MULTIPLE ACCESS INTERFERENCE PLUS NOISE CONSTRAINED LEAST MEAN SQUARE ALGORITHM

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ABSTRACT

In this work, a constrained least-mean-square (LMS) algorithm, which incorporates the knowledge of the number of users, spreading sequence length and additive noise variance, is developed subject to the new combined constraint comprising both the MAI and noise variance. The novelty of this constraint resides in the fact that the MAI variance was never used as a constraint. This constrained optimization technique results in an (MAI plus noise)-constrained LMS (MNCLMS) algorithm. Convergence analysis is carried out of the proposed algorithm in the presence of MAI. Finally, a number of simulations are conducted to compare performance of MNC-LMS algorithm with other adaptive algorithms.

1. INTRODUCTION

In the performance of multiuser environment, the major limiting factor is the multiple access interference (MAI). Hence, a multiuser detection receiver that deals with the effect of both MAI and additive noise must be designed. The rationale behind such a need is explained as follows. Previous research work simply incorporated the MAI as part of the interfering noise; although MAI was a wholly unstructured white Gaussian noise as it was invariably assumed.

Recently an LMS-type algorithm that exploits the knowledge of channel noise variance for identification and tracking of FIR channels, called noise constrained LMS (NCLMS) algorithm, was proposed [1]. Since the NCLMS algorithm does not consider the effect of MAI, a more effective constrained LMS algorithm is developed in this work by adding one more constraint (MAI variance) to the usual noise variance in the algorithm's structure. This constraint optimization procedure results in an MAI-plus-noise-constrained-LMS (MNCLMS) algorithm which is a generalized constrained adaptive algorithm that includes a class of algorithms as special cases such as MAI-constrained-LMS (MCLMS) algorithm, noise-constrained-LMS (NCLMS), and zero noise-constrained-LMS (ZNCLMS) algorithms. Note that MCLMS algorithm is a by-product of the MNCLMS algorithm. Moreover, the variance of MAI is also derived for the case of a synchronous downlink DS-CDMA system that uses BPSK signals and rectangular signature waveforms with random signature sequences.

2. ALGORITHM DEVELOPMENT

In a multiuser CDMA system, the output of the matched filter matched to the desired user's spreading waveform consists of three parts. These are: the desired user's component (b_n), the MAI (m_n) and the additive white Gaussian noise (η_n), i.e.,

$$r_n = b_n + z_n, \tag{1}$$

where z_n is the sum of MAI (m_n) and noise (η_n) with variance σ_z^2. The desired user's component can be written as follows:

$$b_n = \mathbf{h}_n^T \mathbf{x}_n, \tag{2}$$

where $\mathbf{h}_n^T = [h_n \; h_{n-1} \cdots \; h_{n-N+1}]$ corresponds to the time varying channel impulse response and $\mathbf{x}_n^T = [x_n \; x_{n-1} \cdots \; x_{n-N+1}]$ is the input vector to the channel, where N is the length of the adaptive filter. If $\mathbf{w}_n$ represents the impulse response of the adaptive filter, then the mean-squared error (MSE) to be minimized is given by:

$$J(\mathbf{w}_n) = E\{e_n^2\}, \tag{3}$$

where e_n is the error between the output of the matched filter and the adaptive filter and is defined as:

$$e_n = r_n - \mathbf{w}_n^T \mathbf{x}_n. \tag{4}$$

Minimization of the cost function (3) over $\mathbf{w}_n$ gives the optimal weight value at time n, i.e., $\mathbf{w}_{opt} = \mathbf{h}_n$ with $\hat{J}(\mathbf{w}_{opt}) = \sigma_z^2$. It can be shown that knowledge of σ_z^2 is useful in selecting the search direction of an adaptive algorithm in multiuser environment similar to the case of NCLMS algorithm in a single user environment.

Now, consider the following constrained minimization problem that incorporates knowledge of σ_z^2: Minimize $J(\mathbf{w}_n)$ over $\mathbf{w}_n$ subject to constraint $J(\mathbf{w}_n) = \sigma_z^2$. The Lagrangian for this problem is:

$$J_1(\mathbf{w}_n, \lambda) = J(\mathbf{w}_n) + \lambda\big[J(\mathbf{w}_n) - \sigma_z^2\big]. \tag{5}$$

Since, the critical values of λ are not unique in this case, augmented Lagrangian is used to avoid this problem giving the following cost function:

$$J_2(\mathbf{w}_n, \lambda) = J(\mathbf{w}_n) + \gamma\lambda\big[J(\mathbf{w}_n) - \sigma_z^2\big] - \gamma\lambda^2. \tag{6}$$

Solution of the above equation for the augmented Lagrangian is obtained and can be shown to be given by:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \alpha_n e_n \mathbf{x}_n, \tag{7}$$

$$\alpha_n = \alpha(1 + \gamma\lambda_n), \tag{8}$$

$$\lambda_{n+1} = \lambda_n + \beta\Big[\frac{1}{2}(e_n^2 - \sigma_z^2) - \lambda_n\Big], \tag{9}$$

where α and β are positive step-sizes. The adaptive algorithm developed above is named as MAI plus noise constrained LMS (MNCLMS) algorithm. It can be seen from (8) that the LMS algorithm is recovered when $\gamma = 0$. Since, the MNCLMS algorithm depends upon the variance of the MAI, it is therefore expected here to outline a procedure by which this quantity is evaluated. The ensuing analysis details this procedure.

2.1. Variance of MAI

In this work, a synchronous DS-CDMA transmitter model for the downlink of a mobile radio network is considered. Considering a flat fading channel whose complex impulse response for the n^{th} symbol is:

$$H_n(t) = h_n e^{j\phi_n}\delta(t), \tag{10}$$

where h_n is the envelope and ϕ_n is the phase of the complex channel for the n^{th} symbol. Assuming that the receiver is able to perfectly track the phase of the channel, the detector in the receiver observes the signal:

$$y(t) = \sum_{n=-\infty}^{\infty}\sum_{k=1}^{K} A^k b_n^k s_n^k(t) h_n + \eta(t), \tag{11}$$

where K represents the number of users, $s_n^k(t)$ is the rectangular signature waveform with random signature sequence of the k^{th} user defined in $(n-1)T_b \leq t \leq nT_b$, T_b and T_c are the bit period and the chip interval, respectively, related by $N_c = T_b/T_c$ (chip sequence length), $\{b_n^k\}$ is the input bit stream of the k^{th} user ($\{b_n^k\} \in \{-1, +1\}$), h_n is the n^{th} channel tap which introduces flat fading ($h_n = 1$ for AWGN channel), A^k is the transmitted amplitude of the k^{th} user and $\eta(t)$ is the additive white Gaussian noise with zero mean and variance σ_η^2. The cross-correlation between the signature sequences of users j and k for the n^{th} symbol is $\rho_n^{k,j} = \int_{(n-1)T_b}^{nT_b} s_n^k(t) s_n^j(t)dt = \sum_{l=1}^{N_c} c_{n,l}^k c_{n,l}^j$, where $\{c_{n,l}^k\}$ is the normalized spreading sequence (so that the autocorrelations of the signature sequences are unity) of user k for the n^{th} symbol.

The receiver consists of a matched filter at the front end which is matched to the signature waveform of the desired user. In our analysis, the desired user will be user 1. Thus, the matched filter's output for the n^{th} symbol can be written as follows:

$$r_n = A^1 b_n^1 h_n + \sum_{k=2}^{K} A^k b_n^k \rho_n^{k,1} h_n + \eta_n. \tag{12}$$

The second term in the above equation is called MAI, i.e., $m_n = \sum_{k=2}^{K} A^k b_n^k \rho_n^{k,1} h_n$. It can be seen that the cross-correlation $\rho_n^{k,1}$ is in the range $[-1, +1]$ and can be set up to the following relation :

$$\rho_n^{k,1} = (N_c - 2d)/N_c, \quad d = 0, 1, \cdots, N_c, \tag{13}$$

where d is a binomial random variable with equal probability of success and failure, thus, its mean and variance are $E[d] = \frac{1}{2}N_c$ and $\sigma_d^2 = \frac{1}{4}N_c$, respectively. Since the channel taps are independent from the spreading sequences and the data sequences, therefore the interferer's components, $A^k b_n^k \rho_n^{k,1} h_n, \forall k \neq 1$, are independent of each other and have zero mean. Thus, the variance of MAI, σ_m^2, with equal transmitted powers can be shown to be:

$$\sigma_m^2 = \frac{A^2(K-1)}{N_c}\, E[h_n^2], \tag{14}$$

where $E[h_n^2]$ is the second moment of h_n.

2.2. The MNCLMS adaptive algorithm

Ultimately, based on the above analysis, the proposed MNCLMS algorithm defined by (7)-(9), is now modified to incorporate the changes, especially the variance of MAI to finally look like:

$$\lambda_{n+1} = \lambda_n + \beta\Big[\frac{1}{2}\Big(e_n^2 - \frac{A^2(K-1)E[h_n^2]}{N_c} - \sigma_\eta^2\Big) - \lambda_n\Big], \tag{15}$$

and the input vector $\mathbf{x}_n$ in (2) is $[A^1 b_n^1\ A^1 b_{n-1}^1 \cdots\ A^1 b_{n-N+1}^1]^T$. It can be seen from (15) that in the absence of noise, $\sigma_\eta^2 = 0$, the MAI constrained LMS (MCLMS) algorithm is obtained. Moreover, we can recover the noise constrained LMS (NCLMS) and the zero noise constrained LMS (ZNCLMS) algorithms [1] by substituting $K = 1$ (single user scenario) and $K = 1$ with $\sigma_\eta^2 = 0$, respectively. Thus, the MNCLMS algorithm can be considered as a generalized constrained adaptive algorithm that includes the MCLMS, the NCLMS and the ZNCLMS algorithms as special cases.

3. CONVERGENCE ANALYSIS OF THE MNCLMS ALGORITHM

In this section, we consider a multiuser system scenario and carry out the convergence analysis of the proposed MNCLMS algorithm in the presence of both MAI and additive Gaussian noise.

The Gaussian approximation for MAI is well known and has been used in various forms [3, 4, 5]. Since MAI has no dependency on the noise process, consequently MAI plus noise (z_n) is also independent of the input process $\{\mathbf{x}_n\}$.

Note that since α_n and $\mathbf{w}_n$ are functions of $\{x_k, \eta_k : k \leq n\}$, they will, in general, be dependent. However, when parameters are chosen so that the steady-state variance of α_n and/or $\mathbf{w}_n$ is small, then for any fixed time n, the step-size α_n and the weight vector $\mathbf{w}_n$ are statistically independent.

Also, since $\mathbf{x}_k$ and $\mathbf{x}_n$ are uncorrelated for $n \neq k$, they are independent as well. Thus, it can be shown that input sequence $\mathbf{x}_n$ and the weight error vector $\mathbf{u}_n$, that will be defined later, are also independent.

3.1. Convergence in the Mean

The weight update equation for the proposed algorithm is given by (7). If $\mathbf{w}_{opt}$ is the optimum value of the weight according to the Wiener solution, i.e., exact solution for the parameters of the actual system, then we can define the weight error vector, $\mathbf{u}_n$, as follows:

$$\mathbf{u}_n = \mathbf{w}_n - \mathbf{w}_{opt}. \tag{16}$$

Thus, using the above relation it can be shown that in the system identification scenario e_n can be setup into the following relation:

$$e_n = z_n - \mathbf{u}_n^T \mathbf{x}_n. \tag{17}$$

Now subtracting $\mathbf{w}_{opt}$ from both sides of the Equation (7) and taking the expectation by using the aforementioned assumptions, we get:

$$\overline{\mathbf{u}}_{n+1} = (\mathbf{I} - \overline{\alpha}_n \mathbf{R})\overline{\mathbf{u}}_n, \tag{18}$$

where $\mathbf{R} = E[\mathbf{x}_n \mathbf{x}_n^T]$ is the correlation matrix of the input process, $\overline{\mathbf{u}}_n = E[\mathbf{u}_n]$ is the mean weight error vector and $\overline{\alpha}_n = E[\alpha_n]$ is the mean step-size.

Similarly, if we define the mean lagrangian multiplier as $\overline{\lambda}_n = E[\lambda_n]$, it can be shown that:

$$\overline{\alpha}_n = \alpha(1+\gamma\overline{\lambda}_n), \quad (19)$$

$$\text{and} \quad \overline{\lambda}_{n+1} = (1-\beta)\overline{\lambda}_n + \frac{\beta}{2}\epsilon_n, \quad (20)$$

where $\epsilon_n = E[e_n^2] - \sigma_z^2$ is the excess mean square error (EMSE) at time n.

If $\lambda_1^{'}, \lambda_2^{'}, \cdots, \lambda_N^{'}$ represent the eigenvalues of the input correlation matrix $\mathbf{R}$, the necessary condition for convergence in the mean is represented by:

$$\left|1 - \overline{\alpha}_n \lambda_k^{'}\right| < 1, \quad \forall\, k. \quad (21)$$

Thus, the value of $\overline{\alpha}_n$ is bounded in the range:

$$0 < \overline{\alpha}_n < \frac{2}{\lambda_{max}^{'}},$$

where $\lambda_{max}^{'}$ is the maximum eigenvalue of the input correlation matrix $\mathbf{R}$. Using the approach of [6], a strong but simpler sufficient condition for convergence of the mean weight error vector can be written as:

$$\alpha_{max} < \frac{2}{\lambda_{max}^{'}}.$$

Now, defining $\mathbf{K}_n = E[\mathbf{v}_n\mathbf{v}_n^T]$ as weight error vector correlation matrix, the following recursion can be obtained:

$$\mathbf{K}_{n+1} = \mathbf{K}_n - \overline{\alpha}_n\Big(\mathbf{R}\mathbf{K}_n + \mathbf{K}_n\mathbf{R}\Big) + \overline{\alpha_n^2}\Big(2\mathbf{R}\mathbf{K}_n\mathbf{R} + \mathbf{R}\epsilon_n + \sigma_z^2\mathbf{R}\Big), \quad (22)$$

where we have used the fact that $\epsilon_n = Tr(\mathbf{R}\mathbf{K}_n)$.

Similarly, if $\overline{\alpha_n^2} = E[\alpha_n^2]$ and $\overline{\lambda_n^2} = E[\lambda_n^2]$, then using *Gaussian Factoring Moment Theorem* [2], it can be shown that

$$\overline{\alpha_n^2} = \overline{\alpha^2}(1 + 2\gamma\overline{\lambda}_n + \gamma^2\overline{\lambda_n^2}) \quad (23)$$

$$\text{and} \quad \overline{\lambda_{n+1}^2} = (1-\beta)^2\bar{\lambda_n^2} + \beta(1-\beta)\overline{\lambda}_n\epsilon_n + \frac{\beta^2}{4}\Big(3\epsilon_n^2 + 6Tr\big(\mathbf{R}\mathbf{K}_n\mathbf{R}\mathbf{K}_n\big) - 6\big(\overline{\mathbf{v}}_n^T\mathbf{R}\overline{\mathbf{v}}_n\big)^2 + 4\sigma_z^2\epsilon_n + 2\sigma_z^4\Big). \quad (24)$$

3.2. Steady-State Performance

Now, considering the steady-state scenario, i.e., when $n \to \infty$. Let $\overline{\alpha}_s$, $\overline{\lambda}_s$, $\overline{\alpha_s^2}$, and $\overline{\lambda_s^2}$ denote the steady-state values of $\overline{\alpha}_n$, $\overline{\lambda}_n$, $\overline{\alpha_n^2}$, and $\overline{\lambda_n^2}$, respectively. Knowing the fact that at $\mathbf{v}_s = 0$, and using equations (19), (20), (24), and (24), following steady-state values are obtained:

$$\overline{\alpha}_s = \alpha(1+\gamma\epsilon_s/2), \quad (25)$$

$$\overline{\lambda}_s = \frac{\epsilon_s}{2}, \quad (26)$$

$$\overline{\alpha_s^2} = \alpha^2(1+2\gamma\overline{\lambda}_s + \gamma^2\overline{\lambda_s^2}), \quad (27)$$

$$\overline{\lambda_s^2} = \frac{1}{(2-\beta)}\left[\Big(\frac{1}{2}+\frac{\beta}{4}\Big)\epsilon_s^2 + \frac{3}{2}\beta Tr\big(\mathbf{R}\mathbf{K}_s\mathbf{R}\mathbf{K}_s\big) + \beta\sigma_z^2\epsilon_s + \frac{\beta}{2}\sigma_z^4\right]. \quad (28)$$

Now, taking trace of both sides of (22) at steady-state, following recursion can be obtained:

$$2\overline{\alpha}_s\epsilon_s = \overline{\alpha_s^2}\Big[2Tr\big(\mathbf{R}\mathbf{K}_s\mathbf{R}\big) + \epsilon_s Tr(\mathbf{R}) + \sigma_z^2 Tr(\mathbf{R})\Big] \quad (29)$$

Using unitary transformation $\mathbf{R} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H$ with $\mathbf{Q}$ has a set of orthogonal set of eigenvectors associated with the eigenvalues of matrix $\mathbf{R}$ and $\mathbf{\Lambda}$ is a diagonal matrix having eigenvalues of the matrix $\mathbf{R}$. Thus, by applying this unitary transformation, the following relations can be obtained:

$$Tr\big(\mathbf{R}\mathbf{K}_s\mathbf{R}\mathbf{K}_s\big) = k_1\epsilon_s^2, \quad (1/N) \le k_1 \le 1, \quad (30)$$

and

$$Tr\big(\mathbf{R}\mathbf{K}_s\mathbf{R}\big) = k_2 Tr(\mathbf{\Lambda})\epsilon_s, \quad 0 \le k_2 \le 1. \quad (31)$$

Now, substituting the values of $\overline{\alpha}_s$, $\overline{\lambda}_s$, $\overline{\alpha_s^2}$ and $\overline{\lambda_s^2}$ in (29), following quadratic equation in ϵ_s is obtained:

$$A\epsilon_s^3 + B\epsilon_s^2 + C\epsilon_s + D = 0 \quad (32)$$

where

$$A = -\alpha Tr(\Lambda)\left\{\frac{\gamma^2\Big(2k_2+1\Big)}{(2-\beta)}\left[\frac{1}{2} + (1+6k_1)\frac{\beta}{4}\right]\right\}, \quad (33)$$

$$B = \gamma - \alpha Tr(\Lambda)\left\{\frac{\gamma^2\sigma_z^2}{(2-\beta)}\left[\frac{1}{2} + (1+6k_1)\frac{\beta}{4}\right] + \Big(\gamma + \frac{\gamma^2\beta\sigma_z^2}{(2-\beta)}\Big)\Big(2k_2+1\Big)\right\}, \quad (34)$$

$$C = 2 - \alpha Tr(\Lambda)\left[\left(1 + \frac{\gamma^2\beta\sigma_z^4}{2(2-\beta)}\right)\Big(2k_2+1\Big) + \left(\gamma + \frac{\gamma^2\beta\sigma_z^2}{(2-\beta)}\right)\sigma_z^2\right], \quad (35)$$

$$\text{and} \quad D = -\alpha Tr(\Lambda)\sigma_z^2\left(1 + \frac{\gamma^2\beta\sigma_z^4}{2(2-\beta)}\right). \quad (36)$$

Assuming $\alpha Tr(\Lambda) << 1$, which is a well known approximation for the steady-state MSE of LMS algorithm [7], it can be shown that steady-state value of ϵ_s is close to zero. Thus, higher powers ϵ_s can be ignored. Hence, the asymptotic expression for the steady-state EMSE of MNCLMS algorithm can be shown to be:

$$\epsilon_{MNCLMS} \approx \frac{\alpha Tr(\Lambda)\sigma_z^2}{2}\left[1 + \frac{\gamma^2\beta\sigma_z^4}{2(2-\beta)}\right]. \quad (37)$$

4. SIMULATION RESULTS

In this section, the performance analysis of the LMS, the NCLMS, the ZNCLMS, the MCLMS and the MNCLMS algorithms is investigated where an adaptive interference cancellation scenario in a synchronous CDMA multiuser system for a downlink scenario is considered in AWGN and flat Rayleigh fading environments. Random signature sequences of length 31 and rectangular chip waveforms are used. One scenario of 4 users with equal transmitted powers is considered. The signal-to-noise ratio (SNR) is set at 20 dB.

The comparison of the convergence speed of these algorithms for 4 users in an AWGN channel is depicted in Fig. 1. It can be seen from this figure that the MNCLMS algorithm was able to achieve an MSE of around -10 dB at around 150 iterations while the first of the other algorithms converged at this same MSE value after only 300 iterations, hence a two-fold gain in convergence speed. The behavior of the step-size of the proposed algorithm is depicted in Fig. 2 for 4 users under equal transmitted power scenario. This figure shows that initially, in the transient state, the MNCLMS algorithm has the largest step-size value when compared to the other algorithms and thus yields the fastest convergence. Also, in the steady-state, the step-size parameter of the MNCLMS algorithm was reduced to the smallest value amongst all algorithms.

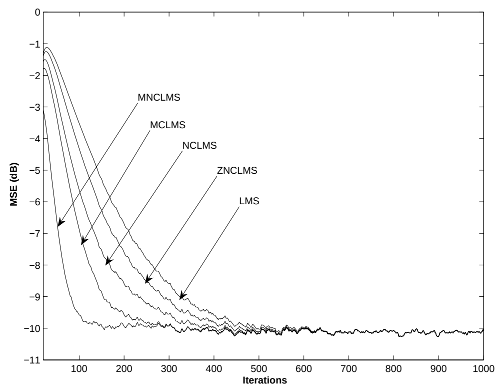

Fig. 1. MSE behaviour for different algorithms in an AWGN Environment under equal transmitted power scenario.

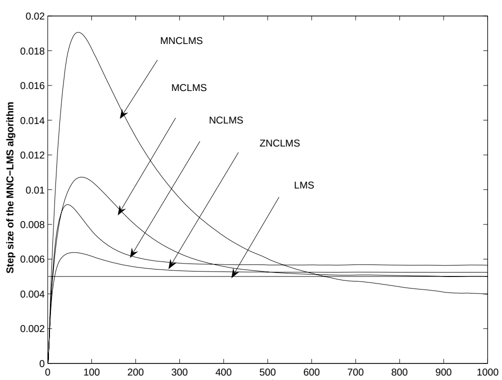

Fig. 2. Behavior of time varying Step size of the MNCLMS algorithm.

The effect of a sudden increase in the number of users on the performance behaviour of these algorithms is reported in Fig. 3. It can be seen from this figure that the proposed algorithm is still able to recover faster than the rest of the algorithms as the number of users is increased from 4 to 10. Therefore, a consistency in the performance of the proposed algorithm is maintained.

In the case of unequal transmitted powers, the user of interest, user one here in this scenario, has a transmitted power equals to

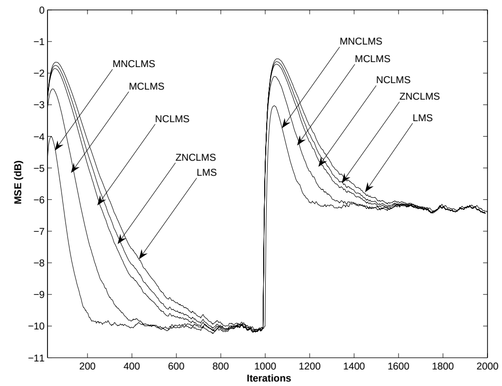

Fig. 3. Effect of a sudden increase in number of users from 4users to 10 users.

one; while the rest of the users, their transmitted powers are uniformly distributed between zero and one. Fig. 4 shows the comparison of the convergence speed for the algorithms under consideration. As depicted in this figure, a consistency in performance of the proposed algorithm is observed. The proposed algorithm was able to achieve an MSE of around -16 dB at around 170 iterations while the first of the other algorithms converged at the same MSE value after only 320 iterations.

The result for unequal transmitted powers, Fig. 4, is better than that of the equal transmitted powers, Fig. 1. The reason is that in the unequal transmitted powers scenario, some users may have transmitted powers less than one (because of uniform power assignment between zero and one) which decreases the effect of MAI in the system. However, in the case of equal transmitted powers all users have equal transmitted powers (one) which obviously increases MAI in the system.

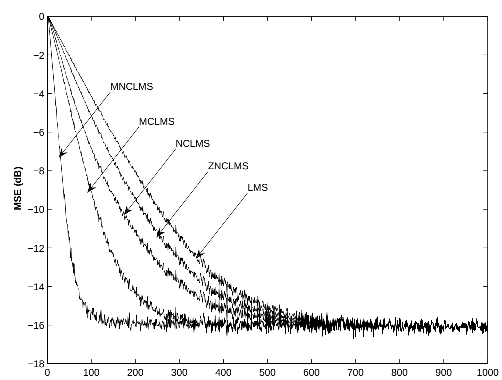

Fig. 4. MSE behaviour for different algorithms in an AWGN Environment under unequal transmitted power scenario.

Figure 5 compares the BER performance of the proposed algorithm with the LMS algorithm for 4 users in an AWGN environment. As expected the proposed algorithm maintains its superiority over the LMS algorithm. The proposed algorithm attains lower error floor than that of the LMS algorithm. As can be depicted from this figure, the LMS algorithm saturates around an error floor of approximately 8×10^{-3} after an SNR greater than 17 dB. While the proposed algorithm saturates around an error floor of approximately 2.5×10^{-4} after an SNR greater than 27 dB. An improvement with the proposed algorithm of approximately 6 dB over the LMS algorithm at a BER of 9×10^{-3} has been clearly achieved.

Figure 6 depicts the analytical result of the MNCLMS algorithm when compared to the experimental one for 4 users. As can be seen from this figure that the analytical result matches quite well the experimental one.

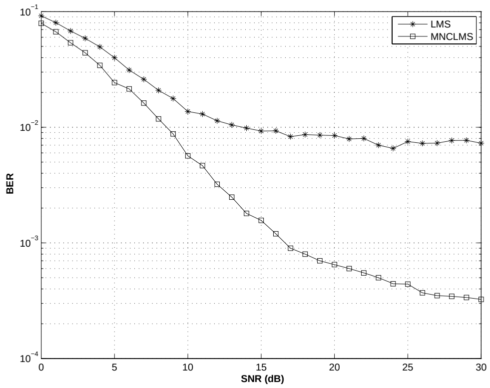

Fig. 5. BER performance of the proposed algorithm with the LMS algorithm.

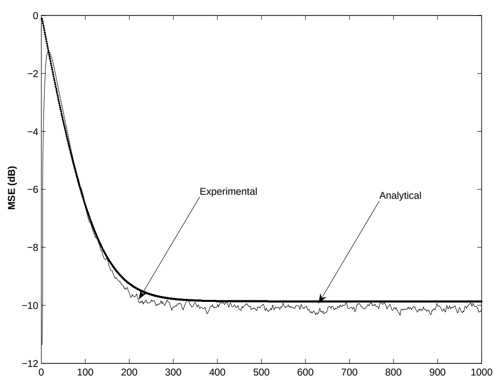

Fig. 6. Analytical and experimental results of the MNCLMS.

Finally, the convergence speed of the proposed algorithm is investigated in flat Rayleigh fading environment. A fast-fading scenario with a doppler frequency (f_d) of 150 Hz and a data rate of 3.84 Mcps is considered here. Figure 7 shows the MSE learning curves for the LMS, the NCLMS, the ZNCLMS, the MCLMS and the MNCLMS algorithms for 4 users in a Rayleigh fading environment. It can be seen from the figure that the MNCLMS algorithm converges faster than the others, thus showing a great performance improvement in terms of convergence rate. Consistency in performance for the MNCLMS algorithm is observed here.

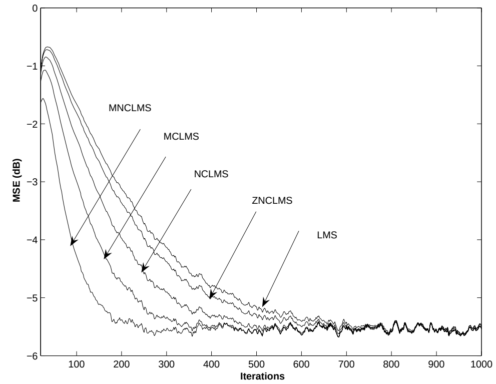

Fig. 7. MSE in flat Rayleigh fading, f_d=150 Hz.

5. CONCLUSION

In this work, we proposed a new constrained LMS-type algorithm (MNCLMS) for multiuser wireless environments and studied its performance both analytically and by simulations. The MNCLMS algorithm can be considered as a generalized constrained adaptive algorithm that includes the MCLMS, the NCLMS and the ZNCLMS algorithms as special cases. Our study included a thorough comparison of the proposed MNCLMS algorithm with a number of other well-established algorithms and showed that, overall, the MNCLMS enjoys a superior performance. Finally, similar behavior is obtained for the case of 20 users.

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