RECURSIVE BLIND IDENTIFICATION AND EQUALIZATION OF FIR CHANNELS FOR CHAOTIC COMMUNICATION SYSTEMS

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ABSTRACT

In this study, an adaptive autoregressive (AR) algorithm is proposed for the blind identification and equalization of finite impulse response (FIR) channels for chaotic communication systems. The AR method is derived by minimizing a nonlinear prediction error function that is calculated by exploiting the short time predictability of a chaotic signal. The adaptive AR algorithm requires a recursion within a recursion. A simplification is provided to remove the costly inner recursion. When convergence takes place, the AR filter coefficients give an approximation of the FIR channel coefficients and the AR filter output is an estimate of the transmitted chaotic signal. Simulation results show that the method gives promising results.

1. INTRODUCTION

Broad-band, orthogonality and complexity aspects of chaotic signals motivated researches in the area of communication and signal processing to investigate if chaos based communication offers advantages over classical communication systems in the last two decades. Potential applications of chaos resulting directly from these three aspects are spread- spectrum, multi-user communication, and cryptography [1, 2]. In chaotic communications, a chaotic sequence is transmitted through the propagation channel. If the channel is not ideal, which is often the case in practice, the transmitted signal is corrupted before it reaches to the receiver. Hence, channel equalization is required to make the bit error rate of the receiver as small as possible. In many practical cases, channel parameters are unknown. Hence, channel equalization must be performed from the corrupted signal alone, and this is called the blind channel equalization.

In classical communication systems, most of the channel equalization algorithms are based on the statistical properties of the transmitted signal. However, since a chaotic sequence is a deterministic signal, the statistics-based equalization techniques will not achieve optimum estimation accuracy for chaotic communication systems because they do not take into account the inherent properties specific to a chaotic signal.

Various chaotic blind identification and equalization techniques based on different properties of the transmitted chaotic signal have been developed recently. Leung et al. introduced

a new complexity measure called the Phase Space Volume (PSV) to describe the finite dimensional property of a chaotic signal. They showed that the parameter estimates could be obtained by minimizing the PSV. Although the Minimum Phase Space Volume (MPSV) technique can offer very high estimation accuracy, the computation time spent for the PSV is very high [3]. Then, Zhu and Leung proposed an identification approach called the Minimum Nonlinear Prediction Error (MNPE). The MNPE approach is derived by considering the short-term predictability of chaotic signal. However, they performed the identification by representing an FIR channel as an infinite-order AR model. The estimation accuracy is limited by the truncation error in approximating the infinite order AR model by a finite order AR model [4]. In addition to above mentioned studies, Zhu and Leung presented an Extended Kalman Filter (EKF) based adaptive equalization algorithm. Compared with the MPSV and MNPE, its computational complexity is very low, but its estimation accuracy is not well [5].

In this study, an adaptive algorithm is proposed for the blind identification and equalization of FIR channels for chaotic communication systems. Equalization of an FIR channel can be performed by using either an FIR filter or an AR filter. When an AR filter is used, we do not need to find the optimum filter support because it is same as the support of the channel. Furthermore, there is no distortion introduced by the finite support of the FIR filter. Hence, in our study we used AR filter. The AR method is derived by minimizing a nonlinear prediction error function that is calculated by exploiting the short time predictability of a chaotic signal. The algorithm requires a recursion within a recursion, which is computationally complex. We propose a simplification that removes the inner recursion in a manner similar to that described in [6, 7]. When convergence takes place, the AR filter coefficients give an approximation of the FIR channel coefficients and the AR filter output is an estimate of the transmitted chaotic signal. Simulation results show that the method gives promising results.

The study is organized as follows. In Section II, we formulate the blind equalization of FIR channels for chaotic communication systems. The proposed adaptive algorithm is derived in Section III. In Section IV, simulation results are presented to evaluate the effectiveness of the proposed algorithm. Concluding remarks are given in Section V.
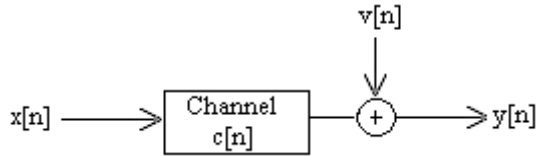
Figure 1- A model that shows the relationship between the received and the transmitted chaotic signals.

Figure 2- The proposed recursive blind channel equalization approach

2. BLIND CHANNEL EQUALIZATION

Consider a chaotic communication system shown in Figure 1, in which a chaotic information signal $x[n]$ is transmitted through a propagation channel $c[n]$ and contaminated further by an additive noise $v[n]$ before it reaches to the receiver. Even though other maps are also possible, without loss of generality, the transmitted chaotic signal $x[n]$ is assumed to be generated by the logistic map given by

$$x[n] = f(x[n-1]) = \lambda x[n-1](1 - x[n-1]). \qquad (1)$$

λ is a constant called the bifurcation parameter. In practice, the ideal channel does not exist. It was found that a typical channel could be modelled by an FIR filter [5]. Thus, the received signal $y[n]$ would be

$$y[n] = x[n] * c[n] + v[n]$$
$$= \sum_{k=0}^{N-1} c[k]x[n-k] + v[n]. \qquad (2)$$

where $c[k]$'s are the channel coefficients, and $v[n]$ is the additive white Gaussian noise with zero mean and variance σ_v^2, and N is the length of the channel. The objective of channel equalization is to recover the transmitted signal $x[n]$ from the received signal $y[n]$ alone.

3. THE PROPOSED METHOD

For the case of non-chaotic transmitted signals, in most blind channel equalization algorithms a priori information about the statistics of the input signal is exploited. In case of chaotic communication, we need to base the equalization strategy to the nonlinearity of the chaotic signal instead of its statistical properties since a chaotic signal is a deterministic signal. Figure 2 illustrates the proposed recursive blind channel equalization approach where the received signal is applied to an adaptive AR filter whose purpose is to estimate the transmitted chaotic signal. A desired signal is required in all adaptive algorithms in order to update the filter coefficients. One possible way of obtaining the desired signal is to use a training sequence. This method is usually not preferred since it is not bandwidth efficient. The other method is to artificially generate the desired signal from the adaptive filter output.

In this study we will adopt the second approach. The adaptive filter output should satisfy Eq. (1) assuming that it
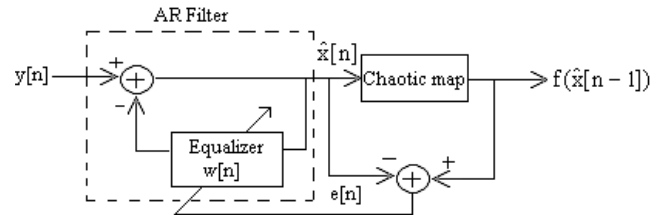
is a reliable estimate of the transmitted chaotic signal. Hence, a plausible cost function J is the squared difference between the adaptive filter output and its prediction obtained by using Eq.(1), i.e.,

$$J = \frac{1}{2}(\hat{x}_j[n] - f(\hat{x}_j[n-1]))^2 \qquad (3)$$

where $f(.)$ is the nonlinear map used to generate the transmitted chaotic signal. When the logistic map is used, the cost function can be written as

$$J = \frac{1}{2}(\hat{x}_j[n] - \lambda \hat{x}_j[n-1](1 - \hat{x}_j[n-1]))^2. \qquad (4)$$

The Gradient Descent (GD) algorithm will be used to update the filter coefficients [8]. Hence, the derivative of J with respect to the adaptive filter coefficients must be determined. Note that the equalizer output at time n at jth iteration is given by

$$\hat{x}_j[n] = y[n] - \sum_{i=1}^{N-1} w_j[i]\hat{x}_j[n-i]. \qquad (5)$$

where $y[n], \hat{x}_j[n], w_j[i]$ are the received signal, the output of the AR filter and the adaptive filter coefficients at jth iteration, respectively. The general form of the GD algorithm for minimizing the proposed cost function is

$$w_{j+1}[k] = w_j[k] - \mu \frac{dJ}{dw_j[k]}$$
$$= w_j[k] - \mu \frac{dJ}{d\hat{x}[n]} \frac{d\hat{x}[n]}{dw_j[k]} \qquad (6)$$

where k=1, 2, …,N-1. The first derivative in Equation (6) is

$$\frac{dJ}{d\hat{x}_j[n]} = (\hat{x}_j[n] - \lambda \hat{x}_j[n-1](1 - \hat{x}_j[n-1])). \qquad (7)$$

It is not possible to write a closed-form expression for the second derivative in Equation (6), but it can be calculated iteratively using regressor filtering. To derive this term, note that $\hat{x}_j[n]$ can be written as

$$\hat{x}_j[n] = y[n] - \sum_{i=1}^{N-1} w_j[i]\hat{x}_j[n-i]$$

$$= y[n] - w_j[k]\hat{x}_j[n-k] - \sum_{\substack{i=1 \\ i \neq k}}^{N-1} w_j[i]\hat{x}_j[n-i]. \quad (8)$$

Taking the derivative of both sides of Equation (8) with respect to $w_j[k]$ gives

$$\frac{d\hat{x}_j[n]}{dw_j[k]} = -\hat{x}_j[n-k] - \sum_{\substack{i=1 \\ i \neq k}}^{N-1} w_j[i]\frac{d\hat{x}_j[n-i]}{dw_j[k]}, \quad (9)$$

Let us define the regressor $\Phi_{j,k}[n]$ as

$$\Phi_{j,k}[n] := -\frac{d\hat{x}_j[n]}{dw_j[k]}.$$

Then, Equation (9) can be written in terms of $\Phi_{j,k}[n]$ as

$$\Phi_{j,k}[n] = \hat{x}_j[n-k] - \sum_{\substack{i=1 \\ i \neq k}}^{N-1} w_j[i]\Phi_{j,k}[n-i]. \quad (10)$$

Substituting Equation (10) and Equation (7) in Equation (6) yields

$$w_{j+1}[k] = w_j[k] + \mu(\hat{x}_j[n] - \lambda\hat{x}_j[n-1](1-\hat{x}_j[n-1]))\Phi_{j,k}[n] \quad (11)$$

Equation (11) gives the update equations for one of the adaptive filter coefficients. It is possible to write update equations for all coefficients in vector form as

$$\mathbf{w}_{j+1} = \mathbf{w}_j + \mu(\hat{x}_j[n] - \lambda\hat{x}_j[n-1](1-\hat{x}_j[n-1]))\,\mathbf{\Phi}_j[n] \quad (12)$$

where $\mathbf{w}_j$ and $\mathbf{\Phi}_j[n]$ are the adaptive filter coefficients vector and the regressor filter vector at time n given by

$$\mathbf{w}_j := [w_j[1], w_j[2],..., w_j[N-1]]^T$$

$$\mathbf{\Phi}_j[n] := [\Phi_{j,1}[n], \Phi_{j,2}[n],..., \Phi_{j,N-1}[n]]^T$$

The presence of regressor filter in Equation (12) makes implementation of the recursive algorithm costly. A simplified algorithm that bypasses the regressor filtering would be preferred. An approximate gradient uses the currently available data vector in place of the regressor filtered version, that is

$$\mathbf{\Phi}_j[n] \approx [\hat{x}_j[n-1], \hat{x}_j[n-2],..., \hat{x}_j[n-N+1]]^T. \quad (13)$$

Equations (12) and (13) constitute the proposed method. The output of the adaptive AR filter is an estimate of transmitted chaotic signal x[n], and the coefficients $w_j[n]$ provide an estimate of the channel coefficients c[n] at convergence.

Conditions under which the approximation made in Equation (13) need to be determined. In classical communication systems, a sufficient condition was obtained according to which the channel must be Strictly Positive Real (SPR) [5,6]. A channel is said to be SPR if the real part of the discrete Fourier transform of its impulse response is positive. In our simulations we used SPR and non-SPR channels to see whether this condition is valid in chaotic communication systems as well. We will not attempt to derive the necessary condition here since the space is limited. We plan to provide the proof (if it exists) in a forthcoming paper.

4. SIMULATION RESULTS

In this section, we use three computer simulations to evaluate the efficiency of the proposed algorithm. In all simulations the transmitted chaotic signal was generated using logistic map given in Equation (1), the bifurcation parameter λ was taken to be 3.8, and the initial value of x[n] was chosen as x[0]=0.78.

In the first experiment, performance of the proposed algorithm is discussed for SPR and non SPR channels. Efficiency of the adaptive algorithm for increasing channel length is investigated in the second experiment. Finally, effect of the adaptive AR filter order on the accuracy of the estimation of the channel coefficients is investigated in the third experiment.

Experiment 1: In this case, the channel coefficients are estimated by using the proposed algorithm for the third-order SPR and non SPR channels, The simulation results are shown in Table I and Figure 3. As shown in Table I, the estimated channel coefficients are very close to the true channel coefficients for the SPR channel. However, the estimated channel coefficients for the non SPR channel are not good approximations of the true channel coefficients. For these two channels the mean square error (MSE) between the true channel coefficients and the estimated channel coefficients are shown in Figure 3 as function of the signal-to-noise ratio (SNR). MSE decreases for the SPR channel when SNR increases, but for the non SPR channel, MSE does not change much with respect to changes in SNR. In addition, MSE values in the SPR case are much better than those for the non SPR case. Improvement might be obtained if the simplification that bypasses the regressor filtering is not ignored.

Experiment 2: In this experiment, it is shown that the proposed algorithm gives good results for different length channels as well. Simulations are performed for the fourth (channel 1) and the fifth order (channel 2) channels. Results are demonstrated in Figure 4. Note that even for SNR values as low as 20 dB the adaptive AR algorithm provides good results for both cases. However, for low SNR values the

Table I- Identification results for SPR and non-SPR channels of lenght 3. The true channel coefficients for SPR channel are c[0] =1, c[1] =0.715, c[2] =0.356 and for those non-SPR channel are c[0] =1, c[1] = 0.85, c[2] =-0.25.

SNR(dB)	SPR channel		Non-SPR channel	
	$\hat{c}[1]$	$\hat{c}[2]$	$\hat{c}[1]$	$\hat{c}[2]$
20	0.7123	0.4148	0.7021	0.088
25	0.7150	0.3860	0.7664	0.003
30	0.6979	0.3590	0.7693	-0.0246
35	0.7110	0.3580	0.7784	-0.0296
40	0.7151	0.3532	0.7863	-0.0856
45	0.7155	0.3546	0.7966	-0.094
50	0.7154	0.3565	0.8155	-0.1056
55	0.7152	0.3563	0.8206	-0.1080
60	0.7149	0.3548	0.8224	-0.1069

Table- II- Effect of adaptive AR filter order on the accuracy of the estimation of the channel coefficients. The true channel coefficients are c[0] =1, c[1] =0.715, c[2] =0.356.

AR filter order	$\hat{c}[1]$	$\hat{c}[2]$	$\hat{c}[3]$	$\hat{c}[4]$	$\hat{c}[5]$	MSE
2	0.715	0.356				2.0487e-26
3	0.715	0.356	1.2772e–10			4.6690e-21
4	0.715	0.356	2.4211e–09	-3.4592e–10		1.9951e-18
5	0.715	0.356	1.2325e–06	-4.0351e–07	1.2456e–06	1.9610e-12

method gives comparable results for the two channels while as the SNR increases the performance difference between the fourth order and the fifth order channel cases increases.

Experiment 3*: Effect of adaptive AR filter order on the accuracy of the estimation of the channel coefficients is investigated in this experiment. For this purpose, a third order channel is used. By assuming that the order of the channel is not known a priori, the order of the adaptive AR filter is varied to see its effect on the identification. The estimated channel coefficients are shown in Table-II for different orders for the AR filter. It is clear from Table-II that the first two AR filter coefficients are close to the true channel coefficients and all extra coefficients take small values that can be ignored. In other words, the proposed method can also be used to estimate the order of the channel if it is not known.

Before concluding this section it is important to mention that in all simulations MSE was computed by

$$MSE = \frac{1}{T}\sum_{i=1}^{T}[(\hat{c}_i[1] - c_i[1])^2 + (\hat{c}_i[2] - c_i[2])^2]$$

where T is the number of iterations used in the adaptive algorithm until convergence takes place. Note that this definition of MSE applies to the length two channels, though it is

easy to extend the definition to orders bigger than two. MSE is the most frequently used measure in order to compare the performance of different channel equalization algorithms when the channel is known. When the channel is not known MSE does not make sense, other performance measures must be considered.

5. CONCLUSIONS

In this study, an adaptive AR algorithm   was developed for blind equalization of FIR channels in chaotic communications. This method exploits the prior knowledge about nonlinear dynamics of the chaotic signal to estimate the transmitted chaotic signal. One limitation of the adaptive AR filter is that it requires a recursion within a recursion, which makes its implementation costly. A simplified algorithm to remove the costly inner recursion was proposed, though a rigorous proof on when this can be done was omitted because of space limitation. The simplified algorithm gives better results for SPR channels than non-SPR channels and it may not even work non-SPR channels. However, it is easy to implement and provides dramatic improvement on MSE compared to the   existing methods.
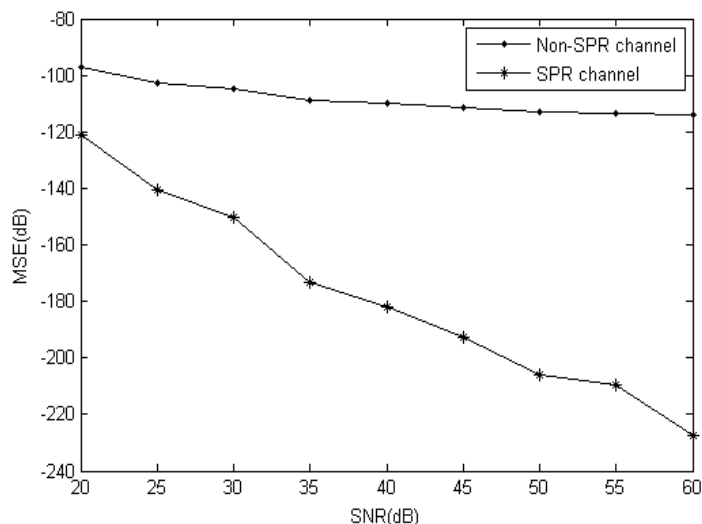
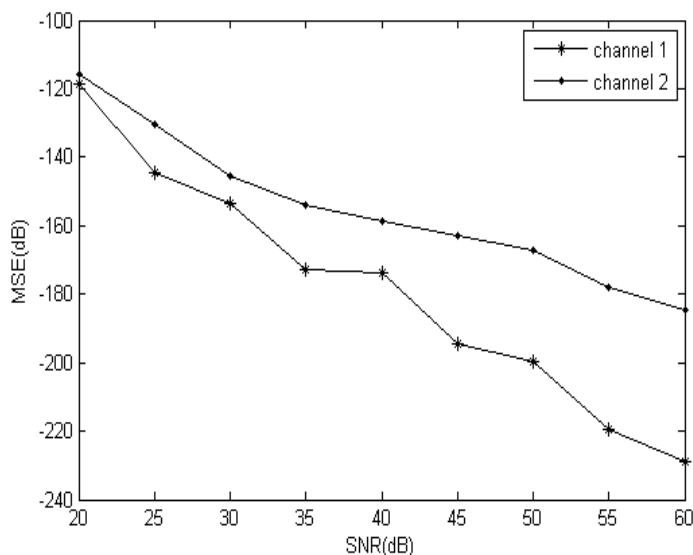Figure 3- MSE versus SNR for the SPR and the non-SPR channels.

Figure 4- MSE versus SNR for two channels with different lengths.

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