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A VSSLMS ALGORITHM BASED ON ERROR AUTOCORRELATION

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ABSTRACT

This paper proposes a new variable step-size (VSS) LMS algorithm. Since the step-size adjusting process and misadjustment parameter are affected by measurement noise, VSS approaches based on the error signal autocorrelation add some immunity against such an undesired signal. Thus, an existing approach uses a lag(1) error signal autocorrelation function obtaining good results, in the proposed one, lag(1),lag(2),...,lag(N) error signal autocorrelation functions are used. The computational complexity increase due to the use of several lags is small. Numerical simulations verify the performance of the proposed algorithm, assessed in a system identification problem.

1. INTRODUCTION

Stochastic gradient-based adaptive algorithms, such as the least-mean-square (LMS) one, are the most popular in adaptive filtering applications, due to its low computational complexity and very good stability characteristic. Moreover, in the LMS algorithm a previous knowledge of the process statistics is not required [1]. Such advantages make the LMS algorithm adequate for system identification, noise canceling, echo canceling, channel equalization, among other applications [2]. The standard LMS uses a fixed adaptation step size, determined by considering a tradeoff between convergence rate and misadjustment. A large step-size value leads to a fast convergence, providing the maximum value to guarantee algorithm stability is not violated, along with a large misadjustment. On the other side, a small step size provides a small misadjustment with a slow convergence rate. To overcome this tradeoff, variable step-size LMS (VSSLMS) algorithms have been proposed in the literature, improving the standard LMS performance for several applications. The basic idea behind these algorithms is to use a large step-size value at the beginning of the convergence process and reducing it as the steady state is achieved. The adjusting law for the step-size parameter in VSSLMS algorithms can be based on square error gradient [3]-[9], instantaneous square error [10]-[12], error autocorrelation function [13], absolute adaptation error [14], error vector normalization [15], absolute values of the weight vector coefficients [16]-[18] and other methods [19]. In [10], a VSSLMS algorithm based on the instantaneous square error is discussed. This one is termed Kwong's algorithm and presents a good performance in the most applications. However, the step-size adjusting and the misadjustment parameter are affected by the noise. A modification in Kwong's algorithm is proposed in [13], which gives rise to Aboulnasr's algorithm, improving the immunity of the VSSLMS algorithm for white noise. For such, Aboulnasr's algorithm uses a lag(1) error autocorrelation function. In this research work an improved VSSLMS algorithm based on the error autocorrelation is proposed. Here, the basic idea for adjusting the step-size parameter is to consider $\mathrm{lag}(1), \mathrm{lag}(2), \ldots, \mathrm{lag}(N)$ error autocorrelation functions, where N denotes the adaptive filter order. The corresponding increase of the computational burden is very small. Numerical simulations, considering a system identification problem, verify the performance of the proposed VSSLMS algorithm.

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2. VSSLMS ALGORITHMS

VSSLMS algorithms have been used extensively in adaptive filtering to improve the performance of the standard LMS one. Common aspects of several VSSLMS algorithms are presented in this section. To this end, let us consider a system identification scheme depicted in Fig. 1.

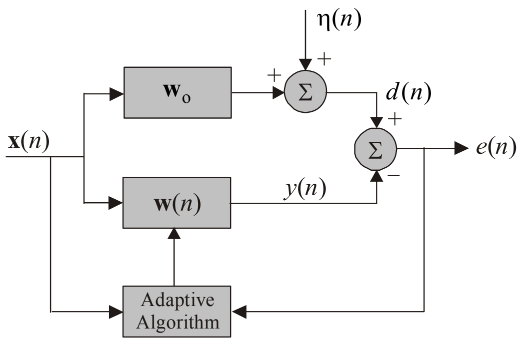

Figure 1 - Block diagram of a system identification.

The unknown system output is

$$d(n) = \mathbf{w}_o^T \mathbf{x}(n) + \eta(n) \quad (1)$$

where $\mathbf{x}(n) = [x(n)\, x(n-1) \cdots x(n-N+1)]^T$ with $x(n)$ being a zero-mean Gaussian process with variance σ_x^2, $\eta(n)$ is an i.i.d. noise process with variance σ_η^2. Vector $\mathbf{w}(n)$ is the *N*-order weight vector and $\mathbf{w}_o$ is the plant of the system to be identified. The error signal is given by

$$e(n) = d(n) - \mathbf{w}^T(n)\mathbf{x}(n). \quad (2)$$

The weight update expression of the VSSLMS algorithm is given by [2]

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu(n)e(n)\mathbf{x}(n) \quad (3)$$

where $\mu(n)$ is the variable step-size parameter.

To guarantee a stable operation in all VSSLMS algorithms, a sufficient condition for the step-size parameter is [1]

$$0 < \mu(n) < \frac{2}{3\mathrm{tr}[\mathbf{R}]} \quad (4)$$

where $\mathbf{R}$ is the input autocorrelation matrix.

3. KWONG'S ALGORITHM

This algorithm uses the instantaneous square error signal to update the step-size parameter, according to the following expression:

$$\mu(n+1) = \alpha\mu(n) + \gamma e^2(n) \quad (5)$$

where α and γ are positive control parameters. The main motivation for this algorithm is that a large prediction error will increase the step size, leading to a faster tracking; while a small prediction error will decrease the step size, also resulting in a smaller misadjustment [10].

In general, Kwong's algorithm is strongly dependent on the additive noise, reducing its performance for a low signal-to-noise ratio (SNR) environment.

4. ABOULNASR'S ALGORITHM

This algorithm includes a change in Kwong's Algorithm, by adjusting the step-size parameter considering the autocorrelation between $e(n)$ and $e(n-1)$, instead of the square error $e^2(n)$. In this way, the algorithm can effectively maintain a reasonable immunity to uncorrelated additive noise. To update the variable step size Aboulnasr's approach [13] considers the square of the error signal autocorrelation estimate obtained through a low-pass filter given by

$$p(n) = \beta p(n-1) + (1-\beta)e(n)e(n-1) \quad (6)$$

where β is a positive control parameter. The setting of the step-size parameter is

$$\mu(n+1) = \alpha\mu(n) + \gamma p^2(n) \quad (7)$$

where α and γ are positive control parameters.

5. PROPOSED ALGORITHM

For several adaptive filtering applications, the autocorrelation function between $e(n)$ and $e(n-1)$ is a poor index of convergence closeness. For correlated inputs and/or some particular kinds of impulse response of the unknown system, the autocorrelation between $e(n)$ and $e(n-2)$, $e(n)$ and $e(n-3)$ or other lags provides more information than simply using lag(1) error autocorrelation. In Aboulnasr's algorithm, lag(1) error autocorrelation function could reduce the step-size value too early in some situations, resulting in a slower convergence.

The proposed modification considers the lags from 1 to N in the error autocorrelation functions, improving the convergence speed and maintaining very good noise immunity. Then, let us consider $p(n)$ as a smooth estimation of the mean-square correlation functions between $e(n)$ and past errors $e(n-1), e(n-2), \ldots, e(n-N)$ given by

$$p(n) = \beta p(n-1) + (1-\beta)\sum_{i=1}^{N}[e(n)e(n-i)]^2 \quad (8)$$

The step-size update equation is given by

$$\mu(n+1) = \alpha\mu(n) + \gamma p(n) \quad (9)$$

where α and γ are positive parameters.

6. SIMULATION RESULTS

In this section, numerical simulations are presented comparing the performance of the proposed VSSLMS algorithm with Kwong's and Aboulnasr's ones. For such, a system identification problem is considered, using the scenario described in [13]. The simulations show the step-size behavior of the excess mean-square error (MSE), given by $E\{[e(n)-\eta(n)]^2\}$. The algorithm behavior is also assessed by considering an abrupt change in the unknown system. The used plant has 4 coefficients, given by vector $\mathbf{w_o} = [5 \;\; 0 \;\; 1 \;\; 8]^T$. The used input signals are both white and correlated zero-mean Gaussian data. The correlated input signal is obtained from an AR(1) process given by $x(n) = ax(n-1) + u(n)$, with $a = 0.9$, $\sigma_x^2 = 5.26$, and $u(n)$ is a white Gaussian noise with variance $\sigma_u^2 = 1$. The eigenvalue spread of the input signal autocorrelation matrix is $\chi = 57.4$ and the additive noise variance is also unity $(\sigma_\eta^2 = 1)$.

6.1. Example A

In this example, a $0\,\mathrm{dB}$ signal-to-noise ratio (SNR) white input signal is used. Numerical results obtained through Monte Carlo (MC) simulations (200 independent runs) comparing Kwong's and Aboulnasr's approaches with the proposed one are presented. For all algorithms, the maximum step size is limited to 0.1. Figures 2, 3, and 4 show the step size, $w_1(n)$ coefficient, and excess MSE behavior, respectively. In Figure 5, the excess MSE behavior considering an abrupt change in the unknown system (plant) parameters is depicted. The change of the plant is achieved by multiplying its coefficients by -1 at iteration 20000. From these figures, a better performance of the proposed VSSLMS algorithm is verified.

In this example, for all simulations, the parameters of the three algorithms are adjusted according to Table 1 for obtaining the same final excess MSE of -33 dB.

Table 1
Used parameter values for the numerical simulations with white input signal

Algorithms	Parameters
Kwong	$\alpha = 0.97$ $\gamma = 7\times10^{-6}$
Aboulnasr	$\alpha = 0.97$ $\beta = 0.99$ $\gamma = 8\times10^{-4}$
Proposed	$\alpha = 0.97$ $\beta = 0.99$ $\gamma = 2\times10^{-6}$

6.2. Example B

In this example, a 7.2 dB SNR correlated input signal is used. Again, a comparison between Kwong's and Aboulnasr's algorithms with the proposed VSSLMS one is performed. For all algorithms, the maximum step-size value is limited to 0.01. Figures 8, 9, and 10 show the step size, $w_1(n)$ coefficient, excess MSE behavior, respectively. In Figure 5, the excess MSE behavior considering an abrupt change (at iteration 20000) in the unknown system parameters is shown. From the numerical results, the proposed VSSLMS algorithm presents a better performance in comparison with the other VSSLMS approaches discussed here.

In this example, the used parameters are adjusted to obtain -24 dB of final excess MSE. Table 2 shows the parameters values used in this numerical example.

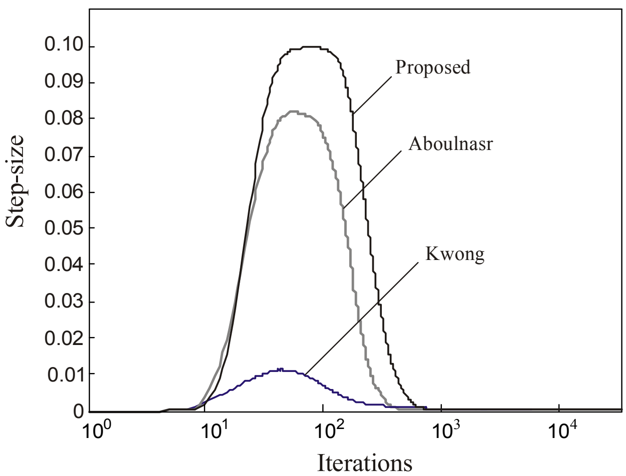

Figure 2 - Step-size evolution for white input signal.

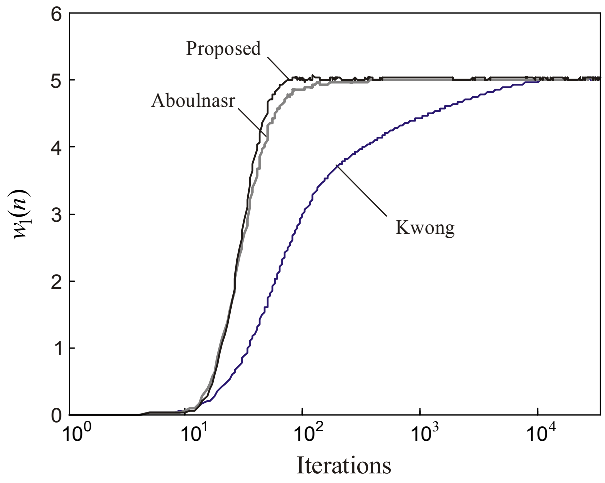

Figure 3 - Evolution of the $w_1(n)$ coefficient for white input signal.

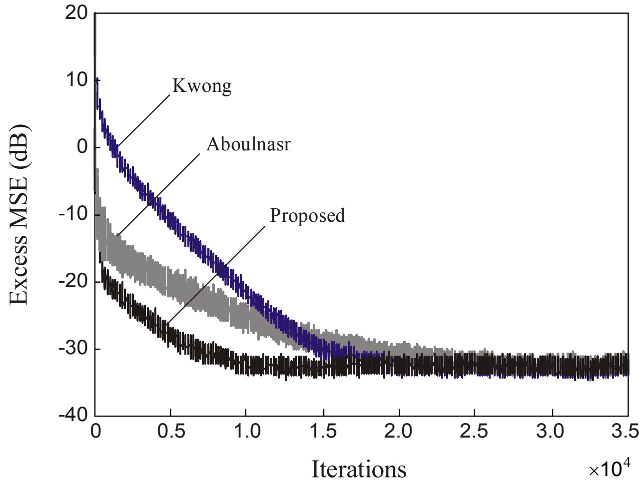

Figure 4 - Excess MSE behavior for white input signal.

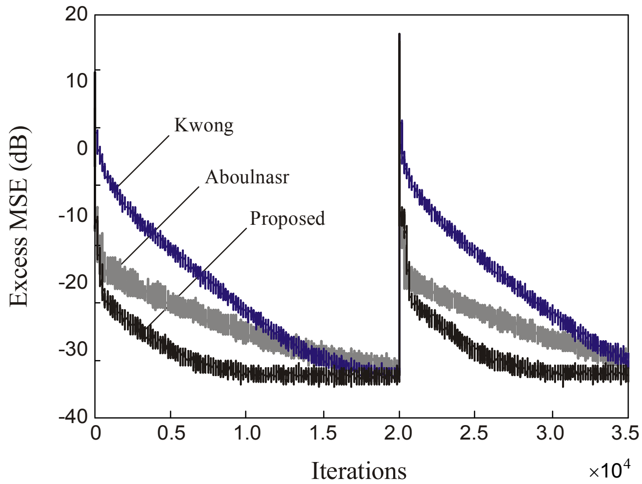

Figure 5 - Excess MSE behavior for white input signal with an abrupt change in the unknown system at iteration 20000.

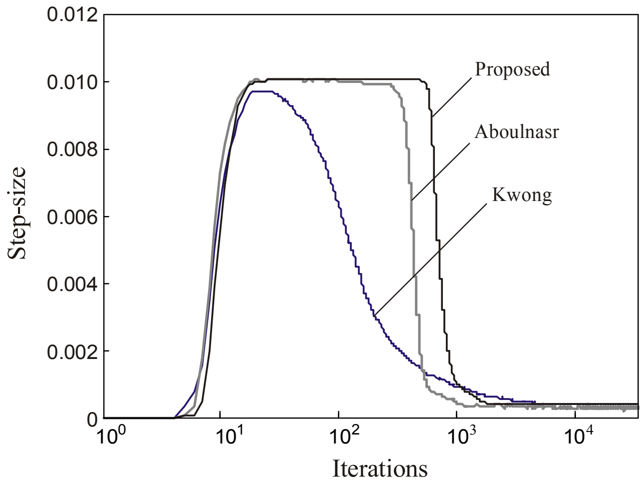

Figure 6 - Step-size evolution for correlated input signal.

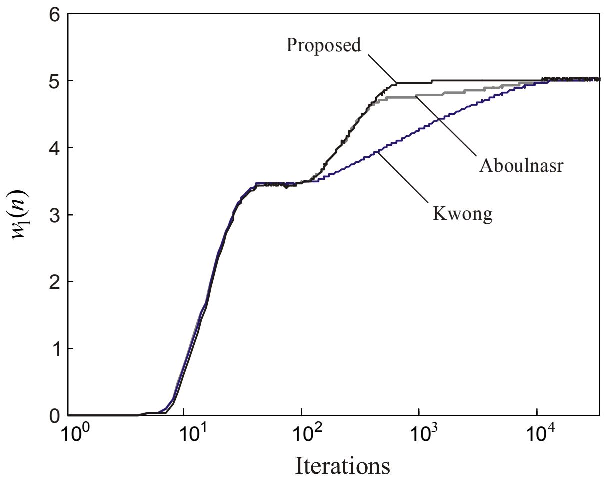

Figure 7 - Evolution of the $w_1(n)$ coefficient for correlated input signal.

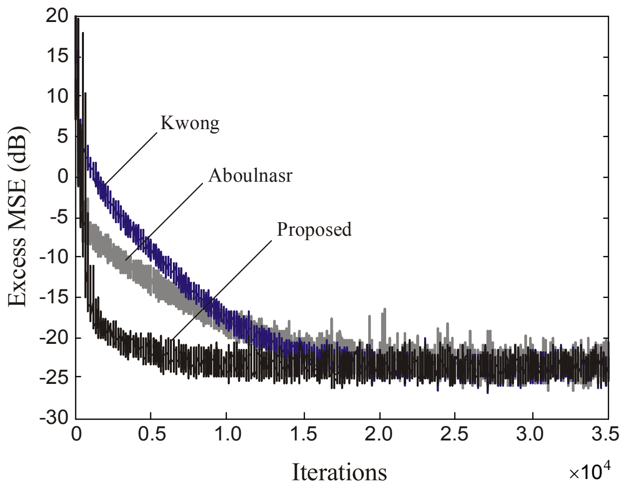

Figure 8 - Excess MSE behavior for correlated input signal.

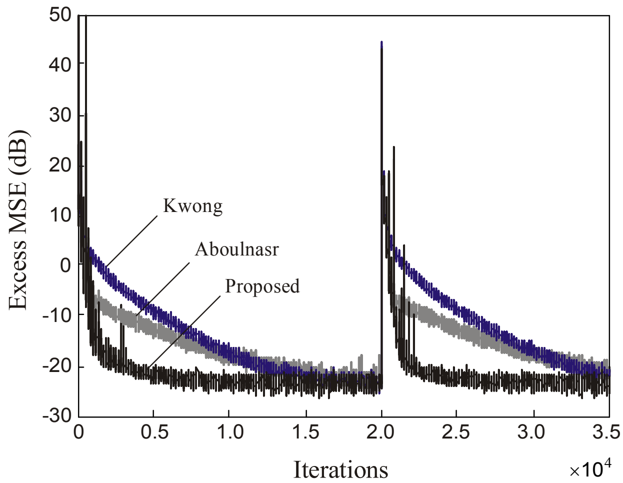

Figure 9 - Excess MSE behavior for correlated input signal with an abrupt change in the unknown system at iteration 20000.

Table 2
Used parameter values for the numerical simulations with correlated input signal

Algorithms	Parameters
Kwong	$\alpha = 0.97$ $\gamma = 1.1\times10^{-5}$
Aboulnasr	$\alpha = 0.97$ $\beta = 0.99$ $\gamma = 1.6\times10^{-3}$
Proposed	$\alpha = 0.97$ $\beta = 0.99$ $\gamma = 3\times10^{-6}$

7. CONCLUSIONS

In this work, a VSSLMS algorithm has been proposed based on the error autocorrelation function and considering several time lags. This fact improves the algorithm performance under noise presence (low SNR environments). From numerical simulations, the proposed algorithm is assessed for both white and colored input signals.

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