  $2 \times 2$  MIMO. The inner code is a Golden code [11] and the outer code is a trellis code, which improves the coding gain. The Viterbi algorithm is used for trellis decoding, where the branch metrics are computed using a sphere decoder (SD) [21, 22] for the inner code. GST-TCM is appropriate for high-data-rate systems, thanks to its flexibility in the choice of the modulation spectral efficiency. However, the GST-TCM scheme suffers from a high decoding complexity.

To reduce the decoding complexity of GST-TCM, while maintaining good performance and high spectral efficiency, we advocate the replacement of the Golden code with the Silver Code (SiLC), which yields Silver Space-Time Trellis Coded Modulation (SST-TCM). SiLC, proposed in [1–4] for  $2 \times 2$  MIMO, has non-vanishing minimum determinant of  $\frac{1}{7}$  [10], full-diversity, cubic shaping, and the fast-decodable capability, i.e., it enables reduced-complexity maximum likelihood decoding (MLD), as specialized in the form of an SD search [5–7]. Other fast-decodable space-time block codes,

presented in [23], suffer from diversity loss due to their construction. We therefore do not consider it for this  $2 \times 2$  MIMO ST-TCM.

In the proposed SST-TCM, lattice set partitioning is combined with a trellis code to increase the minimum determinant. Thanks to the fast-decodable capability of SiLC, we can substantially reduce the branch-metric computation complexity of the SD algorithm used in the Viterbi algorithm. We show a design example of SST-TCM, a 16-state TCM which achieves a power gain of 4.2 dB over the uncoded SiLC and has a performance comparable to that of GST-TCM, for a frame error rate (FER) of  $10^{-3}$  and a spectral efficiency of 6 bits per channel use (bpcu), but with a much lower decoding complexity.

The rest of the paper is organized as follows. Section 2 introduces SiLC, system model, and the general code design criterion for SST-TCM. Section 3 introduces set partitioning rules of SiLC. Section 4 presents the proposed SST-TCM. In Section 5 a code design example of SST-TCM with a spectral efficiency of 6 bpcu is shown. Finally, conclusions are drawn in Section 6.

The following notations are used:  $T$  denotes transpose,  $*$  complex conjugation, and  $\dagger$  Hermitian transpose.  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{C}$  and  $\mathbb{Z}[j]$  denote the ring of rational integers, the field of rational numbers, the field of complex numbers, and the ring of Gaussian integers, where  $j^2 = -1$ , respectively. Let  $\mathbb{Q}(\theta)$  denote an algebraic number field generated by the primitive element  $\theta$ .  $\text{GF}(2) = \{0, 1\}$  denotes the Galois field of degree two. The  $m \times m$  identity matrix is denoted by  $\mathbf{I}_m$ . The  $m \times n$  null matrix is denoted by  $\mathbf{0}_{m \times n}$ .

## 2. SYSTEM MODEL

**Silver Code** – First we recall the Silver code  $\mathcal{S}$ , defined in [1–4]. It has codeword matrices of the form

$$X = X_{1,2}(s_1, s_2) + X_{3,4}(s_3, s_4) \quad (1)$$

where the first (resp., second) component code encodes information symbols  $s_1, s_2$  (resp.,  $s_3, s_4$ ) and  $s_1, s_2, s_3, s_4 \in \mathbb{Z}[j]$ ,  $X_{1,2}(s_1, s_2)$  is chosen as an Alamouti code [8], i.e.,

$$X_{1,2}(s_1, s_2) = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} \quad (2)$$

and  $X_{3,4}(s_3, s_4)$  is chosen as follows:

$$X_{3,4}(s_3, s_4) = \mathbf{T} \begin{bmatrix} z_1 & -z_2^* \\ z_2 & z_1^* \end{bmatrix} \quad (3)$$

which has the Alamouti structure [8], where

$$\mathbf{T} \triangleq \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \mathbf{U} \begin{bmatrix} s_3 \\ s_4 \end{bmatrix} \quad (4)$$

where  $z_1, z_2 \in \mathbb{C}$ , and  $\mathbf{U} \in \mathbb{C}^{2 \times 2}$  is the unitary ‘‘Alamouti’’ matrix

$$\mathbf{U} = \begin{bmatrix} \varphi_1 & -\varphi_2^* \\ \varphi_2 & \varphi_1^* \end{bmatrix}$$

with  $|\varphi_1|^2 + |\varphi_2|^2 = 1$ . The best known code of the form (4) was defined by the following unitary matrix [1–6]

$$\mathbf{U} = \frac{1}{\sqrt{7}} \begin{bmatrix} 1+j & -1+2j \\ 1+2j & 1-j \end{bmatrix}$$

The SilC has the following properties:

1. Full rank: The special cyclic division algebra structure of  $\mathbb{Q}(\sqrt{-7})$  guarantees that all codewords have full rank [10].
2. Full rate: The spectral efficiency is of two  $Q$ -QAM information symbols per channel use, i.e.,  $2 \log_2 Q$  bit/s/Hz, and saturates the two degrees of freedom of  $2 \times 2$  MIMO.
3. Cubic shaping: this relates to the cubic shape of the vectorized eight-dimensional constellation, and guarantees that no shaping loss is incurred by the code [6, 7]. We also say this code is information-lossless.
4. Nonvanishing determinant for increasing  $Q$ -QAM size: this property is derived in [10].
5. Minimum determinant  $1/7$ : this preserves the coding gain for any  $Q$ -QAM size.

**System Model.** We consider a  $2 \times 2$  ( $n_T = 2, n_R = 2$ ) MIMO system over slow fading channels. The received signal matrix  $\mathbf{Y} \in \mathbb{C}^{2 \times 2L}$  ( $2L$  is the frame length), is given by

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}, \quad (5)$$

where  $\mathbf{Z} \in \mathbb{C}^{2 \times 2L}$  is the complex white Gaussian noise matrix with i.i.d. samples  $\sim \mathcal{N}_{\mathbb{C}}(0, N_0)$ , and  $\mathbf{H} \in \mathbb{C}^{2 \times 2}$  is the channel matrix, which is constant during a frame and varies independently from one frame to another. The elements of  $\mathbf{H}$  are assumed to be i.i.d. circularly symmetric Gaussian random variables  $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$ . The channel is assumed to be known at the receiver.

In (5),  $\mathbf{X} = [X_1, \dots, X_t, \dots, X_L] \in \mathbb{C}^{2 \times 2L}$  is the transmitted signal matrix, where  $X_t \in \mathbb{C}^{2 \times 2}$ . The inner codewords  $X_t, t = 1, \dots, L$  are selected as follows:

1.  $X_t$  are independently selected from the SilC  $\mathcal{S}$ .
2. A trellis code is used as the outer code. It encodes across symbols  $X_t$  selected from partitions of  $\mathcal{S}$ .

In this paper, we use  $Q$ -QAM constellations, with  $Q = 2^n$  information symbols in (1). We assume the constellation is scaled to match  $\mathbb{Z}[j] + (1+j)/2$ . This implies that its minimum Euclidean distance is set to 1, and centered at the origin. The average energy  $E_s$  is 0.5, 1.5 and 2.5 for  $Q = 4, 8, 16$ , respectively, and  $E_b = E_s/q$  is the energy per bit, where  $q$  denotes the number of information bits per symbol. We have  $N_0 = 2\sigma^2$ , where  $\sigma^2$  is the noise variance per real dimension, which can be adjusted as  $\sigma^2 = (n_T E_b / 2) 10^{-(\text{SNR}_b/10)}$ , where  $\text{SNR}_b$  denotes the signal-to-noise ratio per bit.

**Design Criterion.** Assuming that a codeword  $\mathbf{X}$  is transmitted, the maximum-likelihood receiver might decide erroneously in favor of another codeword  $\widehat{\mathbf{X}}$ . Let  $r$  denote the rank of the *codeword difference matrix*  $\mathbf{X} - \widehat{\mathbf{X}}$ . Since SilC

is full-rank,  $r = n_T = 2$ . Let  $\lambda_i, i = 1, \dots, r$ , be the eigenvalues of the *codeword distance matrix*  $\mathbf{A} = (\mathbf{X} - \widehat{\mathbf{X}})(\mathbf{X} - \widehat{\mathbf{X}})^\dagger$ . Further, let  $\Delta = \prod \lambda_i$  be the determinant of the codeword distance matrix  $\mathbf{A}$ , and  $\Delta_{\min}$  the corresponding *minimum determinant*  $\Delta_{\min} = \min_{\mathbf{X} \neq \widehat{\mathbf{X}}} \det(\mathbf{A})$ . We call  $n_T n_R$  the *diversity*

*gain* and  $(\Delta_{\min})^{1/n_T}$  the *coding gain* [9]. In the case of linear codes, we can simply consider the all-zero codeword matrix, which yields  $\Delta_{\min} \geq \min_{\mathbf{X} \neq \mathbf{0}_{2 \times 2L}} \det(\mathbf{X}\mathbf{X}^\dagger)$ , where equality holds for infinite codes [11].

In order to compare two coding schemes supporting the same information bit rate, but with different minimum determinants ( $\Delta_{\min,1}$  and  $\Delta_{\min,2}$ ) and different constellation energies ( $E_{s,1}$  and  $E_{s,2}$ ), we define the asymptotic coding gain as

$$\gamma_{as} = \frac{\sqrt{\Delta_{\min,1}/E_{s,1}}}{\sqrt{\Delta_{\min,2}/E_{s,2}}}. \quad (6)$$

When  $L = 1$ , the codeword matrix  $\mathbf{X} = X_1 \in \mathcal{S}$  is square. The SilC  $\mathcal{S}$  has full rate, full rank  $r = 2$ , and nonvanishing minimum determinant  $\delta_{\min} = 1/7$  [10]; thus,  $\Delta_{\min} = \delta_{\min}$  for the uncoded SilC. In all cases, we have

$$\det(\mathbf{X}\mathbf{X}^\dagger) = \det\left(\sum_{t=1}^L (X_t X_t^\dagger)\right). \quad (7)$$

A code design criterion attempting at maximizing  $\Delta_{\min}$  is hard to apply, due to the nonadditive nature of the determinant metric in (7). Since  $X_t X_t^\dagger$  are positive definite matrices, we use the following determinant inequality [19]

$$\Delta_{\min} \geq \min_{\mathbf{X} \neq \mathbf{0}_{2 \times 2L}} \sum_{t=1}^L \det(X_t X_t^\dagger) \triangleq \Delta'_{\min}. \quad (8)$$

Maximization of the lower bound  $\Delta'_{\min}$  will be adopted as a design criterion of our concatenated scheme. In particular, we will design trellis codes that attempt at maximizing  $\Delta'_{\min}$ , by using set partitioning to increase the minimum number of nonzero terms in the sum (8).

### 3. SET-PARTITIONING OF SILC

We use a systematic design approach following Ungerboeck-style set-partitioning rules for coset codes [14–16] and for GST-TCM [20], respectively. As in [20], the design criterion for the trellis code is developed in order to maximize  $\Delta'_{\min}$ , since this yields the maximum lower bound on the asymptotic coding gain of the SST-TCM over the uncoded SilC:

$$\gamma_{as} \geq \frac{\sqrt{\Delta'_{\min}/E_{s,1}}}{\sqrt{\delta_{\min}/E_{s,2}}}. \quad (9)$$

We observe that, since SilC shares with the Golden Code the cubic-shaping property, then in STT-TCM we can simply use the same set-partitioning rule as that in GST-TCM. Let us recall the set partition chain in [20].

**Partitioning SilC.** Let us consider a subcode  $\mathcal{S}_k \subseteq \mathcal{S}$  for  $k = 1, \dots, 4$ , obtained from

$$\mathcal{S}_k = \{X B^k, X \in \mathcal{S}\}, \quad (10)$$

| Level | Subcode                        | Lattice         | Binary code            | $\Delta_{\min}$   |
|-------|--------------------------------|-----------------|------------------------|-------------------|
| 0     | $\mathcal{S}$                  | $\mathbb{Z}^8$  | $C_0 = (8, 8, 1)$      | $\delta_{\min}$   |
| 1     | $\mathcal{S}_1$                | $D_4^2$         | $C_1 = (8, 6, 2)$      | $2\delta_{\min}$  |
| 2     | $\mathcal{S}_2$                | $E_8$           | $C_2 = (8, 4, 4)$      | $4\delta_{\min}$  |
| 3     | $\mathcal{S}_3$                | $L_8$           | $C_3 = (8, 2, 4)$      | $8\delta_{\min}$  |
| 4     | $\mathcal{S}_4 = 2\mathcal{S}$ | $2\mathbb{Z}^8$ | $C_4 = (8, 0, \infty)$ | $16\delta_{\min}$ |

Table 1: *SilC partition chain with corresponding lattices, binary codes, and minimum squared determinants. Here  $\delta_{\min} = 1/7$ .*

where

$$B = \begin{bmatrix} j(1-\theta) & 1-\theta \\ j\theta & j\theta \end{bmatrix}. \quad (11)$$

where  $\theta = (1 + \sqrt{5})/2$ . This yields minimum square determinant  $2^k \delta_{\min}$  (see Table 1). It is shown that the codewords of  $\mathcal{S}_k$ , when vectorized, correspond to different sublattices of  $\mathbb{Z}^8$ . It can be verified that these lattices form the lattice partition chain, similarly to that of GST-TCM [20]

$$\mathbb{Z}^8 \supset D_4^2 \supset E_8 \supset L_8 \supset 2\mathbb{Z}^8 \quad (12)$$

where  $D_4^2$  is the direct sum of two four-dimensional Shläfli lattices,  $E_8$  is the Gosset lattice, and  $L_8$  is a lattice of index 64 in  $\mathbb{Z}^8$ . Any two consecutive lattices  $\Lambda \supset \Lambda'$  in this chain form a four-way partition, since the quotient group  $\Lambda/\Lambda'$  has order 4. Let  $[\Lambda/\Lambda']$  denote the set of coset leaders of the quotient group  $\Lambda/\Lambda'$ . The lattices in the partition chain can be obtained by Construction A [18], using the nested sequence of linear binary codes  $C_k = (8, 8 - 2k, d_{\min})$ , where  $d_{\min}$  is the minimum Hamming distance and  $k = 0, \dots, 4$ :

$$\begin{aligned} C_0 = (8, 8, 1) \supset C_1 = (8, 6, 2) \supset C_2 = (8, 4, 4) \\ \supset C_3 = (8, 2, 4) \supset C_4 = (8, 0, \infty) \end{aligned} \quad (13)$$

Following in the footsteps of [14–16], we consider the lattice partition chain  $\Lambda \supset \Lambda' \supset \Lambda_\ell$ , where  $\Lambda, \Lambda', \Lambda_\ell$  are any three consecutive lattices in the partition chain. We can write

$$\Lambda = \Lambda_\ell + [\Lambda/\Lambda_\ell] = \Lambda_\ell + [\Lambda/\Lambda'] + [\Lambda'/\Lambda_\ell].$$

Let  $C, C'$  and  $C''$  be the corresponding codes in (13). Then we can write<sup>1</sup>

$$\Lambda = \Lambda_\ell + [C/C''] = \Lambda_\ell + [C/C'] + [C'/C'']. \quad (14)$$

The coset leaders in  $[C/C']$  form a group of order 4 ( $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ ), generated by two binary generating vectors  $\mathbf{h}_1$  and  $\mathbf{h}_2$

$$[C/C'] = \{b_1 \mathbf{h}_1 + b_2 \mathbf{h}_2 \mid b_1, b_2 \in GF(2)\}$$

If we consider all the lattices in (12), and the corresponding nested sequence of linear binary codes  $C_k$  in (13), we obtain the same  $[C_i/C_{i+1}]$ ,  $i = 0, \dots, 3$ , as for GST-TCM.

<sup>1</sup>Note that the binary components in  $GF(2)$  of the coset leaders are lifted to the ring of integers (slight notational abuse).

## 4. SST-TCM

**Encoder Structure.** We adopt the same encoder structure and trellis labeling as GST-TCM, shown in [20, Fig. 5]. Specifically, the input bits feed two encoders, an upper trellis encoder and a lower lattice encoder. Generalizing (14), we consider two lattices  $\Lambda$  and  $\Lambda_\ell$  from the lattice partition chain in (12), such that  $\Lambda_\ell$  is a proper sublattice of the lattice  $\Lambda$ . Let  $\ell$  denote the relative partition level of  $\Lambda_\ell$  with respect to  $\Lambda$ . The quotient group  $\Lambda/\Lambda_\ell$  has order  $N_c = |\Lambda/\Lambda_\ell| = 4^\ell$ , which corresponds to the total number of cosets of the sublattice  $\Lambda_\ell$  in the lattice  $\Lambda$ .

The upper encoder is a trellis encoder that operates on  $q_c$  information bits. Given the relative partition depth  $\ell$ , we select a trellis code rate  $R_c = 1/\ell$ . The trellis encoder outputs  $n_c = q_c/R_c$  bits, which are used by the coset mapper to label the coset leaders  $\mathbf{c} \in [\Lambda/\Lambda_\ell]$ . The mapping is obtained by the product of the  $n_c$  bit vector with a binary coset leader generator matrix  $H_c$  with relevant rows. Since the trellis has  $2^{q_c}$  incoming and outgoing branches from each state, we limit ourselves to considering  $q_c = 2$ , so as to preserve a reasonable branch trellis complexity.

The lower encoder is a lattice encoder for  $\Lambda_\ell$ , and operates on  $q_\mu$  information bits. The  $\Lambda_\ell$  encoder generates vectors  $\mathbf{x}$ . The vectors  $\mathbf{x}$  and the binary coset leaders  $\mathbf{c}$  from trellis encoder are added component-wise, and mapped to the SilC codeword  $X_t$ .

We now focus on the structure of the trellis code to be used. We consider linear convolutional encoders over the quaternary alphabet  $\mathbb{Z}_4 = \{0, 1, 2, 3\}$  with mod-4 operations. We assume the natural mapping between pairs of bits, and symbols in  $\mathbb{Z}_4$ . Let  $\beta \in \mathbb{Z}_4$  denote the input symbol, and  $\alpha_1, \dots, \alpha_\ell \in \mathbb{Z}_4$  the  $\ell$  output symbols generated by the polynomials  $g_1(D), \dots, g_\ell(D)$  over  $\mathbb{Z}_4[D]$ .

**Labeling.** In order to increase the potential coding gain, the lower bound  $\Delta'_{\min}$  in (8) should be maximized. This lower bound is determined by the shortest simple error events in the trellis, i.e.,

$$\begin{aligned} \Delta'_{\min} &= \min_{\mathbf{x} \neq \mathbf{0}_{2 \times 2L}} \sum_{t=t_o}^{t_o+L'-1} \det(X_t X_t^\dagger) \\ &\geq \min_{X_{t_o}} \det(X_{t_o} X_{t_o}^\dagger) + \min_{X_{t_i}} \det(X_{t_i} X_{t_i}^\dagger) \end{aligned} \quad (15)$$

where  $L'$  is the length of the shortest simple error event diverging from the zero state at  $t_o$  and merging into the zero state at  $t_i = t_o + L'$ . Therefore, we have the following:

**Design criterion.** We focus on simple error events. The incoming and outgoing trellis paths for each state should belong to different cosets that are as deep as possible in the partition tree. This guarantees that simple error events in the trellis have large  $\Delta'_{\min}$ .

**Fast decoding.** For SST-TCM, the decoder structure is similar to that of GST-TCM, i.e., a Viterbi algorithm using a branch metric computer obtained by SD. The branch metric computer should output the distance of the received symbol from all the cosets of a sublattice in the lattice. The decoding complexity depends on two parameters

1. the total number of distinct parallel branch metrics.
2. the number of states in the trellis.

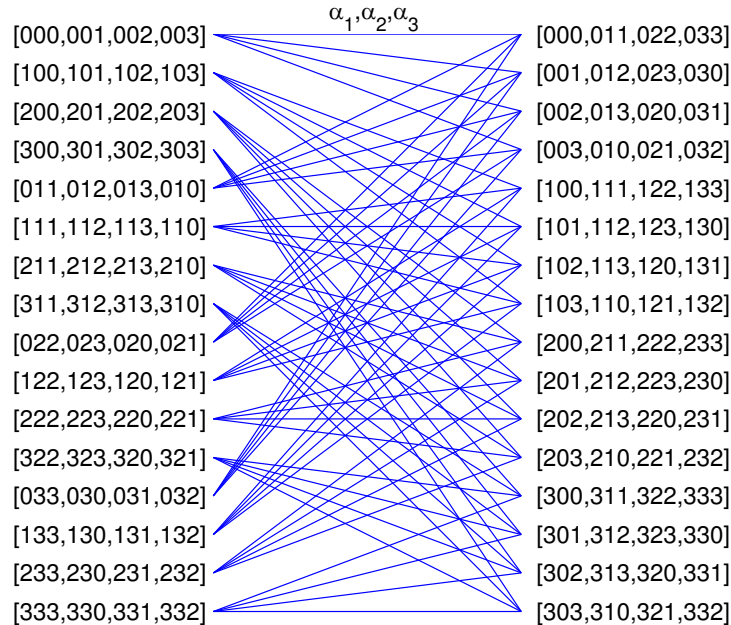


Figure 1: The 16-state trellis corresponding to the generators  $g_1(D) = D$ ,  $g_2(D) = D^2$ , and  $g_3(D) = 1 + D^2$ . Labels on the left are emanating from each state clockwise, labels on the right are merging counterclockwise,  $\Lambda = \mathbb{Z}^8$ ,  $\Lambda_\ell = L_8$ ,  $\ell = 3$ , SST-TCM.

In GST-TCM, the branch metric computer can be realized as a traditional sphere decoder for each branch metric, which requires an  $O(M^4)$  worst-case decoding complexity, where  $M$  is the cardinality of the relevant constellation. For SiIC, it was pointed out in [6, 7] that its worst-case decoding complexity is  $O(M^3)$ .

## 5. CODE DESIGN EXAMPLE

In this section, we exhibit a code design example of SST-TCM with 16-state trellis codes. We assume that each frame contains  $L = 130$  symbols ( $2 \times 2$ ). We first describe uncoded SiIC schemes with the same frame length, to be used as baselines for performance comparison.

**Uncoded Silver code 6 bpcu.** A total of 12 bits must be sent in a Silver codeword: the symbols  $a, b, c, d$  are in a 8-QAM (3bits). This guarantees that the same average energy is transmitted from both antennas. In this case we have  $E_{s,2} = 1.5$  and  $q = 3$  bits.

**Example.** The 16-state trellis codes with 16-QAM gain 4.2 dB over an uncoded transmission scheme at the rate of 6 bpcu and  $\Lambda = \mathbb{Z}^8$ ,  $\Lambda_\ell = L_8$ , where  $\ell_0 = 0, \ell = 3$ . We have  $E_{s,1} = 2.5$  and  $q = 3$  bits.

We consider a 3-level partition with quotient group  $\Lambda/\Lambda_\ell = \mathbb{Z}^8/L_8$  of order  $N_c = 64$ . The quaternary trellis encoders for 16-states with rate  $R_c = 1/3$  have  $q_1 = 2$  input information bits and  $n_c = 6$  output bits, which label the coset leaders. The sublattice encoder has  $q_2 = 2$  and  $q_3 = 8$  input bits, giving a total number of input bits per information symbol  $q = (q_1 + q_2 + q_3)/4 = 12/4 = 3$  bits.

The 16-state GST-TCM has the following generator polynomials:  $g_1(D) = D$ ,  $g_2(D) = D^2$ ,  $g_3(D) = 1 + D^2$ , where  $D$  is a delay operator mod 4. In Fig. 1, for each trellis state, the four outgoing branches with labels  $\alpha_1, \alpha_2, \alpha_3$ , corresponding

to input  $\beta = 0, 1, 2, 3$ , are listed on the left side of the trellis. Similarly, the four incoming trellis branches to each state are listed on the right side of the trellis structure. In such a case,  $\alpha_1$  chooses the cosets from  $D_4^2$  in  $\Lambda = \mathbb{Z}^8$ ,  $\alpha_2$  chooses the cosets from  $E_8$  in  $D_4^2$ , and  $\alpha_3$  chooses the cosets from  $\Lambda_\ell = L_8$  in  $E_8$ .

We can see that the shortest simple error event has length  $L' = 3$ , corresponding to the state sequence  $0 \rightarrow 1 \rightarrow 4 \rightarrow 0$  and to labels 001, 100, 011. This will yield

$$\Delta'_{\min} \geq \min(8\delta_{\min}, 4\delta_{\min} + \delta_{\min} + 2\delta_{\min}) = 7\delta_{\min}.$$

The performances of the proposed codes, GST-TCM, and the uncoded schemes including SiIC and Golden code at 6 bpcu are compared in Fig. 2. We can observe that a 16-state GST-TCM outperforms the uncoded SiIC and the Golden code scheme by 4.2 dB at FER =  $10^{-3}$ . We also observe that the SST-TCM has a performance comparable to GST-TCM, but its decoding complexity is greatly reduced.

## 6. CONCLUSIONS

In this paper, we present SST-TCM, a concatenated coding scheme suitable for slow-fading  $2 \times 2$  MIMO systems. The inner modulation is the SiIC, which provides full diversity, nonvanishing determinant, and fast-decodable properties. Lattice set partitioning is designed specifically to increase the minimum determinant of SiIC codewords, which label the branches of the trellis code. Viterbi algorithm is applied in trellis decoding, where branch metrics are computed by using a lattice decoder. We were able to show that the proposed SST-TCM has a performance very close to that of GST-TCM, yet with a substantial decoding complexity reduction. Future work may consider the use of Diophantine approximation in the decoding [24].

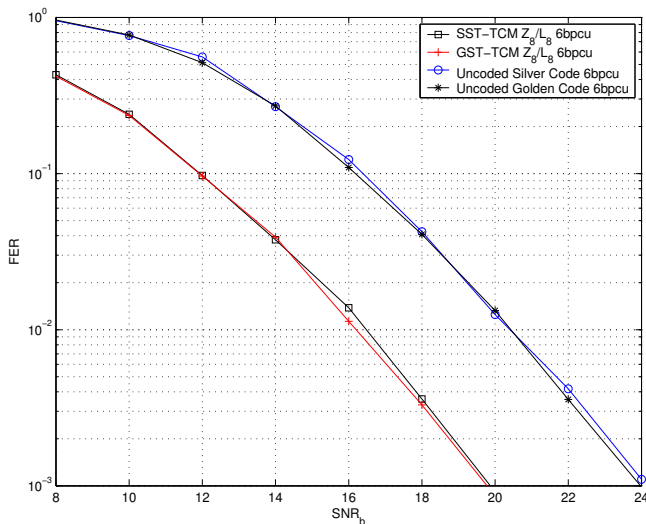


Figure 2: Performance comparison of 16-state trellis codes in SST-TCM and GST-TCM, using 16-QAM constellation, and uncoded Silver code and Golden code transmission at the rate 6bpcu,  $\Lambda = \mathbb{Z}^8$ ,  $\Lambda_\ell = L_8$ ,  $\ell = 3$ .

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