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AN EQUATION ERROR ALGORITHM BASED ON CORRELATION FUNCTION WITH VARIABLE DELAY

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ABSTRACT

Equation error adaptive algorithm (EQ-E) for an adaptive infinite impulse response (IIR) digital filter is one of the IIR adaptive algorithms that exhibit global convergence. In the presence of a disturbance signal, the coefficients of the equation error based adaptive digital filter (ADF) converge to solutions with bias. In this paper, we propose a kind of block adaptive algorithm for the EQ-E IIR ADF. By estimating the correlation functions, the effect of the disturbance signal onto the convergence of the algorithm is reduced.

key words— Equation error, Correlation, IIR ADF, Variable delay

1. INTRODUCTION

Finite impulse response (FIR) ADF has been widely used due to its simplicity and stable convergence characteristics in adaptive signal processing. However, a FIR ADF has a disadvantage in systems where good performance can be attained supposing the degree of a filter is very large. Such a thing is possible at acoustic echo cancellation applications. Consequently, there has been a lot of interest in an IIR ADF, which can achieve the same level of performance as the FIR type, but at the advantage of using only a fewer number of coefficients.

Several approaches have made in order to estimate an unknown system by an IIR ADF [1][2]. In the case of using the output error, it has been shown that there is a possibility of convergence to a local minimum, with no guarantee on system stability [2]. Based on the problems associated with the output error mode of identification, the equation error formulation of an IIR ADF has been actively researched on [3][4]. In its simpler form, where the mean square of the equation error is directly minimized using a gradient based algorithm, estimated parameters contain a bias if there is a disturbance signal [1][2].

Other works have made in order to solve this problem by using correlation function in an adaptive FIR filter [5]. However, this method is needed convolution operations according to the tap order and there is a problem to increase an operand.

In this paper, an equation error algorithm based on correlation function is proposed. We take advantage of the fact that the disturbance signal is independent from the input signal.

This paper is organized as follows. In section 2, an equation error type of IIR ADF is described. In section 3, the proposed algorithm based on estimation of correlation function is described. In section 4, we explain the convergence the proposed algorithm. In section 5, we show some of the simulation results of the proposed algorithm. Finally, we conclude our paper in section 6.

2. EQUATION ERROR IIR ADF

Figure 1 shows an equation error IIR ADF. An unknown system H(z) is assumed to be modeled by an IIR filter, which is defined by

$$H(z) = \frac{B(z)}{A(z)} \tag{1}$$

where $B(z) = b_0 + b_1 z^{-1} + ... + b_M z^{-M}$
$A(z) = a_0 + a_1 z^{-1} + ... + a_M z^{-M}$

and without loss of generality, $a_0 = 1$. In Eq. (1), we have also assumed that the order of the numerator and denominator polynomial of H(z) are the same. Thus, the coefficient vectors of the unknown system $\mathbf{a}$ and $\mathbf{b}$, can be respectively expressed as

$$\mathbf{a} = [a_0 \quad a_1 \cdots a_N] \tag{2}$$

$$\mathbf{b} = [b_0 \quad b_1 \cdots b_N] \tag{3}$$

The IIR ADF has a transfer function $\hat{H}(z)$, which is given by

$$\hat{H}(z) = \frac{\hat{B}(z)}{\hat{A}(z)} \tag{4}$$

where $\hat{B}(z) = \hat{b}_0(n) + \hat{b}_1(n) z^{-1} + ... + \hat{b}_M(n) z^{-M}$
$\hat{A}(z) = \hat{a}_0(n) + \hat{a}_1(n) z^{-1} + ... + \hat{a}_M(n) z^{-M}$.

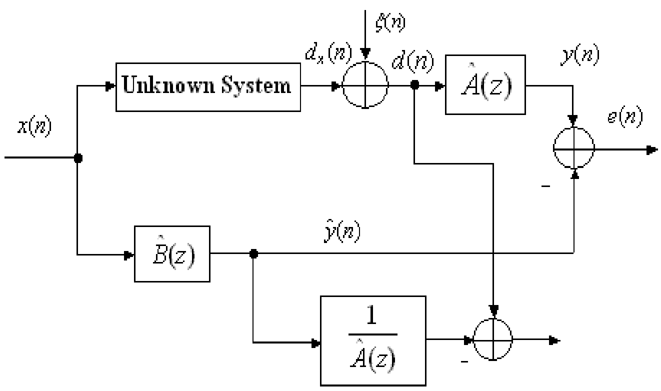

Fig.1 IIR system identification model using the equation error formulation.

The desired signal, which has been corrupted by noise, is represented as follows:

$$d(n) = d_x(n) + \xi(n) \quad (5)$$

where $d_x(n)$ is the output of an unknown system and $\xi(n)$ is the disturbance signal, which is assumed to be statistically independent from an input signal $x(n)$. The disturbance signal is also assumed to be white with a vari-ance of σ_ξ^2.

The error signal $e(n)$ is used for updating the ADF based on the equation error approach. It can also be expressed as

$$\begin{aligned} e(n) &= \mathbf{d}^T(n)\hat{\mathbf{a}}(n) - \mathbf{x}^T(n)\hat{\mathbf{b}}(n) \\ &= \mathbf{d}_x{}^T(n)\hat{\mathbf{a}}(n) + \boldsymbol{\xi}^T(n)\hat{\mathbf{a}}(n) - \mathbf{x}^T(n)\hat{\mathbf{b}}(n) \end{aligned} \quad (6)$$

where
$$\begin{aligned} \mathbf{d}(n) &= [d(n) \quad d(n-1)...d(n-M)]^T \\ \mathbf{x}(n) &= [x(n) \quad x(n-1)...x(n-M)]^T \\ \boldsymbol{\xi}(n) &= [\xi(n) \quad \xi(n-1)...\xi(n-M)]^T \\ \hat{\mathbf{a}}(n) &= [\hat{a}_0(n) \quad \hat{a}_1(n)...\hat{a}_M(n)]^T \\ \hat{\mathbf{b}}(n) &= [\hat{b}_0(n) \quad \hat{b}_1(n)...\hat{b}_M(n)]^T. \end{aligned}$$

The mean square of the estimation error $E[e^2(n)]$ is given by

$$\begin{aligned} E[e^2(n)] &= E[\{\mathbf{d}_x{}^T(n)\hat{\mathbf{a}}(n) - \mathbf{x}^T(n)\hat{\mathbf{b}}(n)\}^2] \\ &\quad + \hat{\mathbf{a}}^T(n)E[\boldsymbol{\xi}(n)\boldsymbol{\xi}^T(n)]\hat{\mathbf{a}}(n) \\ &= E[\{\mathbf{d}_x{}^T(n)\hat{\mathbf{a}}(n) - \mathbf{x}^T(n)\hat{\mathbf{b}}(n)\}^2] + \sigma_\xi^2\hat{\mathbf{a}}^T(n)\hat{\mathbf{a}}(n). \end{aligned} \quad (7)$$

This function is a unimodal function of both $\hat{\mathbf{a}}(n)$ and $\hat{\mathbf{b}}(n)$ [2]. However, when it is minimized directly using a gradient-based algorithm, the second term will contribute to the bias of parameter estimation.

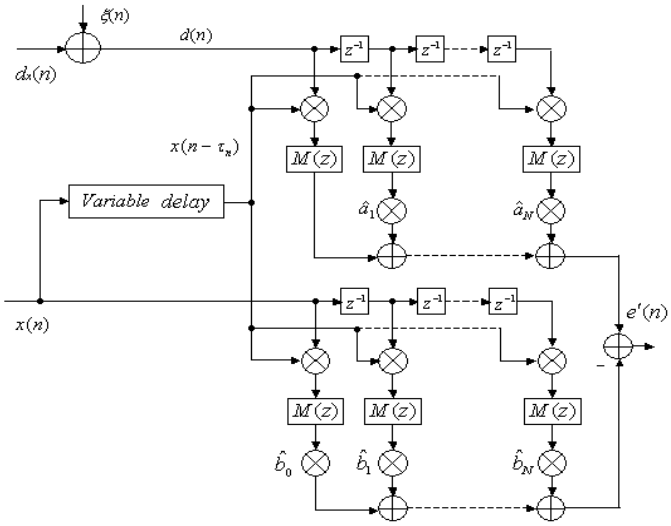

Fig.2 Proposed structure for generating an error signal $e'(n)$.

Several methods have been proposed in order to eliminate the effect of the second term onto the parameter estimation [4]. Song and Shin [6] is known as the unit norm constrained EQ-E IIR ADF. All the coefficients of EQ-E IIR ADF are updated, while keeping the value of $\hat{\mathbf{a}}^T(n)\hat{\mathbf{a}}(n)$ to be unity. However, the validity of all these methods ceases when the disturbance is a color signal. This method would be to minimize $E[e^2(n)] - \lambda\hat{\mathbf{a}}^T(n)\hat{\mathbf{a}}(n)$, by indirectly estimating the variance of the noise such that $\lambda = \sigma_\xi^2$ [6].

3. PROPOSED ADAPTIVE ALGORITHM

Fig. 2 shows the proposed structure for generating an error signal $e'(n)$, where an error signal $e'(n)$ is used to derive an adaptive algorithm for updating the coefficients of the IIR ADF based on estimation of the correlation function. The corrupted desired signal $d(n)$, which has been given in Fig. 2, can be expressed in the vector form as $\mathbf{d}(n)$, which has already been defined in Eq. (6). Similarly, the input signal vector $\mathbf{x}(n)$ has been defined in the same Eq. (6). By multiplying these two vectors with the input signal, which is assumed to be independent from the disturbance signal, we obtain new vectors, whose time average values $\mathbf{d}'(n)$ and $\mathbf{x}'(n)$, respectively, are independent of the disturbance signal.

Let $r_{xd}(l-\tau_n)$ and $r_{xx}(l-\tau_n)$ be the cross correlation function between $x(n-\tau_n)$ and $d(n)$, and the cross corre-lation function between $x(n-\tau_n)$ and $x(n)$, respectively. $r_{xd}(l-\tau_n)$ and $r_{xx}(l-\tau_n)$ are given by

$$r_{xd}(l-\tau_n) = E[x(n-\tau_n)d(n-l)] \quad (8)$$

$$r_{xx}(l-\tau_n) = E[x(n-\tau_n)x(n-l)] \quad (9)$$

respectively. In these equations, the signal $x(n-\tau_n)$ is assumed to be the input signal that has been delayed by an arbitrary number of samples (τ_n), where τ_n is a constant during the time interval between $n-L$ and n. Therefore, $\mathbf{d}'(n)$ and $\mathbf{x}'(n)$ are represented by

$$\mathbf{d}'(n) \approx [r_{xd}(-\tau_n) \quad \mathbf{r}'_{xd}(\tau_n)^T]^T \tag{10}$$

$$\mathbf{x}'(n) \approx [r_{xx}(\tau_n) \quad r_{xx}(1-\tau_n)...r_{xx}(N-\tau_n)]^T \tag{11}$$

where $$\mathbf{r}'_{xd}(\tau_n) = [r_{xd}(1-\tau_n)...r_{xd}(N-\tau_n)]^T . \tag{12}$$

The cost function used to update the coefficients is based on the error function $e'(n)$, which is given by

$$\begin{aligned} e'(n) &= \mathbf{d}'^T(n)\hat{\mathbf{a}}(n) - \mathbf{x}'^T(n)\hat{\mathbf{b}}(n) \\ &\approx r_{xd}(-\tau_n) - \mathbf{u}^T(n)\hat{\mathbf{h}}(n) \\ &\approx \mathbf{u}^T(n)\{\mathbf{h} - \hat{\mathbf{h}}(n)\} \end{aligned} \tag{13}$$

where $$\hat{\mathbf{h}}(n) = [\hat{\mathbf{b}}'(n)^T \quad \hat{\mathbf{a}}'(n)^T]^T \tag{14}$$

$$\mathbf{u}(n) = [\mathbf{x}'(n)^T \quad -\mathbf{r}'_{xd}(\tau_n)^T]^T \tag{15}$$

$$\mathbf{h} = [\mathbf{b}^T \quad \mathbf{a}'^T]^T \tag{16}$$

where $\hat{\mathbf{a}}'(n)$ is part of vector $\hat{\mathbf{a}}(n)$ such that $\hat{\mathbf{a}}(n) =$ $[1 \quad \hat{\mathbf{a}}'(n)^T]^T$ and $\mathbf{a}'$ is part of the coefficient vector of the unknown system, $\mathbf{a} = [1 \quad \mathbf{a}'^T]^T$. Eq. (13) does not have a bias contributing term of the disturbance signal.

Alternatively, it is possible to use a summing circuit with feedback and a reset after every $L+1$ samples of input signal. Such a summation can be implemented by

$$\begin{aligned} \mathbf{d}'(n) &= \alpha\mathbf{d}'(n-1) + \{x(n-\tau_n)\mathbf{d}(n)\} \\ &\quad - \alpha^L\{x(n-\tau_n-L-1)\mathbf{d}(n-L-1)\} \end{aligned} \tag{17}$$

$$\begin{aligned} \mathbf{x}'(n) &= \mathbf{x}'(n-1) + \{x(n-\tau_n)\mathbf{x}(n)\} \\ &\quad - \alpha^L\{x(n-\tau_n-L-1)\mathbf{d}(n-L-1)\} \end{aligned} \tag{18}$$

where α is a forgetting factor and α^L is nearly equal to one but less than one. In this paper we use Eqs. (17) and (18) in order to estimate the correlation functions. The circuit, which performs this estimation is represented in Fig. 2 as a block with label "$M(z)$", where $M(z)$ is a linear filter that calculates Eqs. (17) and (18).

The proposed algorithm, which minimizes the mean square of $e'(n)$ is therefore given by

$$\hat{\mathbf{h}}(n'+1) = \hat{\mathbf{h}}(n') + \mu\frac{\mathbf{u}(n')e'(n')}{\mathbf{u}^T(n')\mathbf{u}(n')} \tag{19}$$

where μ is the step size of adaptation. The value of the step size μ must be such that

$$0 < \mu < 2 . \tag{20}$$

The iteration interval n' is far much less than the sampling rate of the input signal due to the relatively large value of the block length $L+1$. n' is related to n by the following equation

$$n' = div\left(\frac{n}{L+1}\right) \tag{21}$$

where, $div(\upsilon,\omega)$ refers to the biggest integer less or equal to υ/ω. It is also possible to update the coefficients using the recursive least square (RLS) algorithm, which is given by

$$\hat{\mathbf{h}}(n'+1) = \hat{\mathbf{h}}(n') + \mu\frac{\hat{\mathbf{R}}_{uu}^{-1}(n')\mathbf{u}(n')e'(n')}{\mathbf{u}^T(n')\mathbf{R}_{uu}^{-1}(n')\mathbf{u}(n')} \tag{22}$$

where $$\begin{aligned} \hat{\mathbf{R}}_{uu}^{-1}(n') &= \lambda^{-1}\hat{\mathbf{R}}_{uu}^{-1}(n'-1) \\ &\quad - \frac{\hat{\mathbf{R}}_{uu}^{-1}(n'-1)\mathbf{u}(n')\mathbf{u}^T(n')\hat{\mathbf{R}}_{uu}^{-1}(n'-1)}{\lambda\left(\lambda + \mathbf{u}^T(n')\hat{\mathbf{R}}_{uu}^{-1}(n'-1)\mathbf{u}(n')\right)} \end{aligned} \tag{23}$$

λ is a forgetting factor and $\hat{\mathbf{R}}_{uu}(n)$ is an estimate of $\mathbf{R}_{uu}(n)$, where $\mathbf{R}_{uu}(n) = E[\mathbf{u}(n)\mathbf{u}^T(n)]$.

4. CONVERGENCE OF THE ALGORITHM

In this section, we shall explain convergence of the proposed algorithm based on the normalized least mean square algorithm, which is given in Eq. (19). We consider two cases, where the statistics of the input signal $x(n)$ is entirely constant, and the second case where the statistics of the input signal can be assumed to be stationary during intervals of time.

In general, it can be said that after convergence of the algorithm, we would expect that,

$$\begin{aligned} E[\mathbf{u}(n)e'(n)] &= E[\mathbf{u}(n)\mathbf{u}^T(n)\{\mathbf{h} - \hat{\mathbf{h}}_o(n)\}] \\ &= \mathbf{R}_{uu}(n)\{\mathbf{h} - \hat{\mathbf{h}}_o(n)\} \\ &= 0 \end{aligned} \tag{24}$$

where $\hat{\mathbf{h}}_o(n)$ is the convergence value of $\hat{\mathbf{h}}(n)$.

4.1 Case 1: Input signal is a wide-sense stationary process

If the input signal $x(n)$ is a wide-sense stationary process, its mean $E[x(n)]$ will be a constant. Similarly, for the same signal, its autocorrelation $E[x(n+\tau)x(n)]$ depends only on τ. Thus, for a given unknown system, and the delay τ_n defined in Fig. 2 is also fixed, the correlation function $\mathbf{r}'_{xd}(\tau_n)$ and $\mathbf{x}'(n)$ will also be a constant. It will therefore follow from Eq. (15) that the value of $\mathbf{u}(n)$ will also be a constant over time. Under such a condition, $\mathbf{R}_{uu}(n)$ will be given by $\mathbf{R}_{uu}(n) \approx \mathbf{u}(n)\mathbf{u}^T(n)$ since $\mathbf{u}(n)$ is a constant. This will lead to only one of the eigenvalues of $\mathbf{R}_{uu}(n)$ becoming nonzero, while others will all be zero. Therefore, $\mathbf{R}_{uu}(n)$ will not be a full rank matrix, hence its inverse will not exist. Under such a condition $\hat{\mathbf{h}}_o(n)$ will never be equal to $\mathbf{h}$. This result is basically equivalent to an adaptive FIR filter with a constant input signal vector. The above fact can be understood better by expressing $\mathbf{R}_{uu}(n)$ as

$$\mathbf{R}_{uu}(n) = [-r_{xd}(1-\tau_n)\mathbf{u}(n) \cdots -r_{xd}(N-\tau_n)\mathbf{u}(n) \quad r_{xx}(\tau_n)\mathbf{u}(n) \cdots r_{xx}(N-\tau_n)\mathbf{u}(n)] \quad (25)$$

for a constant value of $\mathbf{u}(n)$. From this equation, it is clear that the columns of matrix $\mathbf{R}_{uu}(n)$ will not have an inverse. In order to avoid the situation where the correlation vector $\mathbf{u}(n)$ is entirely constant over time, we make the value of the delay τ_n to change after certain time interval. This can be achieved by setting τ_n to be a function of time index (n) such that

$$\tau_n = \mathrm{mod}(div(n, D_1), D_2) \quad (26)$$

where, $\mathrm{mod}(\upsilon, \omega)$ is a remainder when υ is divided by ω, D_1 is an integer which determines the duration of iteration when τ_n is fixed. When D_1 is kept very small, the transition of $x(n-\tau_n)$ from say $x(n)$ to $x(n-1)$ occurs after a very short interval and fairly good estimate of the correlation functions will not be achieved. On the other hand, very large values of D_1 will result in $\mathbf{d}'(n)$ and $\mathbf{x}'(n)$ being constant over a long period of time. In this paper, we shall fix the value of D_1 to be the same as $L+1$.

D_2 is an integer which determines the maximum value of the delay τ_n, which we shall represent as $\tau_{\max}$, which is given by

$$\tau_{\max} = D_2 - 1. \quad (27)$$

4.2 Case 2: Input signal is not a stationary process

The input signal is considered to as non-stationary if its statistics changes with time. Under this condition, the value of the correlation vector $\mathbf{u}(n)$ will not be constant, but it will be a function of time (n). However, when a change in the statistics of input signal $x(n)$ does not result in a significant change in the relative values of the entries of vector $\mathbf{u}(n)$, matrix The value of $\mathbf{R}_{uu}(n)$ may have an unexpected value. Thus, to avoid this, τ_n is set to be variable as explained in Case 1.

In summary, introduction of a variable delay τ_n creates a combination of correlation vectors $\mathbf{u}(n)$ whose cross correlation matrix $\mathbf{R}_{uu}(n)$ is positive definite and has an inverse. The equation error algorithm is then transformed to an FIR gradient based algorithm, which after convergence will result in $\hat{\mathbf{h}}_o(n) \approx \mathbf{h}$, since $\mathbf{R}_{uu}^{-1}(n)$ exists. How close $\hat{\mathbf{h}}_o(n)$ approaches $\mathbf{h}$ will entirely depend in the block length $(L+1)$.

5. SIMULATION AND RESULTS

Performance of proposed system was evaluated. The unknown system that was considered was a fourth order IIR filter with the numerator $B(z)$ and the denominator $A(z)$ polynomial defined by,

$$B(z) = 1 + 0.9z^{-1} + 0.81z^{-2} + 0.729z^{-3} + 0.6561z^{-4} \quad (28)$$

$$A(z) = 1 + 0.36z^{-2} + 0.1296z^{-4} \quad (29)$$

respectively. In order to evaluate the performance, the normalized vector error $V_e(n)$ was plotted as a function of time. $V_e(n)$ is given by

$$V_e(n) = 10\log_{10} E\left[\frac{\left\|\mathbf{h} - \hat{\mathbf{h}}(n)\right\|^2}{\left\|\mathbf{h}\right\|^2}\right]^T \quad (30)$$

Signal to noise ratio (S/N) was set to 10dB throughout the simulation. S/N is defined as

$$S/N = 10\log_{10}\frac{E[d_x^2(n)]}{E[\xi^2(n)]}. \quad (31)$$

The length of the block D_1 was set to 800, while maximum delay $\tau_{\max}$ was set to 9. A forgetting factor α was set to 0.999. In all simulations, 200 independent runs were performed and the average value of the performance function was plotted as a function of iteration number.

In the first simulation, the disturbance signal was a zero mean white signal, which was generated independent from the disturbance signal. Fig. 3 shows comparison of performance with a white disturbance signal. Fig. 3 (a) represents the NLMS (Normalized LMS) algorithm. Fig. 3 (b) represents the RLS algorithm. Fig. 3 (c) represents the proposed algorithm based on NLMS algorithm. Fig. 3 (d) represents the proposed algorithm based on RLS algorithm. In this simulation, the step size μ was respectively set to0.005, 0.005, 0.80, 0.30 in Fig. 3 (a), (b), (c) and (d). From this result we observe that the proposed method gives a better result, due to the time averaging nature of the algorithm. In the next simulation, the disturbance signal was a color

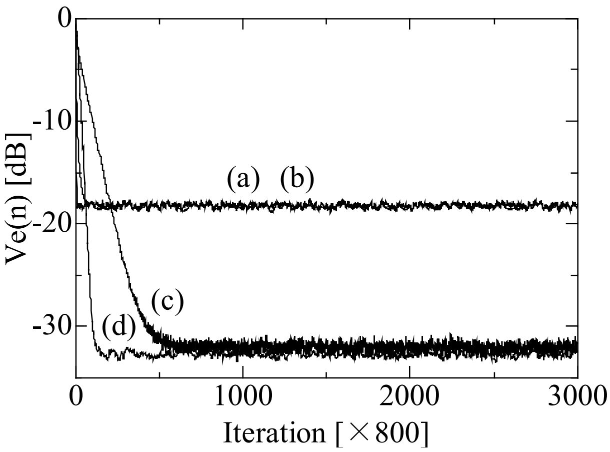

Fig.3 Comparison of performance with a white disturbance signal.
[(a) NLMS algorithm (b) RLS algorithm
(c) Proposed NLMS algorithm with a variable delay τ_n
(d) Proposed RLS algorithm with a variable delay τ_n]

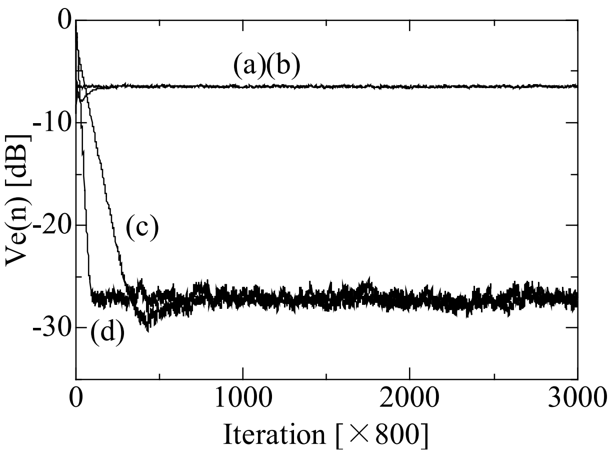

Fig.4 Comparison of performance with a color disturbance signal.
[(a) NLMS algorithm (b) RLS algorithm
(c) Proposed NLMS algorithm with a variable delay τ_n
(d) Proposed RLS algorithm with a variable delay τ_n]

signal that was generated using a coloring filter with the transfer function $H_c(z)$ and a zero mean white signal. $H_c(z)$ is given by

$$H_c(z) = \frac{1}{1+0.95z^{-1}}. \quad (33)$$

Fig. 4 shows comparison of performance with a color disturbance signal. Fig. 4 (a) represents the NLMS algorithm. Fig. 4 (b) represents the RLS algorithm. Fig. 4 (c) represents the proposed algorithm based on NLMS algorithm. Fig. 4 (d) represents the proposed algorithm based on RLS algorithm. In this simulation, the step size μ was respectively set to 0.005, 0.005, 0.80, 0.30 in Fig. 4 (a), (b), (c) and (d). From this figure, it can be seen that the proposed method gives better performance even in the presence of color disturbance signal.

6. CONCLUSION

We have proposed a new algorithm for an IIR ADF formulated using the equation error mode of estimation. The proposed algorithm, which is based on the non-correlated characteristic of the disturbance signal and input signal, achieves better estimation accuracy in comparison to the existing algorithm for EQ-E IIR ADF. This kind of proposed performance is achieved although the disturbance signal is either white or color signal. Simulation results confirm the superiority of the proposed algorithm. In further study, we need to search the convergence speed of proposed method.

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