16th European Signal Processing Conference (EUSIPCO 2008), Lausanne, Switzerland, August 25-29, 2008, copyright by EURASIP

NOVEL CHARACTERISTICS OF EVEN-STACKED COSINE-MODULATED FILTER BANKS: NON-REDUNDANT DIRECTIONAL TRANSFORM AND SHIFT-INVARIANCE

Seisuke Kyochi and Masaaki Ikehara

Department of Electrical and Electronics Engineering, Keio University
3-14-1 Hiyoshi Kohoku-ku, 223-8522, Yokohama, Japan
phone: +81-45-566-1530, email: kyochi@tkhm.elec.keio.ac.jp

ABSTRACT

In this paper, we present non-redundant directional transform and shift-invariant transform based on even-stacked cosine-modulated filter banks (ECFBs). M-band dual-tree wavelet transform (MDTWT) recently proposed has rich directional selectivity and near shift-invariance. However, it has two problems. First, it is not critically-sampled transform. Second, it cannot achieve shift-invariance at all decomposition levels. For these problems, we show a solution based on ECFBs. Critically-sampled ECFBs provide non-redundant transform with rich directional selectivity. Furthermore, oversampled ECFBs with oversampling ratio of 2 can assure the shift-invariance at any decomposition levels. Moreover, ECFBs can be designed from only one prototype filter and guarantee linear phase. In this paper, we verify these facts by theoretical analysis and simulations.

1. INTRODUCTION

Recently, M-band dual-tree wavelet transform (MDTWT) has been paid much attention in signal processing [1][2]. It achieves near shift-invariance when $M = 2$ and rich directional selectivity for $2(M^2-1)$ directions. Due to these properties, it works successfully in several applications [1]. Although it is a powerful tool for practical signal processing, there still remain two problems. First, it cannot be designed with the critical sampling. For some applications, such as image coding, non-redundant transforms are more desired than redundant ones. Second, it cannot achieve shift-invariance at all decomposition levels. In addition, its shift-invariant property has not been discussed in the case of $M > 2$.

In this paper, we solve these problems by using even-stacked cosine-modulated filter banks (ECFBs) [3][4]. ECFBs were proposed as linear-phase CFBs initially. However, we show they have more attractive properties which have been unknown previously. Those are rich directional selectivity and shift-invariance. In fact, critically-sampled ECFBs act as a non-redundant directional transform like the MDTWT. Furthermore, oversampled ECFBs with oversampling ratio of 2 and an additional constraint on their prototype filter assure the shift-invariance at any decomposition level.

We organized the rest of this paper as follows. In section 2, we review about MDTWT and ECFB. In section 3, we explain the directional selectivity and shift-invariance of ECFB. In section 4, we check the good directional selectivity and evaluate shift-invariance of ECFBs by simulations. Finally, we conclude this paper in section 5.

Notations: $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{C}$ indicate integer, real number, and complex number. $j := \sqrt{-1}$, $W_M^k := e^{j\frac{2\pi}{M}k}$. $\overline{\alpha}$ is the conjugate of $\alpha \in \mathbb{C}$. $\hat{\psi}$ is the fourier transform of function ψ. $H(z)$, $H^*(z)$, and $\tilde{H}(z)$ are defined as $H(z) := \sum_n h(n)z^{-n}$, $H^*(z) := \sum_n \overline{h(n)}z^{-n}$, and $\tilde{H}(z) := H^*(z^{-1})$.

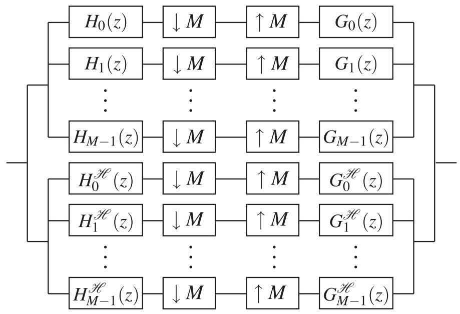


Figure 1: The structure of the M-band DTWT

2. REVIEW

2.1 M-band Dual-Tree Wavelet Transform

In this section, we briefly review MDTWT. Fig. 1 illustrates the system of MDTWT. It consists of two critically-sampled M-channel FBs. In this paper, we call the upper FB "primal FB" and the lower one "dual FB". Let ψ_m and $\psi_m^{\mathscr{H}}$ $(m = 0, \ldots, M-1)$ be the scaling $(m = 0)$ and wavelet $(m = 1, \ldots, M-1)$ functions corresponding to the lowpass and highpass filters in the primal and dual FBs, respectively. MDTWT is constructed such that each pair of scaling and wavelet functions $\{\psi_m, \psi_m^{\mathscr{H}}\}$ forms Hilbert transform pair formulated as follows [1]:

$$\hat{\psi}^{\mathscr{H}}(\omega) = -j\,\mathrm{sgn}(\omega)\hat{\psi}(\omega), \tag{1}$$

$$\mathrm{sgn}(\omega) = \begin{cases} 1 & \omega > 0 \\ 0 & \omega = 0 \\ -1 & \omega < 0 \end{cases}. \tag{2}$$

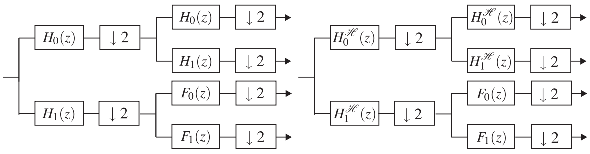
Note that if ψ and $\psi^{\mathscr{H}}$ form Hilbert transform pair, $\psi_A := \psi + j\psi^{\mathscr{H}}$ becomes an analytic function, i.e. $\hat{\psi}_m^A(\omega) \equiv 0,\ \omega < 0$. In such case, the condition for $\{H_m(z), H_m^{\mathscr{H}}(z)\}_{0 \le m \le M-1}$ is represented as follows [2]:

$$\begin{aligned} H_m^{\mathscr{H}}(e^{j\omega}) &= e^{-j\theta_m(\omega)}H_m(e^{j\omega}), \\ \theta_0(\omega) &= \left(d + \frac{1}{2}\right)(M-1)\omega - p\pi, \end{aligned} \tag{3}$$

where $d \in \mathbb{Z}$ denotes the delay between the primal and dual FBs, $\forall p \in \{0, \ldots, \lceil \frac{M}{2} \rceil - 1\}, \forall \omega \in \left[\frac{2\pi}{M}p, \frac{2\pi}{M}(p+1)\right)$, and

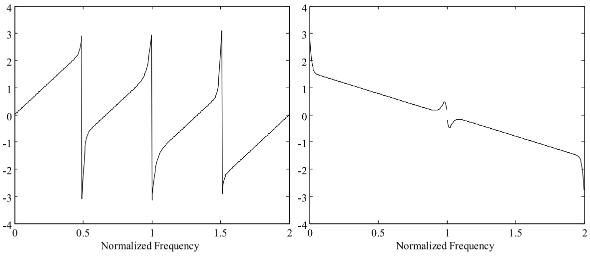
$$\theta_m(\omega) = \begin{cases} \frac{\pi}{2} - \left(d + \frac{1}{2}\right)\omega & \omega \in (0, 2\pi) \\ 0 & \omega = 0 \end{cases}, \tag{4}$$

in which $m \in \{1, \ldots, M-1\}$.



(a) Primal FB (b) Dual FB

Figure 2: The structure of 4-band DTWT



(a) $k=0$ (b) $k=1$

Figure 3: $\theta_k(\omega)$ of the designed 4DTWT

Unfortunately, the conventional MDTWT has two problems. First, it is the overcomplete transform whose oversampling ratio is 2. In other words, it outputs twice the number of samples as the input signal. Some applications, e.g. image coding, require non-redundant transforms rather than redundant ones. Second, the Hilbert transform pair condition cannot guarantee shift-invariance at all decomposition level.

Let us explain about the second problem by simulation. We consider a 4DTWT. It is realized based on the method introduced by Selesnick in [5]. It can be obtained by cascading a 2DTWT and a 2-channel paraunitary FB $\{F_0(z), F_1(z)\}$ [6], as shown in Fig. 2. 2DTWT $\{H_m(z), H_m^{\mathscr{H}}(z)\}_{m=0,1}$ is designed by the method based on Thiran filter [1] and $\{F_0(z), F_1(z)\}$ set as $F_0(z) := H_0(z)$, $F_1(z) := H_1(z)$. The filter length of the designed 4DTWT is 94. Fig. 3 (a) and (b) indicate the functions $\theta_k(\omega)$ $(k=0,1)$ in (3) and (4) of the designed 4DTWT (the case of $k=2,3$ is omitted). It can be observed that the Hilbert pair condition can be satisfied approximately.

For an original step function (2048 samples), we apply the 4DTWT to the original and its 4-sample shifted version, and obtain only the third subband signals both of primal and dual FB, indicated in Fig. 4. Fig. 5 shows two output signals. These signals have not the same form. Therefore, the Hilbert pair condition can not always assure shift-invariance.

2.2 Even-Stacked Cosine-Modulated Filter Banks

Lin *et al.* and Bölcskei *et al.* have proposed critically-sampled and oversampled ECFBs [3][4]. They are well known for their small implementation cost. All bandpass filters can be designed by the modulation of one prototype filter. As shown in Fig. 6, a $2M$-channel ECFB with the downsampling of $2D$ consists of two subsystem, where $H_0(z),\ldots,H_M(z)$ and $G_0(z),\ldots,G_M(z)$ are the analysis and the synthesis filters of the first system respectively, and $H'_1(z),\ldots,H'_{M-1}(z)$ and $G'_1(z),\ldots,G'_{M-1}(z)$ are those of the second one. Fig. 7 depicts frequency responses of ECFBs. In this paper, we consider the paraunitary case and the definition of filters introduced in [3] for the simple discussion in the following sections.

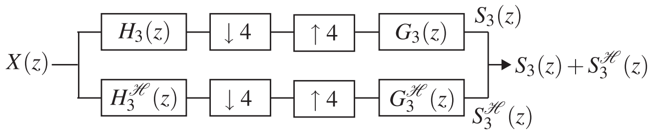
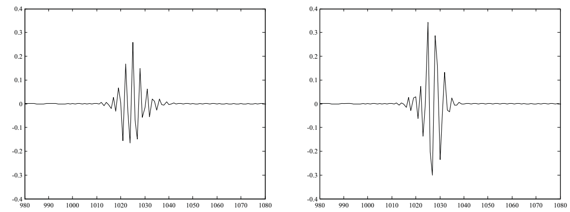


Figure 4: Configuration: the third subband signal retained from both of primal and dual FB.



(a) (b)

Figure 5: Output signals; (a) is obtained from the original signal, and (b) is from 4-sample shifted version of the original one.

These filters are as follows [3]:

$$
\begin{cases}
H_0(z) := 2U_0(z) \\
H_k(z) := \sqrt{2}\,(U_k(z)+U_{-k}(z)) \\
H_M(z) := 2U_M(z) \\
H'_k(z) := -\sqrt{2}jz^{-M}\,(U_k(z)-U_{-k}(z))
\end{cases}
$$

$$
\begin{cases}
G_k(z) := z^{-(N+M)}\tilde{H}_k(z) \\
G'_k(z) := z^{-(N+M)}\tilde{H}'_k(z)
\end{cases}
\tag{5}
$$

where $1 \le k \le M-1$, $U_k(z) := P(zW_{2M}^k)$, and $P(z)$ is a prototype filter. If a prototype filter with its order $N=(2m_0+1)M$ is symmetric, all analysis and synthesis filters have linear phase.

3. DIRECTIONAL SELECTIVITY AND SHIFT-INVARIANCE OF EVEN-STACKED COSINE-MODULATED FILTER BANKS

3.1 Relationship of ECFBs with MDTWT

In order to generate an analytic function structurally, primal and dual FBs of a MDTWT are imposed the Hilbert transform pair condition. On the other hand, although all the pairs $\{H_m(z), H'_m(z)\}_{1\le m\le M-1}$ of ECFBs which are defined as in Sec. 2.2 cannot satisfy the Hilbert transform pair condition, ECFBs act like the MDTWT. Specifically, ECFBs also generate analytic functions. Let ψ_m and ψ'_m $(m=1,\ldots,M-1)$ be the wavelet functions corresponding to the subband filters in ECFBs. In terms of $\{H_m(z), H'_m(z)\}_{1\le m\le M-1}$, $\{\psi_m, \psi'_m\}$ can be expressed by the following equations:

$$
\begin{aligned}
\hat{\psi}_m(\omega) &= \frac{1}{\sqrt{2M}} H_m\left(e^{j\frac{\omega}{2M}}\right)\prod_{k=2}^{\infty} H_0\left(e^{j\frac{\omega}{(2M)^k}}\right), \\
\hat{\psi}'_m(\omega) &= \frac{1}{\sqrt{2M}} H'_m\left(e^{j\frac{\omega}{2M}}\right)\prod_{k=2}^{\infty} H_0\left(e^{j\frac{\omega}{(2M)^k}}\right).
\end{aligned}
\tag{6}
$$

Define $\psi_m^{\mathrm{Re}} := \psi_m$, $\psi_m^{\mathrm{Im}} := e^{jM\omega}\psi'_m$, and $\psi_m^A := \psi_m^{\mathrm{Re}} + j\psi_m^{\mathrm{Im}}$, then $\hat{\psi}_m^A(\omega)$ can be expressed as follows:

$$
\begin{aligned}
\psi_m^A(\omega) &= \hat{\psi}_m(\omega) + je^{jM\omega}\hat{\psi}'_m(\omega) \\
&= \frac{2}{\sqrt{M}} U_m\left(e^{j\frac{\omega}{2M}}\right)\prod_{k=2}^{\infty} H_0\left(e^{j\frac{\omega}{(2M)^k}}\right).
\end{aligned}
\tag{7}
$$

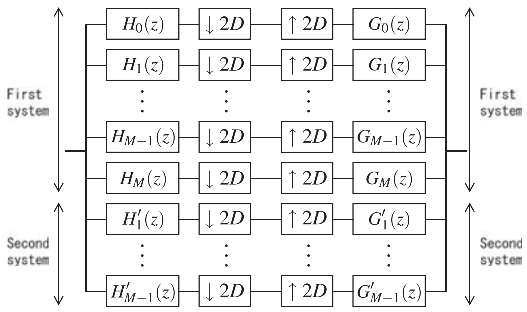


Figure 6: The structure of even-stack CMFB

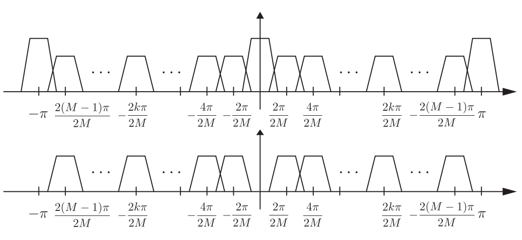
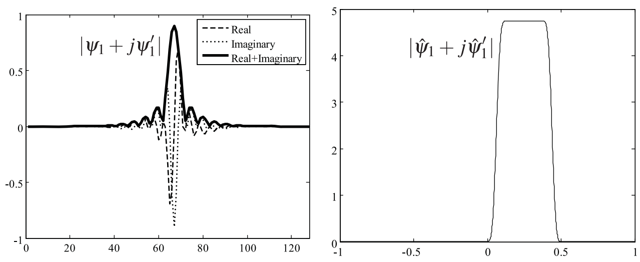


Figure 7: Distribution of frequency responses (even-stack CMFB)



(a) Time domain (b) Frequency domain

Figure 8: Wavelet functions corresponding to the $\{H_1(z), H'_1(z)\}$.

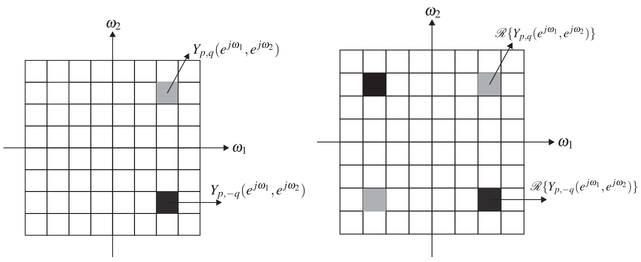


Figure 9: Directional selectivity in the 2D frequency plane

$U_m(z)$ has one-sided spectrum in positive frequency domain. Let us assume that $P(z)$ is a prototype filter whose frequency support $\mathrm{supp}\left[P(e^{j\omega})\right]$ satisfies $\mathrm{supp}\left[P(e^{j\omega})\right] \subset \left[-\frac{\pi}{M}, \frac{\pi}{M}\right]$. From (5), the support of $U_m\left(e^{j\frac{\omega}{2M}}\right)$ $(1 \le m \le M-1)$ and $\prod_{k=2}^{\infty} H_0\left(e^{j\frac{\omega}{(2M)^k}}\right)$ are as follows:

$$\mathrm{supp}\left[U_m\left(e^{j\frac{\omega}{2M}}\right)\right] \subset [2\pi(m-1), 2\pi(m+1)]$$
$$\mathrm{supp}\left[\prod_{k=2}^{\infty} H_0\left(e^{j\frac{\omega}{(2M)^k}}\right)\right] \subset [-4M\pi, 4M\pi]. \tag{8}$$

Therefore $\mathrm{supp}[\hat{\psi}(\omega)] \subset [2\pi(m-1), 2\pi(m+1)]$. In other words, if the frequency support of the prototype filter $P(z)$ is restricted into $\left[-\frac{\pi}{M}, \frac{\pi}{M}\right]$, ψ_m^A defined in (7) becomes an analytic function and $\psi_{m,\ell,k}^A := \psi_{m,\ell,k} + j\psi'_{m,\ell,k}$ $(0 \le m \le M-1,\ \ell \in \mathbb{Z},\ k \in \mathbb{Z})$ forms an analytic wavelet function. As a consequence, ECFBs can be regarded as alternative MDTWTs. Fig. 8(a) shows the example of wavelet function ψ_1, ψ'_1, and $|\psi_1 + j\psi'_1|$ obtained from $\{H_1(z), H'_1(z)\}$ and Fig. 8(b) shows the magnitude of the Fourier transform of $\psi_1 + j\psi'_1$. As shown in Fig. 8(b), the spectrum in only positive frequency domain is left.

On the other hand, in the case of $k = 0$ and M, $H_k(z)$ does not have the dual filter, as in the conventional MDTWT. However ECFBs act as MDTWT in many applications, such as image processing. Most of the energy of natural images is concentrated on the lowest frequency which does not usually have the directionality. Moreover, the energy of the highest frequency is quite small. Therefore, it is not necessary that lowpass filter $H_0(z)$ and highpass filter $H_M(z)$ possess their dual filters.

3.2 Directional Selectivity

As well as the MDTWT, ECFBs have rich directional selectivity. Here, we consider $2M$-channel ECFBs. For $k \in \{1, \ldots, M-1\}$, define $H_k(e^{j\omega}), H'_k(e^{j\omega})$ as in (5). Then, $U_k(e^{j\omega})$ and $U_{-k}(e^{j\omega})$ can be represented as follows:

$$U_{\pm k}(e^{j\omega}) = \frac{1}{2\sqrt{2}}\left(H_k(e^{j\omega}) \pm jz^M H'_k(e^{j\omega})\right). \tag{9}$$

Since $U_k(e^{j\omega})$ and $U_{-k}(e^{j\omega})$ have one-sided spectrum in positive and negative frequency domain, they can extract only the positive and negative frequency information.

Let $Y_{\pm p,\pm q}(e^{j\omega_1}, e^{j\omega_2})$ be $(\pm p, \pm q)$-subband signal of input signal $X(e^{j\omega_1}, e^{j\omega_2})$ and $U_{\pm p,\pm q}(e^{j\omega_1}, e^{j\omega_2}) = U_{\pm p}(e^{j\omega_1})U_{\pm q}(e^{j\omega_2})$ be the 2-D filter whose passband corresponds to $(\pm p, \pm q)$-frequency domain, where $p, q = 1, \ldots, M+1$. $Y_{p,q}(e^{j\omega_1}, e^{j\omega_2})$ can be represented as follows:

$$\begin{aligned}
&Y_{p,q}(e^{j\omega_1}, e^{j\omega_2}) \\
=&U_p(e^{j\omega_1})U_q(e^{j\omega_2})X(e^{j\omega_1}, e^{j\omega_2}) \\
=&(H_p(e^{j\omega_1})H_q(e^{j\omega_2}) - H_{-p}(e^{j\omega_1})H_{-q}(e^{j\omega_2}))X(e^{j\omega_1}, e^{j\omega_2}) \\
&-j(H_p(e^{j\omega_1})H_q(e^{j\omega_2}) + H_{-p}(e^{j\omega_1})H_{-q}(e^{j\omega_2}))X(e^{j\omega_1}, e^{j\omega_2})
\end{aligned} \tag{10}$$

The spectrum $Y_{p,q}(e^{j\omega_1}, e^{j\omega_2})$ is supported in only one quadrant of the 2-D frequency plane. If we take the real part of this spectrum, then the spectrum is supported in two quadrants of the 2-D frequency plane, as illustrated in Fig. 9.

Since $H_0(z)$ and $H_M(z)$ in ECFBs do not possess dual filters, $2M$-channel ECFBs can extract 0°, 90° and other $2(M-1)^2$ directions.

3.3 Non-Redundant Directional Transform Arising from Critically-Sampled ECFBs

ECFBs can be designed with critical sampling [3]. Therefore non-redundant directional transform can be realized. A prototype filter with its order $N = (2m_0+1)M$ and the support of its frequency response $\mathrm{supp}[P(e^{j\omega})] \subset [-\frac{\pi}{M}, \frac{\pi}{M}]$ leads to pseudo QMF. In addition to this constraint, if the prototype filter satisfy the following equations, an ECFB becomes a perfect reconstruction FB [3];

$$\begin{aligned}
&\tilde{G}_k(z)G_k(z) = 1 \quad (k = 0, M) \\
&\tilde{G}_k(z)G_k(z) + \tilde{G}_{k+M}(z)G_{k+M}(z) = 2 \\
&(k = 1, \ldots, M-1),
\end{aligned} \tag{11}$$

where $G_k(z)$ is a polyphase component of $P(z)$, i.e., $G_k(z) := \sum_n p(2Mn+k)z^{-n}$.

3.4 Shift-Invariant Transform Arising from Oversampled ECFBs

In this section, we derive shift-invariance transform based on oversampled ECFBs. Let $X(z)$ and $Y(z)$ be the z-transforms of input and output signal, respectively. In z-domain, the shift-invariance can be formulated as follows;

$$X(z) \to z^{-r}X(z) \Rightarrow Y(z) \to z^{-r}Y(z), \tag{12}$$

where $r \in \mathbb{Z}$ denotes the number of shift.

Concerning the shift-invariance on oversampled ECFBs, the following theorem gives an important result.

Theorem 1. *If an ECFB with the oversampling ratio of 2 and the frequency response of its prototype filter $P\left(e^{j\omega}\right)$ restricted into $[-\frac{\pi}{M}, \frac{\pi}{M}]$ can always achieve the shift-invariance at any decomposition level.*

Before deriving the theorem, we remark an additional notation of the above statement. Let $H_k^{(J)}(z)$, $H_k^{\prime(J)}(z)$, $G_k^{(J)}(z)$ and $G_k^{\prime(J)}(z)$ be the transfer functions of subband filters after the J-level decomposition represented as follows [6]:

$$\begin{aligned}
H_k^{(J)}(z) =& H_0(z)H_0(z^M)\ldots H_0(z^{M^{J-2}})H_k(z^{M^{J-1}}) \\
H_k^{\prime(J)}(z) =& H_0^{(J-1)}(z)H_k^{\prime}(z^{M^{J-1}}) \\
G_k^{(J)}(z) =& G_0(z)G_0(z^M)\ldots G_0(z^{M^{J-2}})G_k(z^{M^{J-1}}) \\
G_k^{\prime(J)}(z) =& G_0^{(J-1)}(z)G_k^{\prime}(z^{M^{J-1}}).
\end{aligned} \tag{13}$$

Let $S_k^{(J)}$ and $S_k^{\prime(J)}$ be the k-th subband signals of the first and the second system, shown as follows:

$$\begin{aligned}
S_k^{(J)}(z) =& \frac{1}{M^J}\sum_{\ell=0}^{M^J-1} H_k^{(J)}(zW_{M^J}^{\ell})G_k^{(J)}(z)X_k^{(J)}(zW_{M^J}^{\ell}) \\
S_k^{\prime(J)}(z) =& \frac{1}{M^J}\sum_{\ell=0}^{M^J-1} H_k^{\prime(J)}(zW_{M^J}^{\ell})G_k^{\prime(J)}(z)X_k^{(J)}(zW_{M^J}^{\ell}),
\end{aligned} \tag{14}$$

As illustrated in Fig. 10, "shift-invariance" in the theorem means that $S_k^{(J)}(z)$ is shift-invariant in the case of $k = 0, M$ and so is $S_k^{(J)}(z) + S_k^{\prime(J)}(z)$ in the case of $k = 1, \ldots, M-1$.

Proof. First, we consider the case of $k = 1, \ldots, M-1$. In this case the shift-invariance can be formulated as the following equations:

$$\begin{aligned}
X(z) \to& z^{-r}X(z) \\
\Rightarrow S_k^{(J)}(z) + S_k^{\prime(J)}(z) \to& z^{-r}(S_k^{(J)}(z) + S_k^{\prime(J)}(z)),
\end{aligned} \tag{15}$$

where $\forall r \in \mathbb{Z}$. Since $W_{M^J}^{\ell} \neq 1$ for some ℓ, $S_k^{(J)}(z) + S_k^{\prime(J)}(z)$ is shift-invariant if and only if the following equation is true:

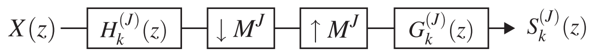
$$H_k^{(J)}(zW_{M^J}^{\ell})G_k^{(J)}(z) + H_k^{\prime(J)}(zW_{M^J}^{\ell})G_k^{\prime(J)}(z) \approx 0 \tag{16}$$

Let $U_k^{(J)}(z) := H_0^{(J-1)}(z)U_k(z^{M^{J-1}})$, then (13) can be rewritten as follows:

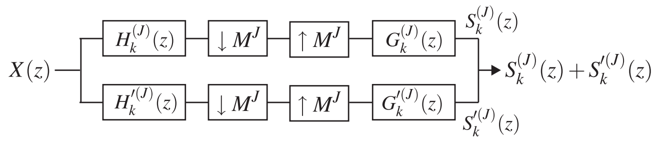
$$\begin{aligned}
H_k^{(J)}(z) =& U_k^{(J)}(z) + U_{-k}^{(J)}(z), \\
H_k^{\prime(J)}(z) =& j(U_k^{(J)}(z) - U_{-k}^{(J)}(z)).
\end{aligned} \tag{17}$$

Substituting (17) into (16), necessary and sufficient condition for shift-invariance can be rewritten as follows:

$$U_k^{(J)}(zW_{M^J}^{\ell})\tilde{U}_k^{(J)}(z) + U_{-k}^{\prime(J)}(zW_{M^J}^{\ell})\tilde{U}_{-k}^{\prime(J)}(z) \approx 0. \tag{18}$$



(a) The case of $k = 0, M$



(b) The case of $k = 1, \cdots, M-1$

Figure 10: Configuration: the k-th subband signal retained from just J-level.

Since the assumption on the frequency support of the prototype filter, i.e. supp $\left[P(e^{j\omega})\right] \subset [-\frac{\pi}{M}, \frac{\pi}{M}]$, the supports of $U_k^{(J)}(zW_{M^J}^{\ell})$ and $\tilde{U}_k^{(J)}(z)$ are,

$$\begin{aligned}
&\text{supp}\left[U_k^{(J)}\left(e^{j\omega}W_{M^J}^{\ell}\right)\right] \subset \left[\frac{\pi}{M^J}(k-1+2\ell), \frac{\pi}{M^J}(k+1+2\ell)\right] \\
&\text{supp}\left[\tilde{U}_k^{(J)}\left(e^{j\omega}\right)\right] \subset \left[\frac{\pi}{M^J}(k-1), \frac{\pi}{M^J}(k+1)\right].
\end{aligned} \tag{19}$$

From these equations, the first term of the left-hand side "$U_k^{(J)}(zW_{M^J}^{\ell})\tilde{U}_k^{(J)}(z)$" in (18) vanishes for $1 \le k \le M^J - 1$. Similarly, it can be verified that the second term of the left-hand side "$U_{-k}^{(J)}(zW_{M^j}^{\ell})\tilde{U}_{-k}^{(J)}(z)$" in (18) vanishes for $1 \le \ell \le M^J - 1$. Therefore, (16) is obtained.

For the case of $k = 0, M$, since $H_0(z)$ and $H_M(z)$ do not have the dual filters, (16) becomes as follows:

$$H_k^{(J)}(zW_{M^J}^{\ell})G_k^{(J)}(z) \approx 0. \tag{20}$$

In similar discussion, it also can be verified that $H_k^{(J)}(zW_{M^J}^{\ell})G_k^{J}(z)$ vanishes for $1 \le \ell \le M^J - 1$. Consequently, ECFBs with oversampling ratio of 2 have shift invariance in all the subbands. This is the end of the proof. □

4. SIMULATION

In this section, we validate that ECFBs have rich directional selectivity and shift-invariance by simulations.

4.1 Design

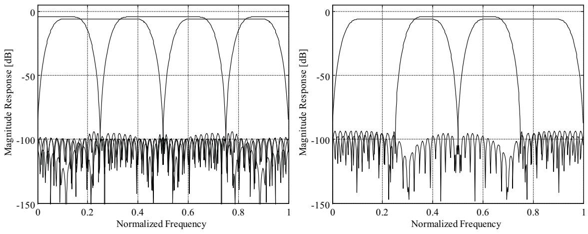
We design a critically-sampled ECFB as a pseudo QMF. For the design of prototype filter $p(n)$, we minimized the following stopband energy Φ.

$$\Phi = \int_{\frac{\pi}{M}}^{\pi} |P(e^{j\omega})| d\omega \tag{21}$$

For that purpose, we used the "*remez*" function in MATLAB. Fig. 11 shows the design example of the frequency responses of the ECFB (the number of channel: 8, filter length: 93).

4.2 Directional Selectivity

The 2-D wavelet basis functions obtained in a 3-level FB structure are shown in Fig. 12. In this case, the designed ECFB can extract 20 directions (horizontal, vertical and the other angles).



(a) First system (b) Second system

Figure 11: Frequency responses of the designed ECFB

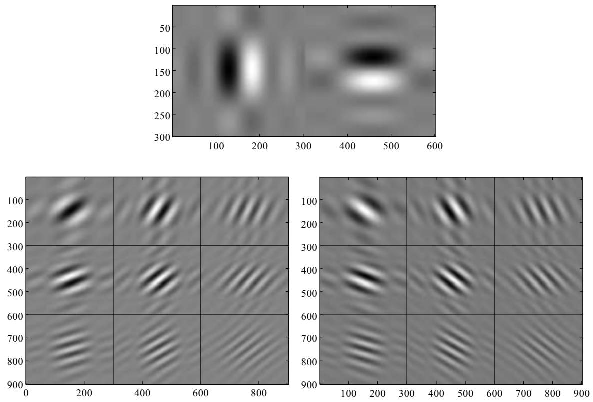


Figure 12: Directional selectivity of the designed ECFB (channel:8)

4.3 Evaluation of Shift-Invariance

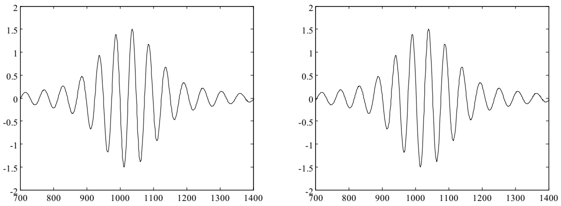
In this section, we evaluate the shift-invariance performance of oversampled ECFBs. We use the critically-sampled ECFB designed in the previous subsection as a oversampled ECFB by replacing downsampling factor 8 to 4. As a comparison, 4DTWT based on the method [5] shown in Sec. 2.1 is used. The step function (2048 samples) illustrated in Fig. 4 is used as an input signal. We apply the oversampled ECFBs and 4DTWT to the input signal and its shifted versions, up to 3 level. Let $s_{k,r}^{(J)}$ and $s_{k,r}'^{(J)}$ be the J-level k-th subband signals of the first and the second system which are obtained from the r-sample shifted version of original signal (the original one is denoted by $r = 0$). Then, we evaluate the shift-invariance by the correlation of the output signals as follows:

$$\Phi_C(J,k,r) = \begin{cases} \left| \left\langle \frac{s_{k,0}^{(J)}(\cdot - r)}{\|s_{k,r}^{(J)}(\cdot - r)\|}, \frac{s_{k,r}^{(J)}}{\|s_{k,r}^{(J)}\|} \right\rangle \right| \quad (k = 0, M) \\ \left| \left\langle \frac{(s_{k,0}^{(J)}(\cdot - r) + s_{k,0}'^{(J)}(\cdot - r))}{\|(s_{k,0}^{(J)}(\cdot - r) + s_{k,0}'^{(J)}(\cdot - r))\|}, \frac{(s_{k,r}^{(J)} + s_{k,r}'^{(J)})}{\|(s_{k,r}^{(J)} + s_{k,r}'^{(J)})\|} \right\rangle \right| \\ (k = 1, \ldots, M-1) \end{cases} \tag{22}$$

where $\langle f, g \rangle := \int f(x)\overline{g(x)}dx$ and $\|f\| := \sqrt{\langle f, f \rangle}$. Fig. 13(a) and (b) shows the output signals of $s_{3,0}^{(3)} + s_{3,0}'^{(3)}$ and $s_{3,4}^{(3)} + s_{3,4}'^{(3)}$. They are almost the same form (in contrast to the MDTWT based on conventional design method shown in Fig. 5). Table. 1 shows the averaged correlation values $\frac{1}{64}\sum_{r=1}^{64} \Phi_C(J,k,r)$. From Table. 1, it can be seen that ECFBs satisfying the theorem always can achieve the shift-variance.

5. CONCLUSION

In this paper, we presented the non-redundant directional transform and the shift-invariant transform based on ECFBs. Our contribution is summarized as three points. First, ECFBs have rich directional selectivity. Second, their critically-sampled version is regarded as a non-redundant directional transform. Third, their oversampled version is a shift-invariant transform. Oversampled ECFBs with the oversampling ratio of 2 and the additional constraint on prototype filter can always guarantee the shift-invariance. In addition to the theoretical discussion, we verified these facts by the experimental results. Therefore the problems on the conventional MDTWT can be solve by using ECFBs.



(a) (b)

Figure 13: Output signals ((a) and (b))

Table 1: Result (Correlation)

	Conventional 4DTWT (filter length: 94)				
Channel	$k=0$	$k=1$	$k=2$	$k=3$	
Level: 1	0.9999	0.9143	0.9091	0.7759	
Level: 2	0.9999	0.9847	0.9087	0.9580	
Level: 3	0.9999	0.9922	0.9471	0.9710	
	8-Channel ECFB (filter length :93)				
Channel	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$
Level: 1	0.9999	0.9999	0.9999	0.9999	0.9999
Level: 2	0.9999	0.9999	0.9999	0.9999	0.9999
Level: 3	0.9999	0.9999	0.9999	0.9999	0.9999

6. ACKNOWLEDGMENT

This work is supported in part by a Grant-in-Aid for the Global Center of Excellence for high-Level Global Cooperation for Leading-Edge Platform on Access Spaces from the Ministry of Education, Culture, Sport, Science, and Technology in Japan.

REFERENCES

[1] I. W. Selesnick, R. G. Baraniuk and N. C. Kingsbury, "The dual-tree complex wavelet transform," *IEEE Signal Process. Magazine*, vol. 22 pp. 123–151, Nov. 2005.

[2] C. Chaux, L. Duval and J.-C. Pesquet, "Image analysis using a dual-tree *M*-band wavelet transform," *IEEE Trans. Signal Process.*, vol. 15, no. 8, pp. 2397–2412, Aug. 2006.

[3] Y.-P. Lin and P. P. Vaidyanathan, "Linear phase cosine modulated maximally decimated filter banks with perfect reconstruction", *IEEE Trans. Signal Process.*, vol. 42, no. 11, pp. 2525–2539, Nov. 1995.

[4] H. Bölcskei and F. Hlawatsch, "Oversampled cosine modulated filter banks with perfect reconstruction", *IEEE Trans. Circuits and Syst. II*, vol. 45, no. 8, Aug. 1998.

[5] I. Bayram and I. W. Selesnick, "A simple construction for the *M*-band dual-tree complex wavelet transform" *Digital Signal Processing Workshop, 12th - Signal Processing Education Workshop*, 4th, pp.596-601, Sept. 2006

[6] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[7] N. G. Kingsbury, "Complex wavelets for shift invariant analysis and filtering of signals" *Appl. and Comput. Harmon. Anal.*, vol. 10, no. 3, pp. 234–253, May 2001.