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DIRECT DESIGN OF LINEAR-PHASE NONUNIFORM FILTER BANKS WITH ARBITRARY INTEGER DECIMATION FACTORS*

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ABSTRACT

In this paper, we study the direct design of M-channel nonuniform filter banks (NUFBs) and propose a novel method for the NUFBs with linear-phase (LP) property. The filter banks we are concerned with have integer decimation factors and satisfy the maximal decimation condition. We derive a design criterion for the LP NUFBs. In addition, we propose certain modification on the direct structure, achieving LP NUFBs with arbitrary integer decimation factors. A detailed analysis on the new structure is presented to show that the significant aliasing can also be cancelled. Design examples are given to show that the proposed method is capable of achieving near-perfect-reconstruction NUFBs with LP property.

1. INTRODUCTION

M-channel maximally decimated filter banks with linear-phase (LP) property have been successfully employed in some image processing applications. For more flexibility in partitioning subbands, nonuniform filter banks (NUFBs) are highly desired. Tree structure by cascading uniform filter banks [1], is an easy way to achieve LP NUFBs. However, the choice of the decimation factors is very limited. Recombination structure (indirect structure) can also produce LP NUFBs [2], where certain subbands of a uniform filter bank are merged by sets of transmultiplexers. The system delay of the NUFBs is long due to the two-stage architecture. For the NUFBs with single-stage structure (direct structure), the design methods can be classified as the direct and indirect designs. The NUFBs by using the indirect design method are obtained based on uniform filter banks [3-5]. Unfortunately, the LP property of the resulting filters cannot be achieved by this method because of the highly complicated phase distribution. The direct design of NUFBs is proposed [6-13]. In [8-10] the effective design for NUFBs based on cosine-modulation is provided, however, the LP property of filters is not considered. The work described in [11] is focused on the design of LP NUFBs. The design is limited to two-channel NUFBs. Although the designs in [12, 13] aim at *M*-channel NUFBs, highly nonlinear optimization procedures are involved with a considerable number of parameters and the quality of the filters cannot be maintained. Additionally, the complex filters in [12] are involved even in NUFB systems with integer decimation factors, thereby making the implementation more costly.

In this study, we propose a novel method for the direct design of LP NUFBs, which is based on the direct structure. The decimation factors, we focus on, are integers and satisfy maximal decimation condition. With the direct structure, the partitioning schemes of the frequency bandwidth can be either feasible or nonfeasible depending on the sampling factors used for the NUFBs. In the feasible partitioning case, the significant aliasing, appearing in the transition bands of analysis filters, can be cancelled by the proper design of analysis and synthesis filters. In contrast, in the nonfeasible partitioning case, the large aliasing, appearing in the passband regions of analysis filters, cannot be cancelled even with ideal filters. In our work, we first derive a design criterion for the LP NUFBs with feasible frequency partitioning. Then, we propose a modified structure for the NUFBs with nonfeasible partitioning. The structure is based on the one proposed in [3, 13] where the LP property of the analysis and synthesis filters is lost. In our modified structure, the phase distortion is compensated, thereby maintaining the LP property of the filters. A detailed analysis on the resulting NUFBs shows that the significant aliasing can be cancelled. With the modified structure and the derived criterion, the design can be applied to the realization of LP NUFBs with arbitrary integer decimation factors. Further, we employ the Parks-McClellan algorithm to design the filter because the desired magnitude response of the filter can be specified as an optimization objective in the algorithm, leading to a less design effort. Design examples show that the performance of the designed near-perfect-reconstruction NUFB in terms of stopband attenuation is excellent with the expected LP property.

The rest of this paper is organized as follows: in Section II, the proposed LP NUFBs with integer decimation factors are investigated. The nonfeasible case is discussed in Section III. Examples are given in Section IV, followed by conclusions in Section V.

2. PRINCIPLE OF THE PROPOSED LINEAR PHASE NONUNIFORM FILTER BANKS

Figure 1 shows a typical direct structure of an *M*-channel NUFB, where the decimation/interpolation factors, n_k, $k = 0,\ldots,M-1$, are integers. In this paper, we are only concerned with the maximally decimated NUFBs, and all analysis and synthesis filters are LP with real coefficients.

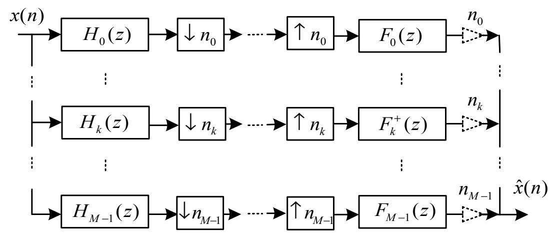

Figure 1: Typical structure of integer decimated NUFBs

For the LP analysis filter $H_k(z)$, its impulse response $h_k(n)$ has the form of

$$h_k(n) = \begin{cases} h_k(N-n), & \text{for symmetry,} \\ -h_k(N-n), & \text{for anti-symmetry.} \end{cases} \tag{1}$$

where N is the order of the filter. In this work, we assume that all filters have the same length, and the synthesis filters are the time

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reversed versions of the analysis filters as

$$f_k(n) = h_k(N-n)\,,\ 0 \le k \le M-1\,. \qquad (2)$$

By the time-reverse relation in (2), we get

$$f_k(n) = \begin{cases} h_k(n), & \text{for symmetry,} \\ -h_k(n), & \text{for anti-symmetry.} \end{cases} \qquad (3)$$

To compensate the energy loss caused by the decimation n_k, we multiply the constants n_k to the outputs of the corresponding channels as shown in Fig. 1, $k = 0,\ldots,M-1$. Subsequently, the output of the system can be expressed as

$$\hat{X}(z) = \sum_{k=0}^{M-1}\sum_{l=0}^{n_k-1} (-1)^{p_k} H_k(zW_{n_k}^l) H_k(z) X(zW_{n_k}^l) \qquad (4)$$

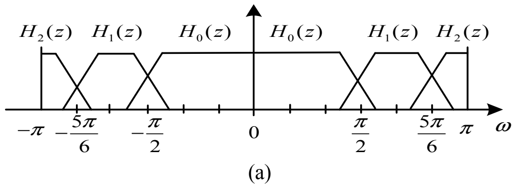
where $W_{n_k} = e^{-j2\pi/n_k}$ and $p_k = \begin{cases} 0, & h_k(n) \text{ is symmetric,} \\ 1, & h_k(n) \text{ is anti-symmetric.} \end{cases}$ From (4), it can be verified that the cutoff frequencies of $H_k(z)$ must be located at the points of $\pi n/n_k$, $n = 1,2,\cdots n_k - 1$, to avoid large aliasing. Otherwise, there must be large amount of aliasing between $H_k(zW_{n_k}^l)$ and $F_k(z)$, $l \in \{1,2,\cdots n_k - 1\}$, which cannot be cancelled even with ideal filters. We introduce the term "feasible frequency partitioning" to describe such NUFB systems where the large aliasing cannot appear and "nonfeasible frequency partitioning" to denote the others with large aliasing.

To help understand this better, let us take an example of the NUFB with decimation factors (2, 3, 6). Since this work considers only near-perfect-reconstruction filter banks, we require that the stopband attenuation of each analysis filter is significantly high. Figure 2(a) shows the ideal magnitude response sketches of the analysis filters. Obviously, the cutoff frequencies of $H_1(z)$ are $\pi/2$ and $5\pi/6$, not $n\pi/3$ for any integer n. Figure 2(b) shows the large alias components of $H_1(zW_3^1)F_1(z)$ and $H_1(zW_3^2)F_1(z)$ with respect to $X_1(zW_3^1)$ and $X_1(zW_3^2)$. It can be verified that these large alias components cannot be cancelled because there is no pair for each component. In this case, we say the NUFB is with nonfeasible frequency partitioning. It can be also easily verified that the NUFBs with such permutation of the decimation factors as (3, 2, 6), (6, 2, 3) and (6, 3, 2) result in nonfeasible partitioning cases.

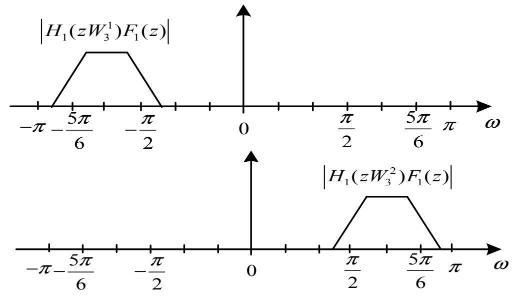
For the feasible frequency partitioning NUFBs with sampling factors such as (2, 6, 3) and (3, 6, 2), the aliasing terms can be cancelled. Figure 3 shows the alias components of the NUFB with the decimation factors (2, 6, 3). As an illustration, only the components of $H_0(zW_2^1)F_0(z)$ and $H_1(zW_6^3)F_1(z)$ with respect to $X(zW_2^1)$ (or $X(zW_6^3)$) are shown. It is evident that those aliasing terms can be cancelled.

The above analysis is based on two constraints: (i) each pair of analysis and synthesis filters satisfies the time-reverse relation, (ii) all the filters have the same length. Under the two constraints, we summarize a design criterion for LP NUFBs as

Design criterion: The significant aliasing terms can be cancelled if and only if the cutoff frequency of each analysis $H_k(z)$ is located at the points of $\pi n/n_k$, $n = 1,2,\cdots,n_k - 1$, $0 \le k \le M-1$, and further, the analysis filters are symmetric and anti-symmetric alternatively with $H_0(z)$ symmetric.

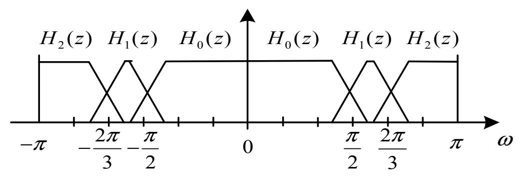


(a)

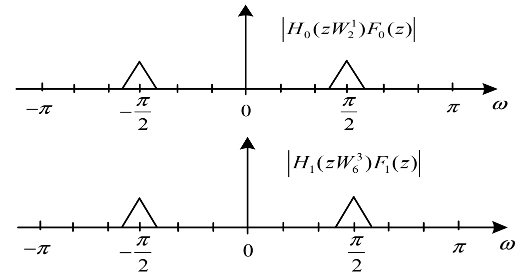


(b)

Figure 2: The (2, 3, 6) NUFB. (a) Magnitude response sketches of analysis filters, (b) alias components of $H_1(zW_3^1)F_1(z)$ with respect to $X_1(zW_3^1)$, and $H_1(zW_3^2)F_1(z)$ to $X_1(zW_3^2)$.



(a)



(b)

Figure 3: The (2, 6, 3) NUFB. (a) Magnitude response sketches of analysis filters, (b) alias components of $H_0(zW_2^1)F_0(z)$ with respect to $X(zW_2^1)$, and $H_1(zW_6^3)F_1(z)$ to $X(zW_6^3)$.

3. LINEAR-PHASE NONUNIFORM FILTER BANKS WITH NONFEASIBLE PARTITIONING

With the direct structure, the large aliasing caused by the adjacent filters cannot be cancelled in nonfeasible partitioning NUFBs because in this case the cutoff frequency of at least one analysis filter $H_k(z)$ is not located at the points of $n\pi/n_k$, $n = 1,2,\cdots n_k - 1$, $0 \le k \le M-1$. If such invalid filter is shifted to a suitable location to satisfy the feasible condition, the large aliasing may be avoided. Here, we will propose a modified structure, where the large aliasing is avoided and the LP property of analysis/synthesis filters is obtained.

3.1 The Modified Structure

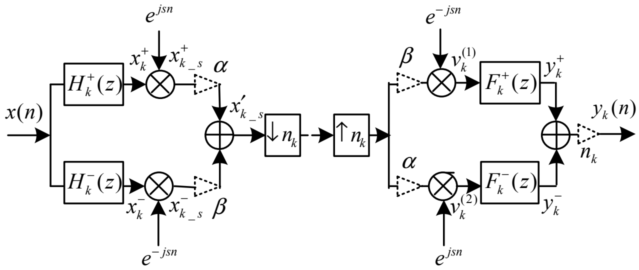


Figure 4: The *k*-th channel of the modified NUFBs with nonfeasible partition.

Assume that $H_k(z)$ occupies an invalid frequency location. It should be shifted to a suitable position to satisfy the feasible condition. The proposed structure is shown in Figure 4. Since $H_k(z)$ is LP with real-valued coefficients, it can be expressed as $H_k(e^{j\omega}) = H_k^+(e^{j\omega}) + H_k^-(e^{j\omega})$. The frequency supports of the two parts are $\left[-\sum_{i=0}^{k}\frac{\pi}{n_i}, -\sum_{i=0}^{k-1}\frac{\pi}{n_i}\right]$ and $\left[\sum_{i=0}^{k-1}\frac{\pi}{n_i}, \sum_{i=0}^{k}\frac{\pi}{n_i}\right]$ in $[-\pi, \pi]$. Since $H_k(e^{j\omega})$ cannot satisfy the feasible condition, $\sum_{i=0}^{k-1}\frac{\pi}{n_i}$ and $-\sum_{i=0}^{k-1}\frac{\pi}{n_i}$ can be expressed as

$$\sum_{i=0}^{k-1}\frac{\pi}{n_i} = \frac{n}{n_k}\pi - s\ , \quad -\sum_{i=0}^{k-1}\frac{\pi}{n_i} = -\frac{n}{n_k}\pi + s \tag{5}$$

where $n \in \{0, 1, \cdots, n_k - 1\}$ and s can be a positive or negative number.

To ensure the cutoff frequencies of the resulting real-coefficient filter being at the points of $n\pi/n_k$, $n \in \{0, 1, \cdots, n_k - 1\}$, $H_k^+(z)$ and $H_k^-(z)$ should be separately shifted by s and $-s$ in the positive direction. Thus, the shifted versions of $H_k^+(z)$ and $H_k^-(z)$ are $H_{k\_s}^+(z) = H_k^+(ze^{-js})$ and $H_{k\_s}^-(e^{j\omega}) = H_k^-(ze^{js})$, respectively. We denote the resulting filter by $H_{k\_s}(z)$

$$H_{k\_s}(z) = H_{k\_s}^+(z) + H_{k\_s}^-(z) = \begin{cases} e^{-j\omega N/2}\left(e^{jsN/2}\cdot\left|H_k^+(z)\right| + e^{-jsN/2}\cdot\left|H_{k\_s}^-(z)\right|\right), \\ \text{for symmetry ;} \\ je^{-j\omega N/2}\left(e^{jsN/2}\cdot\left|H_{k\_s}^+(z)\right| - e^{-jsN/2}\cdot\left|H_{k\_s}^-(z)\right|\right), \\ \text{for anti-symmetry .} \end{cases} \tag{6}$$

From (6), we can see that due to the elements $e^{jsN/2}$ in $H_{k\_s}^+(e^{j\omega})$ and $e^{-jsN/2}$ in $H_{k\_s}^-(e^{j\omega})$, the shifted filter $H_{k\_s}(e^{j\omega})$ might loss the LP property. To maintain the LP property of the analysis filter, we introduce the phase modification factors α and β in our proposed structure as shown in Figure 4,

$$\alpha = e^{-jsN/2}\ , \quad \beta = e^{jsN/2}\ . \tag{7}$$

By multiplying α and β with $H_{k\_s}^+(e^{j\omega})$ and $H_{k\_s}^-(e^{j\omega})$ respectively, the modified filter denoted as $H_{k\_s}^{'}(e^{j\omega})$, obtained from $H_{k\_s}(e^{j\omega})$ in (6), is as follows

$$H_{k\_s}^{'}(z) = \alpha\cdot H_{k\_s}^+(z) + \beta\cdot H_{k\_s}^-(z) = \begin{cases} e^{-j\omega N/2}\left(\left|H_k^+(z)\right| + \left|H_{k\_s}^-(z)\right|\right), & \text{for symmetric} \\ je^{-j\omega N/2}\left(\left|H_{k\_s}^+(z)\right| - \left|H_{k\_s}^-(z)\right|\right), & \text{for antisymmetric} \end{cases} \tag{8}$$

Thus, it is evident that $H_{k\_s}^{'}(z)$ does possess the LP property.

3.2 Alias Cancellation

Consider the k-th channel of the filter bank shown in Figure 4. The input signal $x(n)$ is passing through the filters $H_k^+(z)$ and $H_k^-(z)$, resulting in $X_k^+(z)$ and $X_k^-(z)$. After shifting to the right and left sides by s, respectively, we have

$$X_{k\_s}^+(z) = H_k^+(zS)X(zS)\ , \tag{9a}$$

$$X_{k\_s}^-(z) = H_k^-(zS^{-1})X(zS^{-1})\ , \tag{9b}$$

where $S = e^{-js}$. Multiplying with the factors α and β to $X_{k\_s}^+(z)$ and $X_{k\_s}^-(z)$, respectively, for the correction of the phase distortion, we get the sum of the two signals

$$X_{k\_s}^{\prime}(z) = \alpha\cdot H_k^+(zS)X(zS) + \beta\cdot H_k^-(zS^{-1})X(zS^{-1})\ . \tag{10}$$

In the synthesis section, the reversed operations of the analysis section are taken. After decimation and interpolation, the received signal in the k-th channel are respectively processed with the three steps: i) modified by β and α, ii) shifted back to their original positions, iii) filtered by $F_k^+(z)$ and $F_k^-(z)$, forming,

$$\begin{aligned} Y_k(z) &= Y_k^+(z) + Y_k^-(z) \\ &= \beta\sum_{i=0}^{n_k-1} X_{k\_s}^{\prime}(zW_{n_k}^i S^{-1})F_k^+(z) + \alpha\sum_{i=0}^{n_k-1} X_{k\_s}^{\prime}(zW_{n_k}^i S)F_k^-(z) \\ &= A_{k,0}X(z) + \sum_{i=1}^{n_k-1} A_{k,i}X(zW_{n_k}^i) \\ &+ \sum_{i=0}^{n_k-1} A_{k,i}^{s^-}X(zW_{n_k}^i S^{-2}) + \sum_{i=0}^{n_k-1} A_{k,i}^{s^+}X(zW_{n_k}^i S^2), \end{aligned} \tag{11}$$

where

$A_{k,0} = H_k^+(z)F_k^+(z) + H_k^-(z)F_k^-(z)$

$A_{k,i} = H_k^+(zW_{n_k}^i)F_k^+(z) + H_k^-(zW_{n_k}^i)F_k^-(z),\ i = 1, \cdots, n_k - 1$;

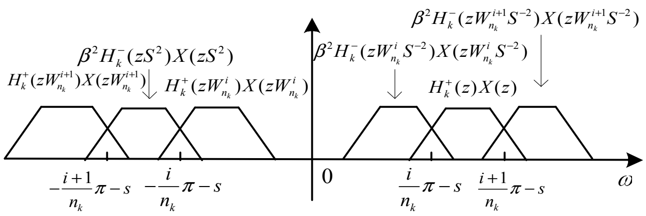
$A_{k,i}^{s^-} = \beta^2 H_k^-(zW_{n_k}^i S^{-2})F_k^+(z),\ A_{k,i}^{s^+} = \alpha^2 H_k^+(zW_{n_k}^i S^2)F_k^-(z)$,

$i = 0, \cdots, n_k - 1$.

In (11), $X(zW_{n_k}^i)$, $i = 1, \cdots, n_k - 1$, $X(zW_{n_k}^i S^{-2})$ and $X(zW_{n_k}^i S^2)$, $i = 0, \cdots, n_k - 1$, are aliasing terms in the k-th channel; and $A_{k,i}$, $A_{k,i}^{s^-}$ and $A_{k,i}^{s^+}$ are the gains of those aliasing terms. In our analysis, we assume that the stopband attenuation of filters is sufficiently high and only the significant aliasing, appearing in the transition bands, is caused by the adjacent filters. Thus, we have $A_{k,i} = 0$, $i = 1, \cdots, n_k - 1$. Hence, (11) can be simplified as

$$Y_k(z) = A_{k,0}X(z) + \sum_{i=0}^{n_k-1} A_{k,i}^{s^-}X(zW_{n_k}^i S^{-2}) + \sum_{i=0}^{n_k-1} A_{k,i}^{s^+}X(zW_{n_k}^i S^2)\ . \tag{12}$$

To help understand the point better, we present Figure 5 as an illustration of the aliasing terms. For the analysis simplicity, we assume that s is a positive number and the input signal has unit amplitude and zero phase. Figure 5(a) shows the magnitude responses of $x_{k\_s}^{'}$ and its various shifted versions to be filtered by $F_k^+(e^{j\omega})$, Figure 5(b) shows those to be filtered by $F_k^-(e^{j\omega})$. Figure 5(c) shows the magnitude responses of $F_k^+(e^{j\omega})$ and $F_k^-(e^{j\omega})$. From Figure 5(a) and (c), we can see that $H_k^+(zW_{n_k}^i)X(zW_{n_k}^i)F_k^+(z)$ is zero. Similarly, $H_k^-(zW_{n_k}^i)X(zW_{n_k}^i)F_k^-(z)$ is also zero from Figure 5(b) and (c). This indicates that $A_{k,i} = 0$, $i = 1, \cdots, n_k - 1$. Figure 5(d) shows the residual aliasing gain $A_{k,i}^{s^-} = \beta^2 H_k^-(zW_{n_k}^i S^{-2})F_k^+(z)$ and $A_{k,i}^{s^+} = \alpha^2 H_k^+(zW_{n_k}^i S^2)F_k^-(z)$.



(a)

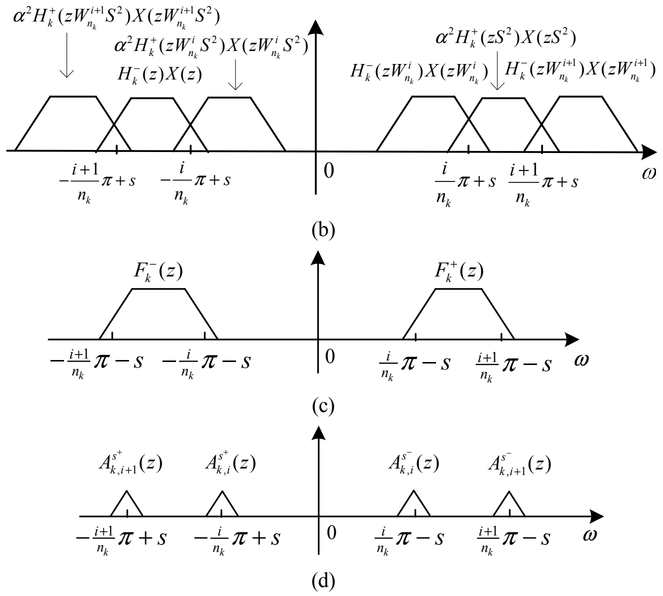


Figure 5: Demonstration of the aliasing term in the k-th channel. (a) Magnitude response sketches of x'_{k_s} and various shifted versions to be filtered by $F_k^+(z)$, (b) those to be filtered by $F_k^-(z)$, (c) magnitude response sketches of synthesis filter, and (d) the residual aliasing gain.

It can be easily verified that the aliasing terms as shown in Figure 5(d) can be canceled by those appearing in the adjacent channels.

3.3 Choice of the Filter Length

To remove the effect of the phase modification factor $\alpha = e^{jsN/2}$ and $\beta = e^{-jsN/2}$ on aliasing gains $A_{k,i}^{s^-} = \beta^2 H_k^-(zW^i S^{-2}) F_k^+(z)$ and $A_{k,i}^{s^+} = \alpha^2 H_k^+(zW^i S^2) F_k^-(z)$, it is necessary to set

$$sN = 2\pi i, \quad i \in \mathbb{Z} \tag{13}$$

Since s can be expressed as $s = q\pi/p$, p and q are integers. From (13), we conclude that the order of the filter should satisfy the following constraint

$$N = 2ip/q, \quad i \in \mathbb{Z}. \tag{14}$$

Different from the feasible partitioning NUFB, the choice of the length of the filter in the nonfeasible NUFB is limited due to the requirement constraint given in (14). The second example in the next section will numerically illustrate this point.

4. DESIGN EXAMPLES AND PERFORMANCE ANALYSIS

In the filter design, we employ the Parks-McClellan algorithm, implemented by `firpm` function in Matlab, which is optimal in minimax sense. More importantly, the desired characteristic of the filter can be specified and used as the design target. We can achieve our target by letting the transition band of the filter follow approximately a cosine roll-off characteristic and keeping the intersection point of the two filters to have the value of $1/\sqrt{2}$. Since all the analysis/synthesis filters are of the same length, they can be designed to have the similar transition band while keeping the identical amount of passband ripple and stopband attenuation.

To demonstrate the proposed method, we provide two examples. The first one is to show the 3-channel LP NUFB with feasible partitioning whereas the second is to show the nonfeasible partitioning. Note that the proposed method is also useful for filter banks with more than 3 channels.

Example 1: Assume the NUFB has decimation factors (2, 6, 3). We choose the length of each filter being 97. According to the design criterion described in Section II, the filters are alternatively symmetric. The filters are designed individually by using the Parks-McClellan algorithm. By specifying each filter to have the similar transition band that follows a cosine roll-off function, we get the desired LP NUFB. Figure 6 shows the magnitude responses of the analysis filters, amplitude distortion and aliasing error of the system. In this example, the maximum peak to peak ripple of amplitude distortion and the maximum aliasing error are respectively $E_{pp} = 2.774 \times 10^{-3}$ and $E_a = 9.096 \times 10^{-4}$.

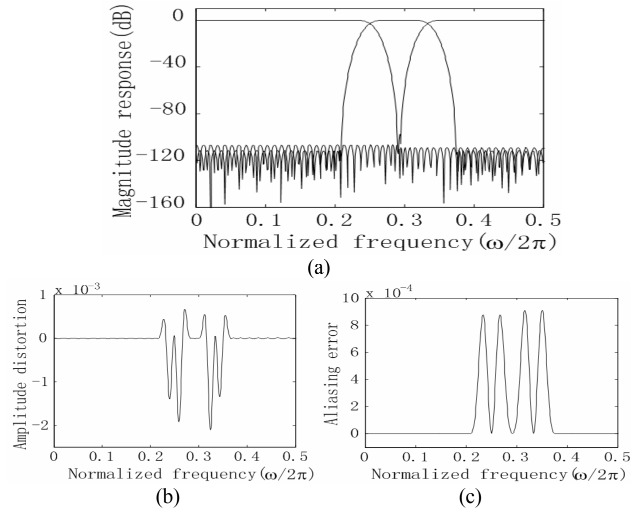
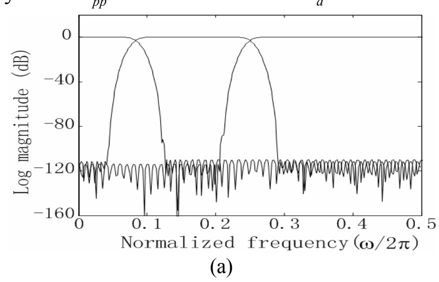
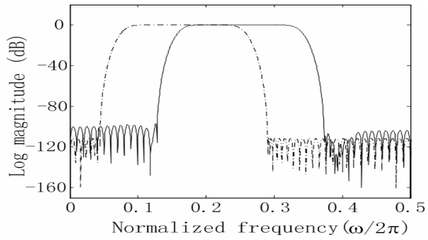


Fig. 6. The (2,6,3) LP NUFB. (a) Magnitude response of analysis filters, (b) amplitude distortion, and (c) aliasing error.

Example 2: In this example, we show the LP NUFB has decimation factors of (6, 3, 2). The second channel occupies an invalid frequency band. According to the description in Section III, this band is separated into two regions. With the modified structure, the two regions are then shifted to a suitable location so that the significant aliasing can be cancelled. Here, we choose the shifting amount being $s = \pi/6$. According to (14), the length of each filter must be $N + 1 = 12i + 1$, where $p = 6$ and $q = 1$. By choosing $i = 8$, the filter has a length of 97. The frequency responses of the original and shifted analysis filters are illustrated in Figure 7 by inputting a sinusoidal signal at certain frequency with unit amplitude and zero phase. Figure 7(a) shows the magnitude responses of three analysis filters. In Figure 7(b), dashed and real line respectively denote the magnitude responses of x_k^+ and $x_{k\_s}^+$ where $k = 1$ as shown in Figure 4, respectively, corresponding to the original and shifted positive versions of the outputs in the second channel. Figure 7(c) shows the phase response of x'_{k_s} and it is obvious that x'_{k_s} has linear phase property. The amplitude distortion and the aliasing error are shown in Figure 7(d) and (e) respectively with $E_{pp} = 2.309 \times 10^{-3}$ and $E_a = 1.401 \times 10^{-3}$.



(a)



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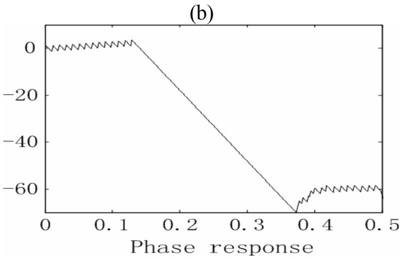

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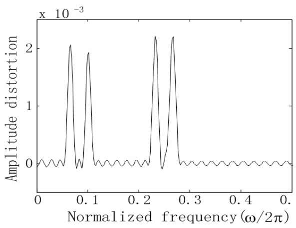

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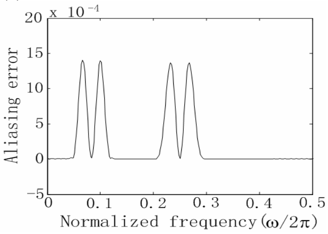

(e)

Figure 7: The (6, 3, 2) LP NUFBs. (a) Magnitude response of analysis filters, (b) magnitude responses of x_k^+ and $x_{k\_s}^+$, $k=1$, respectively corresponding to the original and shifted positive versions of the outputs in the second channel, (c) amplitude distortion, and (d) aliasing error.

5. CONCLUSION

In this paper, we proposed a new approach to the design of near-perfect-reconstruction LP NUFBs with integer decimation factors. A design criterion has been derived aiming at the cancellation of alias components. With a certain modification on the typical structure of NUFBs, we can obtain LP NUFBs with arbitrary integer decimation factors. For further consideration, the possibility of the realization of LP NUFBs with arbitrary rational decimation factors based on the rational decimated NUFBs [15, 16] may be studied.

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