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HARMONIC EXTENSION OF AN ADAPTIVE NOTCH FILTER FOR FREQUENCY TRACKING

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ABSTRACT

Adaptive frequency tracking is useful in a broad range of applications, and many schemes have been introduced in the recent years for that purpose. Starting from the observation that, in some situations, an harmonic component is also present in the signal of interest, we propose in this paper to extend frequency tracking to harmonic frequencies in order to improve convergence properties. We apply this idea to a specific adaptive frequency tracking algorithm and obtain through theoretical analysis and computer simulations the performance gain of this new scheme.

1. INTRODUCTION

Adaptive frequency tracking of noisy sinusoidal components with time-varying amplitudes and frequencies [1, 2, 3] is a useful tool for many applications such as communications, speech processing or biomedical engineering. In many cases, the signal contains a sinusoidal component at a fundamental frequency, and also some harmonic content. Usually, this harmonic component is not used by the frequency tracking algorithms and is considered as a noisy component which can decrease the performances of the algorithm. However, this harmonic component obviously contains information related to the fundamental frequency of the signal. If this information is exploited, it can certainly improve the tracking performance in terms of convergence speed and frequency estimation variance. In this paper, we present an algorithm using this additional information for the frequency tracking of the sinusoidal component of a signal. This algorithm is an extension of the algorithm proposed by Liao [1]. It is based on the combination of the discrete oscillator model and a line-enhancer filter using a mean-square error (MSE) cost function. We propose to add to the basic algorithm a constraint linking the fundamental frequency to its harmonics. Nehorai et al. [5] proposed a comb filter algorithm for harmonic enhancement and estimation. However, the rigid constraint proposed in this work imposes the initial conditions to be close to the steady-states values, due to a possible frequency mismatch at convergence. The flexibility of the constraint in our algorithm makes it able to converge from almost all initial values. Moreover, the use of a soft constraint is surely advantageous during transitions or non-stationary periods where the harmonic relation between the fundamental and second harmonic may be not perfect.

2. OSCILLATOR BASED ADAPTIVE NOTCH FILTER

The algorithm extended in this paper is called the OSC-MSE ANF (oscillator based mean square error adaptive notch filter) algorithm [1]. Its structure is shown in Figure 1. The input signal, $u(n)$, can be represented as

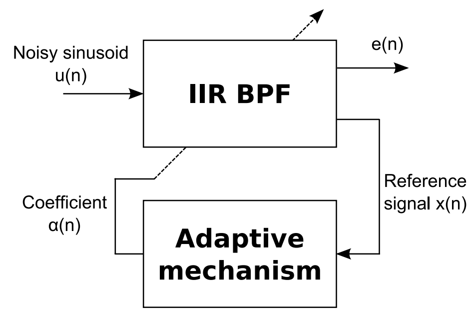

Figure 1: Structure of the OSC-MSE ANF algorithm.

$$u(n) = d(n) + w(n) \tag{1}$$

where $d(n)$ is the sinusoid at pulsation ω_0 and $w(n)$ is an additive, zero-mean, i.i.d. noise. The sinusoidal component of $u(n)$ should satisfy the oscillator equation

$$\begin{aligned} d(n) &= 2\cos\omega_0 d(n-1) - d(n-2) \\ &\equiv 2\alpha_0 d(n-1) - d(n-2). \end{aligned} \tag{2}$$

The adaptive coefficient $\alpha(n)$, which tracks $\alpha_0 = \cos\omega_0$, determines the central frequency of the bandpass filter (BPF). Its transfer function is defined as

$$H_{BP}(z;n) = \frac{1-\beta}{2} \frac{1-z^{-2}}{1-\alpha(n)[1+\beta]z^{-1}+\beta z^{-2}} \tag{3}$$

where β $(0 < \beta < 1)$ controls the bandwidth. The reference signal, $x(n)$, on which the adaptive mechanism is based, is defined as the recursive part of the BPF:

$$x(n) = \alpha(n)[1+\beta]x(n-1) - \beta x(n-2) + u(n). \tag{4}$$

In short, the adaptive algorithm is driven by a line-enhanced version of the input signal. In the OSC-MSE, the goal is to determine the value for $\alpha(n+1)$ which satisfies the discrete oscillator model. This is done by minimizing the following cost function:

$$J = \mathrm{E}\left\{[x(n) - 2\alpha(n+1)x(n-1) + x(n-2)]^2\right\}. \tag{5}$$

By setting $\partial J/\partial\alpha(n+1) = 0$, the optimal solution for this MSE criterion is

$$\alpha(n+1) = \frac{\mathrm{E}\{x(n-1)[x(n)+x(n-2)]\}}{\mathrm{E}\{2x^2(n-1)\}}. \tag{6}$$

However this expression for $\alpha(n+1)$ is not real-time computable. Thus the numerator and the denominator are replaced by their exponentially weighted time-average estimation, and the coefficient-updating algorithm becomes

$$\alpha(n+1) = \frac{Q_x(n)}{2P_x(n)} \tag{7}$$

where

$$\begin{aligned} Q_x(n) &= \delta Q_x(n-1) + (1-\delta)x(n-1)[x(n)+x(n-2)] \\ P_x(n) &= \gamma P_x(n-1) + (1-\gamma)x^2(n-1). \end{aligned}$$

The parameters δ and γ are the forgetting factors ($0 < \delta, \gamma < 1$). They control the convergence rate of this adaptive algorithm.

3. EXTENSION TO HARMONIC FREQUENCIES

Suppose now that the input signal is defined as follows,

$$u(n) = d_1(n) + d_2(n) + w(n) \tag{8}$$

where $d_1(n)$ and $d_2(n)$ are two sinusoids at pulsation ω_{10} and ω_{20} with arbitrary initial phases, and $w(n)$ is an additive, zero-mean, i.i.d. noise. The frequencies of the sinusoidal components are harmonic, so they should satisfy $\omega_{20} = 2\omega_{10}$.

Moreover the discrete oscillator model (2) should be satisfied by $d_1(n)$ and $d_2(n)$:

$$d_i(n) = 2\cos\omega_{i0} d_i(n-1) - d_i(n-2) \ , \ i = 1,2. \tag{9}$$

The input signal is filtered by two different BPFs (one for each sinusoid) to obtain two reference signals $x_1(n)$ and $x_2(n)$. Each BPF has a notch at the other BPF central frequency in order to avoid the disturbance caused by the other sinusoid, an idea introduced in [4] in a slightly different context. The central frequencies and the notch positions of the BPFs are determined by the two adaptive coefficients $\alpha_1(n)$ and $\alpha_2(n)$. The transfer functions of the filters are

$$H_i(z;n) = \frac{1 - 2\alpha_{3-i}(n)z^{-1} + z^{-2}}{1 - \alpha_i(n)[1+\beta]z^{-1} + \beta z^{-2}} \ , \ i = 1,2,$$

where the $3-i$ index is used to indicate 1 for $i = 2$ and 2 for $i = 1$ (same notation throughout the text).

Thus the reference signals are

$$\begin{aligned} x_i(n) &= \alpha_i(n)[1+\beta]x_i(n-1) - \beta x_i(n-2) \\ &+ u(n) - 2\alpha_{3-i}(n)u(n-1) + u(n-2) \ , \ i = 1,2. \end{aligned}$$

The relation between the harmonic frequencies becomes $\alpha_{20} = 2\alpha_{10}^2 - 1$, where $\alpha_{10} = \cos\omega_{10}$ and $\alpha_{20} = \cos\omega_{20}$. This relationship is used to impose the following constraint on the adaptive coefficients:

$$\left[2\alpha_1^2(n) - 1 - \alpha_2(n)\right]^2 = 0. \tag{10}$$

This extended algorithm will try to minimize the following cost function,

$$\begin{aligned} J &= \mathrm{E}\left\{[x_1(n) - 2\alpha_1(n)x_1(n-1) + x_1(n-2)]^2\right\} \\ &+ \mathrm{E}\left\{[x_2(n) - 2\alpha_2(n)x_2(n-1) + x_2(n-2)]^2\right\} \\ &+ \lambda\left[2\alpha_1^2(n) - 1 - \alpha_2(n)\right]^2 \end{aligned} \tag{11}$$

where λ ($\lambda \geq 0$) determines the influence of the constraint. This constraint, which does not impose strictly the equality in (10), gives more flexibility, for instance in the initialization phase (it is possible to initialize the algorithm with $\omega_1(0) = 0$ and $\omega_2(0) = \pi$). The adaptive algorithm is extended as

$$\alpha_i(n+1) = \frac{Q_{x_i}(n)}{2P_{x_i}(n)} - \lambda C_i(n) \ , \ i = 1,2 \tag{12}$$

where

$$\begin{aligned} Q_{x_i}(n) &= \delta Q_{x_i}(n-1) + (1-\delta)x_i(n-1) \\ &\quad \times [x_i(n) + x_i(n-2)] \ , \ i = 1,2 \\ P_{x_i}(n) &= \gamma P_{x_i}(n-1) + (1-\gamma)x_i^2(n-1) \ , \ i = 1,2 \\ C_1(n) &= \left[2\alpha_1^2(n) - 1 - \alpha_2(n)\right] 4\alpha_1(n) \\ C_2(n) &= -\left[2\alpha_1^2(n) - 1 - \alpha_2(n)\right]. \end{aligned}$$

The parameters δ and γ are the forgetting factors, $C_1(n)$ and $C_2(n)$ are obtained by differentiation of the constraint.

4. PERFORMANCE ANALYSIS

The following vectorial notation is introduced:

$$\alpha(n) = \begin{pmatrix} \alpha_1(n) \\ \alpha_2(n) \end{pmatrix}, \quad \alpha_0 = \begin{pmatrix} \alpha_{10} \\ \alpha_{20} \end{pmatrix}.$$

The coefficient-updating algorithm (12) can be rewritten, by letting $\gamma = \delta$, in a recursive form, i.e. $\alpha(n+1) = F(\alpha(n))$ with

$$F(\alpha(n)) = \begin{pmatrix} \frac{P_{x_1}(n-1)}{P_{x_1}(n)}\delta\alpha_1(n) + \frac{1-\delta}{2P_{x_1}(n)}\chi_1(n) - \lambda C_1(n) \\ \frac{P_{x_2}(n-1)}{P_{x_2}(n)}\delta\alpha_2(n) + \frac{1-\delta}{2P_{x_2}(n)}\chi_2(n) - \lambda C_2(n) \end{pmatrix}$$

where $\chi_i(n) = x_i(n-1)[x_i(n) + x_i(n-2)]$, $i = 1,2$.

The cost function (11) has a global minimum. Indeed if $\alpha(n) = \alpha_0$, it will be equal to zero, since the oscillator model and the constraint will be satisfied.

Assuming that the algorithm is near convergence, it is possible to use a Taylor expansion of $F(\alpha(n))$ around α_0. Thus the coefficients update is approximated as

$$\alpha(n+1) \approx F(\alpha_0) + J(\alpha_0)[\alpha(n) - \alpha_0] \tag{13}$$

where $J(\alpha(n))$ is the Jacobian matrix of $F(\alpha(n))$. The values of $J(\alpha_0)$ and $F(\alpha_0)$ are expressed as

$$J(\alpha_0) = \begin{pmatrix} \frac{P_{x_1}(n-1)}{P_{x_1}(n)}\delta - 16\lambda\alpha_{10}^2 & 4\lambda\alpha_{10} \\ 4\lambda\alpha_{10} & \frac{P_{x_2}(n-1)}{P_{x_2}(n)}\delta - \lambda \end{pmatrix},$$

$$F(\alpha_0) = \begin{pmatrix} \alpha_{10} + \frac{1-\delta}{2P_{x_1}(n)}x_1(n-1) \\ \quad \times [x_1(n) - 2\alpha_{10}x_1(n-1) + x_1(n-2)] \\ \alpha_{20} + \frac{1-\delta}{2P_{x_2}(n)}x_2(n-1) \\ \quad \times [x_2(n) - 2\alpha_{20}x_2(n-1) + x_2(n-2)] \end{pmatrix}.$$

4.1 Bias Analysis

By using the facts that the correlation between $[x_i(n) - 2\alpha_{i0}x_i(n-1) + x_i(n-2)]$ and $x_i(n-1),\ i = 1,2$ is negligible and $\mathrm{E}\{x_i(n) - 2\alpha_{i0}x_i(n-1) + x_i(n-2)\} = 0,\ i = 1,2,$ and the assumption that $P_{x_i}(n) \to R_{x_i}(0),\ i = 1,2$ for n large, as it was done in [1], (13) becomes

$$\mathrm{E}\{\alpha(n+1) - \alpha_0\} \approx \widehat{J}\,\mathrm{E}\{\alpha(n) - \alpha_0\}, \tag{14}$$

where

$$\widehat{J} = \begin{pmatrix} \delta - 16\lambda\alpha_{10}^2 & 4\lambda\alpha_{10} \\ 4\lambda\alpha_{10} & \delta - \lambda \end{pmatrix}.$$

The eigenvalues of $\widehat{J}$ are $\mu_1 = \delta$ and $\mu_2 = \delta - \lambda\left[1 + 16\alpha_{10}^2\right]$. The algorithm will converge if $|\mu_1|$ and $|\mu_2|$ are smaller than one. This condition on the convergence yields the following constraint on the parameters:

$$\lambda\left[1 + 16\alpha_{10}^2\right] < 1 + \delta. \tag{15}$$

Thus, if this constraint is satisfied, the coefficient-updating algorithm will converge to α_0. Hence it is unbiased, like the original algorithm.

4.2 Variance Analysis

In order to perform the variance analysis for this extended algorithm, (13) is rewritten as

$$\alpha_d(n+1) \approx J(\alpha_0)\alpha_d(n) + \Gamma(n) \tag{16}$$

where $\alpha_d(n) = \alpha(n) - \alpha_0$ and $\Gamma(n) = F(\alpha_0) - \alpha_0$. Assuming that $P_{x_i}(n) \to R_{x_i}(0),\ i = 1,2$ for n large and applying the change of basis $\alpha_d(n) = V\chi(n)$, (16) becomes

$$\chi(n+1) \approx V^T\widehat{J}V\chi(n) + V^T\Gamma(n) = M\chi(n) + V^T\Gamma(n) \tag{17}$$

where M is the diagonal matrix composed of the eigenvalues μ_1 and μ_2, and V is the matrix of corresponding eigenvectors.

By using the same assumptions as for the bias analysis, and the assumption that $\chi_i(n)$ and $x_j(m)$ are not correlated, $\forall n,m$ and $i,j = 1,2$, and applying the same approximations as in [1], the variances of $\chi_1(n)$ and $\chi_2(n)$ are evaluated as

$$\begin{aligned} R_{\chi_1}(0) &= \frac{(1-\delta)^2}{1-\delta^2}\frac{\Delta\omega_{neq}^3}{6}\frac{1}{1+16\alpha_{10}^2} \\ &\times \left[\frac{1-\alpha_{10}^2}{\frac{A_1^2}{2\sigma^2} + 2\Delta\omega_{neq}} + 16\alpha_{10}^2\frac{1-\alpha_{20}^2}{\frac{A_2^2}{2\sigma^2} + 2\Delta\omega_{neq}}\right] \end{aligned} \tag{18}$$

$$\begin{aligned} R_{\chi_2}(0) &= \frac{(1-\delta)^2}{1-[\delta-\lambda(1+16\alpha_{10}^2)]^2}\frac{\Delta\omega_{neq}^3}{6}\frac{1}{1+16\alpha_{10}^2} \\ &\times \left[16\alpha_{10}^2\frac{1-\alpha_{10}^2}{\frac{A_1^2}{2\sigma^2} + 2\Delta\omega_{neq}} + \frac{1-\alpha_{20}^2}{\frac{A_2^2}{2\sigma^2} + 2\Delta\omega_{neq}}\right] \end{aligned} \tag{19}$$

where A_i is the amplitude of the sinusoid $d_i(n)$, σ^2 is the noise variance, and $\Delta\omega_{neq} = \pi\frac{1-\beta}{1+\beta}$ is the noise equivalent bandwidth of the BPFs. This value is computed with respect to (3), under the hypothesis that the notches added in the extension have almost no influence on $\Delta\omega_{neq}$.

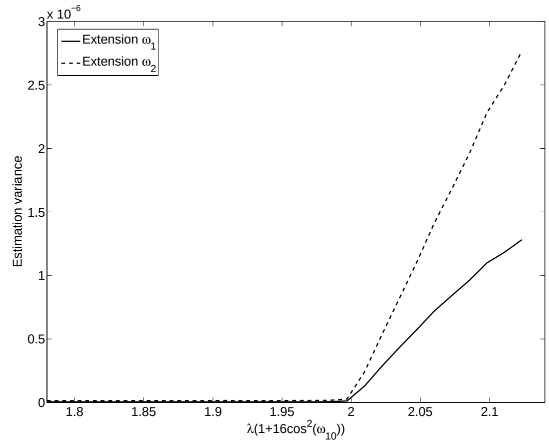

Figure 2: Estimation variance of the extended algorithm around the bound (15) for an SNR value of 5 dB ($\beta = 0.95$, $\delta = 0.99$, $\omega_{10} = 0.4\pi$, $\omega_{20} = 0.8\pi$, $A_1/A_2 = \sqrt{2}$).

The variances of $\alpha_{d_1}(n)$ and $\alpha_{d_2}(n)$ are obtained as linear combinations of $R_{\chi_1}(0)$ and $R_{\chi_2}(0)$:

$$R_{\alpha_{d_i}}(0) = \frac{1}{1+16\alpha_{10}^2}\left[R_{\chi_i}(0) + 16\alpha_{10}^2 R_{\chi_{3-i}}(0)\right] \tag{20}$$

for $i = 1,2$. And using the same approximation as in [1], the variances of the frequency estimates are

$$R_{\omega_{d_i}}(0) = \frac{R_{\alpha_{d_i}}(0)}{\sin^2\omega_{i0}}\ ,\ i = 1,2 \tag{21}$$

where $\omega_{d_i}(n) = \omega_i(n) - \omega_{i0},\ i = 1,2.$

5. SIMULATION RESULTS

In the simulations, the steady-state performance of the OSC-MSE ANF algorithm (7) and its extension to harmonic frequencies are evaluated. The input signal is a single sinusoid embedded in white noise for OSC-MSE ANF, and a sum of two sinusoids at fundamental and second harmonic frequencies with additive white noise for the extension. For both algorithms, the noise variance and the amplitude of the fundamental frequency sinusoid are the same. For the extended algorithm, the amplitude of the harmonic component is set to $A_2 = A1/\sqrt{2}$, were A_1 is the amplitude of the fundamental component. The frequencies of the fundamental and harmonic sinusoidal components are 0.4π and 0.8π respectively. Estimation variance and bias were computed on the last 2000 of 10000 frequency estimated values and averaged over 1000 runs. For each simulation, the forgetting factors δ and γ of the algorithms (7) and (12) were equal.

The first point to verify is the condition for convergence (15). The value of $\lambda\left[1 + 16\alpha_{10}^2\right]$ is increased from 1.78 to 2.13 in steps of 0.01 by varying λ (λ varies from 0.7 to 0.85). Thus the algorithm behavior around the bound $1+\delta$ for $\delta = 0.99$ can be investigated. The SNR is equal to 5 dB. The estimation variance is shown in Figure 2. The simulations confirm that the condition for stability (15) is correct.

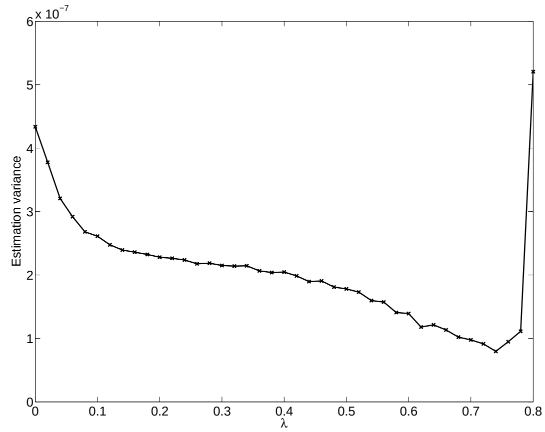

Figure 3: Evolution of the estimation variance of the harmonic extension for several λ values for an SNR value of 5 dB ($\beta = 0.9$, $\delta = 0.95$, $\omega_{10} = 0.4\pi$, $A_1/A_2 = \sqrt{2}$).

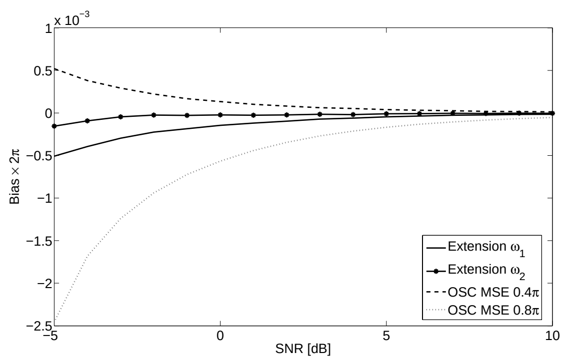

Figure 4: Frequency estimation bias comparison between OSC-MSE ANF and its harmonic extension for different SNR values ($\beta = 0.9$, $\delta = 0.95$, $\lambda = 0.1$, $\omega_{10} = 0.4\pi$, $\omega_{20} = 0.8\pi$, $A_1/A_2 = \sqrt{2}$).

We have to choose λ between zero (no constraint) and the bound given by (15), i.e. $\lambda_{max} = 0.78$. To do so, we studied the influence of the λ value on the estimation variance of the fundamental frequency component for the extended algorithm. The results obtained for λ values ensuring stability are shown on Figure 3 ($\beta = 0.9$, $\delta = 0.95$, $SNR = 5$ dB). First, we can see that the variance increases near the bounds. For a small λ value, the trackings of the fundamental and the harmonic frequency become almost independent and thus the variance increases. For λ values close to the bound obtained in (15), the algorithm is nearly instable, so the estimation variance increases, as shown in Figure 3. For intermediary values, the estimation variance is almost constant. For all the following simulations, we use $\lambda = 0.1$.

The estimation bias of the extended algorithm for the fundamental and harmonic frequencies is displayed on Figure 4. The estimation bias of the OSC-MSE ANF is also displayed for frequencies of 0.4π and 0.8π. The estimation bias for the fundamental frequency is almost the same for both algorithm, but for the harmonic frequency, the bias is a lot smaller

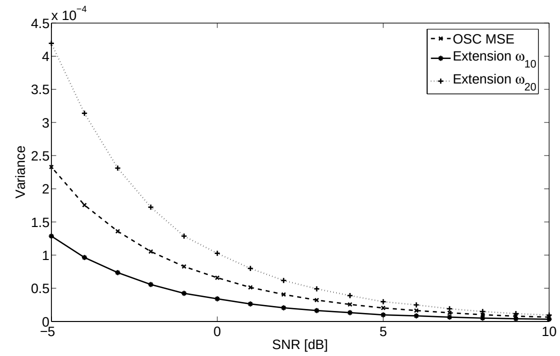

Figure 5: Frequency estimation variance comparison between OSC-MSE ANF and its harmonic extension for different SNR values ($\beta = 0.9$, $\delta = 0.95$, $\lambda = 0.1$, $\omega_{10} = 0.4\pi$, $\omega_{20} = 0.8\pi$, $A_1/A_2 = \sqrt{2}$).

for the extended algorithm. This can be explained by the constraint linking the estimated frequencies of the fundamental and harmonic components. Indeed, the constraint tends to force a harmonic relationship between the estimations and thus, as the bias of the OSC-MSE ANF for 0.4π and 0.8π are of opposite signs, they tend to compensate.

Figure 5 shows that the estimation variance for the extension is almost two times smaller than OSC-MSE ANF for the fundamental frequency, but it is greater for the harmonic frequency. Nevertheless it is the fundamental frequency that is really important. Thus, the information present in the harmonic component and used by the extended algorithm is useful in order to obtain a more robust frequency estimate.

Another important aspect to evaluate is the rate of convergence. A frequency shift from 0.3π and 0.6π to 0.4π and 0.8π is applied after both algorithms have converged, for the fundamental and harmonic components respectively. As before the SNR is computed with respect to the fundamental sinusoid and is equal to 5 dB. The parameter δ is different for each algorithm. Indeed its value is chosen in order to ensure that both algorithms have approximately the same estimation variance for the fundamental frequency estimate. Thus δ is set to 0.95 for the OSC-MSE ANF algorithm and 0.922 for the extension, and λ is equal to 0.1. The frequency estimates averaged over 1000 runs with the frequency shift after 500 samples are shown in Figure 6. One can observe that the convergence of the extended algorithm is faster than the one of the original one.

6. EXTENSION TO THE THIRD HARMONIC FREQUENCY

Sometimes it is not the second harmonic frequency that is present, but the third one. The relation between the frequencies is then $\omega_{30} = 3\omega_{10}$. Thus the constraint on the adaptive coefficients becomes

$$\left[4\alpha_1^3(n) - 3\alpha_1(n) - \alpha_3(n)\right]^2 = 0. \tag{22}$$

An algorithm similar to (12) can be derived by using the same arguments, and the same performance analysis as before can be performed using again a Taylor expansion. Only the condition for stability is presented here. This condition

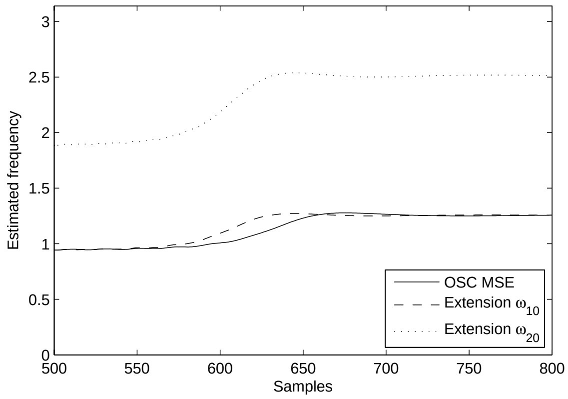

Figure 6: Convergence comparison between OSC-MSE ANF and its harmonic extension for an SNR value of 5 dB ($\beta = 0.95$, $\delta = 0.95$ for OSC-MSE ANF, $\delta = 0.922$ for the extension, $\lambda = 0.1$, $A_1/A_2 = \sqrt{2}$).

on the parameters is

$$\lambda\left[1+\left(12\alpha_{10}^2-3\right)^2\right] < 1+\delta. \tag{23}$$

Thus this extended algorithm can be generalized for any harmonic frequency (with increasing complexity for higher harmonics).

7. CONCLUSION

In this paper, we have extended an existing frequency tracking algorithm to take advantage of the information contained in an harmonic component. The obtained estimate of the fundamental frequency is less sensitive to the noise and has an almost twofold reduction of the estimation variance. The estimation bias of the fundamental frequency is almost the same as for the basic algorithm. Indeed, the constraint linking fundamental and harmonic frequency estimates can yield reduction in estimation bias. In both cases, the small bias confirms the theoretical analysis predicting unbiased estimates. Thus, in presence of a harmonic component in the signal of intereset, the extended algorithm performs better than the OSC-MSE ANF algorithm. The flexibility given by the soft constraint can be an advantage if the initialization is arbitrary. The algorithm was given for the presence of the second or third harmonic and can easily extended for other harmonic components. The proposed approach can be generalized to other frequency tracking algorithms.

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