

# ANGLE-OF-ARRIVAL ESTIMATION FOR LOCALIZATION AND COMMUNICATION IN WIRELESS NETWORKS

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## ABSTRACT

*As one of the major methods for location positioning, angle-of-arrival (AOA) measurement is a significant technology in wireless sensor networks (WSN), which can be explored for node and target localization, improving communication quality, location-based routing, sensor management, and other diverse applications. Due to the size, cost and energy constraints typical of a sensor node and the sophisticated resident environment, popular high resolution AOA estimation methods may malfunction and perform poorly. In this paper, we propose an algorithm for bearing estimation in unknown noise fields and harsh WSN scenarios. By modeling the noise covariance as a linear combination of known weighting matrices, a maximum likelihood (ML) criterion is established. And a particle swarm optimization (PSO) paradigm is presented for optimization of the ML cost function. Simulation results demonstrate that the paired estimator PSO-ML significantly outperforms other popular techniques and produces superior bearing estimates, especially in unfavourable scenarios typical of a sensor network.*

## 1. INTRODUCTION

Wireless sensor networks (WSNs) have drawn increasing attention due to their potential utilization in a variety of civilian and military domains [1]. As one of the major methods for location positioning, angle-of-arrival (AOA) measurement is an important technology of great practical interest in WSN. Node self-localization capability is a highly desirable characteristic for a sensor network, because the sensor readings are meaningless without a coordinate tag and the standard positioning techniques such as global positioning system (GPS) are often prohibitive due to cost, energy and environment constraints. Moreover, localization of targets of interest is usually a central task in sensor networks.

Locations of nodes or objects are mainly derived from two types of measurements: AOA and distance measured from the reference points. The widely used distance estimation models include received signal strength (RSS), time of arrival (TOA) and time difference of arrival (TDOA), where cooperation between the transmitter and receiver is mandatory, e.g., the RSS model requires the receiver to know the signal strength at the source, and TOA and TDOA models require accurate time synchronization between transmitters and receivers [2]. On the contrary, the AOA model potentially localizes targets in a non-cooperative, stealthy, and

passive manner, which is highly desirable in many sensor network applications.

Location derivation using bearing-only measurements [3]-[4] and bearing-distance hybrid measurements [5]-[7] has been an active research area. Yip et al [4] discuss localization and classification of multiple targets in WSN using AOA measurements. Ash and Potter [5] present a node localization scheme in WSN using AOA and RSS measurements and demonstrate that sub-meter location accuracy is achievable. Gu and Gunawan [6] consider mobile user localization in a cellular network using AOA and TOA measurements. Li and Zhuang [7] address mobile user localization using TDOA and AOA measurements in a macro-cell system.

Besides position location, AOA measurements potentially improve the communication quality in terms of lower bit error rate (BER), larger range coverage, reduced multipath fading, and lower power consumption by directional transmission and interference suppression [8], and deliver diverse network level benefits. In [9], a sensor management algorithm using AOA at the central nodes is proposed to save energy consumption and prolong the network lifetime. In [10], the bearings of the sink are used to route the local transmission for each node to improve efficiency and reduce energy consumption.

One of the major criticisms of using AOA measurements in WSN is that AOA capability is usually achieved using an array of sensors such as antennas, microphones, and ultrasonic sensors, and arrays have for long been considered unsuitable for integration in sensor nodes due to size and cost constraints. However, as radio communications move to higher frequencies, antenna dimensions shrink, and micro-electro-mechanical systems (MEMS) advance, the integration becomes feasible and array-enabled node platforms have been built [8], [10]-[11]. A platform equipped with a four-element antenna array is reported in [11]. Kalis et al [10] design a platform integrating multiple antennas in the same dimensions as the counterpart with single antenna, and increase the total cost by only 3%.

On the other hand, the array may be formed in a distributed manner by sensors on different nodes if the clocks are accurately synchronized. The node collaboration for formation of wireless antenna arrays is presented from a statistical viewpoint in [12]. AOA estimation of a single acoustic source using four distributed Berkeley Mica2 motes is considered in [13].

A variety of applications [3], [5]-[7], [9]-[11] under the assumption that nodes have the AOA capability have been developed. However, AOA estimation in WSN scenarios is not a trivial problem, but the research is relatively scarce. In recent years, AOA estimation has received extensive attention from radar and communication communities, and several high resolution algorithms have been proposed based on the white Gaussian noise model, such as MUSIC [14], maximum likelihood (ML) [15], and others [16]-[17].

Nevertheless, these algorithms are not directly applicable to WSN-specific scenarios. The sensor nodes usually reside on the ground in a “radio hostile” environment due to the presence of multipath, reflections and scattering of the signal; and the noise fields tend to be spatially correlated due to the dominant ambient noise. On the other hand, sensor nodes are often forced to work under unfavourable conditions involving low signal-to-noise ratio (SNR), short data samples, highly correlated signals, small array with few elements due to the inherent constraints on the cost, energy and size.

In this paper, we propose an algorithm for accurate narrowband AOA estimation in unknown noise fields and harsh WSN scenarios. By modelling the noise covariance as a linear combination of known weighting matrices, a maximum likelihood criterion is derived with respect to AOA and unknown noise parameters. ML criteria may yield superior statistical performance, but the cost function is multimodal, nonlinear and high-dimensional. To tackle it accurately and efficiently, we propose to use the particle swarm optimization (PSO) paradigm as a robust and fast global search tool. PSO is a recent addition to evolutionary algorithms first introduced by Eberhart and Kennedy [18]. Most of the applications demonstrated that PSO could give competitive or even better results in a much faster and cheaper way, compared to other heuristic methods such as genetic algorithms (GA) [20].

The PSO is a combination of the problem-independent kernel and some newly introduced problem-specific features, which make the algorithm highly flexible while being specific and effective in the current application. The pairing of PSO and ML is feasible in terms of computing cost and energy, because i) fast but not real-time AOA measurement is usually allowed, e.g., if the nodes are static after deployment, the inter-node bearings are merely measured during network initialization; ii) In this approach, processing of the data is performed locally within each node and no global data communication is needed. In WSN, communication consumes much more energy than computation, e.g., the energy required for transmitting a single bit could be used to execute 1000-2000 instructions [2]. Furthermore, the optimization can be implemented in a distributed and parallel manner to enhance the computation. Via extensive numerical studies, we demonstrate that the proposed algorithm yields superior performance over other popular methods, especially in unfavourable scenarios involving low SNR, highly correlated signals, short data samples, and small arrays, which are typical conditions of sensor nodes.

## 2. DATA MODEL AND PROBLEM FORMULATION

We consider an array of  $M$  elements arranged in an arbitrary geometry on a sensor platform and  $N$  narrowband far-field sources at unknown locations. The complex  $M$ -vector of array outputs is modelled by the standard equation

$$\mathbf{y}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{n}(t), \quad t = 1, 2, \dots, L \quad (1)$$

where  $\boldsymbol{\theta} = [\theta_1, \dots, \theta_N]^T$  is the source AOA vector, and the  $k$ th column of the complex  $M \times N$  matrix  $\mathbf{A}(\boldsymbol{\theta})$  is the so called steering vector  $\mathbf{a}(\theta_k)$  for the angle  $\theta_k$ . The  $i$ th element  $a_i(\theta_k)$  models the gain and phase adjustments of the  $k$ th signal at the  $i$ th element. Furthermore, the complex  $N$ -vector  $\mathbf{s}(t)$  is composed of the emitter signals, and  $\mathbf{n}(t)$  models the additive noise.

The vectors of signals and noise are assumed to be stationary, temporally white, zero-mean complex Gaussian random processes with second-order moments given by

$$\begin{aligned} E\{\mathbf{s}(t)\mathbf{s}^H(s)\} &= \mathbf{P}\delta_{ts} \\ E\{\mathbf{s}(t)\mathbf{s}^T(s)\} &= 0 \\ E\{\mathbf{n}(t)\mathbf{n}^H(s)\} &= \mathbf{Q}\delta_{ts} \\ E\{\mathbf{n}(t)\mathbf{n}^T(s)\} &= 0 \end{aligned} \quad (2)$$

where  $\delta_{ts}$  is the Kronecker delta,  $(\cdot)^H$  denotes complex conjugate transpose,  $(\cdot)^T$  denotes transpose, and  $E[\cdot]$  stands for expectation. Assuming that the noise and signals are independent, the data covariance matrix is given by

$$\mathbf{R} = E\{\mathbf{y}(t)\mathbf{y}^H(t)\} = \mathbf{A}\mathbf{P}\mathbf{A}^H + \mathbf{Q}. \quad (3)$$

The problem addressed herein is the estimation of  $\boldsymbol{\theta}$  (and if necessary, along with the parameters in  $\mathbf{P}$  and  $\mathbf{Q}$ ), from a batch of  $L$  measurements  $\mathbf{y}(1), \dots, \mathbf{y}(L)$ .

Under the assumption of additive Gaussian noise and Gaussian distributed signals, the normalized (with  $L$ ) negative log-likelihood function of the data vectors takes the form (ignoring the parameter independent terms) [19]

$$I(\boldsymbol{\theta}, \mathbf{P}, \mathbf{Q}) = \log|\mathbf{R}| + \text{tr}\{\mathbf{R}^{-1}\hat{\mathbf{R}}\}, \quad (4)$$

where  $\text{tr}\{\cdot\}$  stands for trace,  $\log|\cdot|$  denotes the natural logarithm of the determinant, and  $\hat{\mathbf{R}}$  is the covariance matrix of the measured data

$$\hat{\mathbf{R}} = \frac{1}{L} \sum_{t=1}^L \mathbf{y}(t)\mathbf{y}^H(t). \quad (5)$$

In the follows, we focus on the ML criterion derived using parameterization of the noise covariance, because this assumption applies no constraints to the signals, thus both cooperative and non-cooperative scenarios can be modelled.

Based on a Fourier series expansion of the spatial noise power density function, the noise covariance  $\mathbf{Q}$  is assumed to be modelled by the following linear parameterization [19]:

$$\mathbf{Q}(\boldsymbol{\eta}) = \sum_{j=1}^J \eta_j \boldsymbol{\Sigma}_j \quad (6)$$

where  $\boldsymbol{\eta} = [\eta_1, \dots, \eta_J]^T$  is a vector of unknown noise Fourier coefficients,  $\boldsymbol{\Sigma}_j$  is a known function of the array geometry given by

$$\boldsymbol{\Sigma}_j = \begin{cases} \bar{\boldsymbol{\Sigma}}_{(j-1)/2} & j \text{ odd} \\ \tilde{\boldsymbol{\Sigma}}_{j/2} & j \text{ even} \end{cases} \quad (7)$$

where

$$\begin{aligned} \bar{\boldsymbol{\Sigma}}_l &= \int_{-\pi}^{\pi} \mathbf{a}(\theta) \mathbf{a}^H(\theta) \cos(l\theta) d\theta \\ \tilde{\boldsymbol{\Sigma}}_l &= \int_{-\pi}^{\pi} \mathbf{a}(\theta) \mathbf{a}^H(\theta) \sin(l\theta) d\theta \end{aligned} \quad (8)$$

$l = 0, 1, 2, \dots$ . It is assumed that  $J$  is known or has been estimated.

By solving for  $\mathbf{P}$  in terms of  $\boldsymbol{\theta}$  and  $\mathbf{Q}(\boldsymbol{\eta})$  and substituting back to (4), we get the equation (9) [19], and the ML estimates of  $\boldsymbol{\theta}$  and  $\boldsymbol{\eta}$  are obtained by minimizing (9).

$$I_1(\boldsymbol{\theta}, \boldsymbol{\eta}) = \log|\mathbf{Q}| + \log|\mathbf{G}\bar{\mathbf{R}}\mathbf{G} + \mathbf{H}| + \text{tr}\{\mathbf{H}\bar{\mathbf{R}}\}, \quad (9)$$

where

$$\begin{aligned} \bar{\mathbf{A}} &= \mathbf{Q}^{-1/2} \mathbf{A} \\ \mathbf{G} &= \bar{\mathbf{A}} (\bar{\mathbf{A}}^H \bar{\mathbf{A}})^{-1} \bar{\mathbf{A}}^H \\ \bar{\mathbf{R}} &= \mathbf{Q}^{-1/2} \hat{\mathbf{R}} \mathbf{Q}^{-1/2} \\ \mathbf{H} &= \mathbf{I} - \mathbf{G} \end{aligned} \quad (10)$$

### 3. PSO-ML AOA ESTIMATION AND PARAMETER SELECTION

Particle swarm optimization is a stochastic optimization paradigm, which mimics animal social behaviours such as flocking of birds and the methods by which they find roosting places or food sources [18]. As illustrated in Fig. 1, the algorithm starts by initializing a population of particles in the “normalized” search space with random positions  $\mathbf{x}$  and random velocities  $\mathbf{v}$ , which are constrained between zero and one in each dimension. The position vector of the  $i$ th particle takes the form  $\mathbf{x}_i = [\tilde{\theta}_1, \dots, \tilde{\theta}_N, \tilde{\eta}_1, \dots, \tilde{\eta}_J]$ , where  $0 \leq \tilde{\theta}_n, \tilde{\eta}_j < 1$ ,  $n = 1, \dots, N$ ,  $j = 1, \dots, J$ ,  $N \geq 1$ ,  $J \geq 1$ . A particle position vector is converted to a candidate solution vector in the problem space through a mapping. The score of the mapped vector evaluated by the likelihood function  $I_1(\boldsymbol{\theta}, \boldsymbol{\eta})$  (9) is regarded as the fitness of the corresponding particle. The  $i$ th particle’s velocity is updated according to (11)

$$\mathbf{v}_i^{k+1} = \omega^k \mathbf{v}_i^k + c_1 \mathbf{r}_1^k \odot (\mathbf{p}_i^k - \mathbf{x}_i^k) + c_2 \mathbf{r}_2^k \odot (\mathbf{p}_g^k - \mathbf{x}_i^k), \quad (11)$$

where  $p_i$  is the best previous position of the  $i$ th particle,  $p_g$  is the best position found by any particle in the swarm,  $\odot$  denotes element-wise product,  $k = 1, 2, \dots$ , indicates the iterations,  $\omega$  is a parameter called the inertia weight,  $c_1$  and  $c_2$  are positive constants referred to as cognitive and social

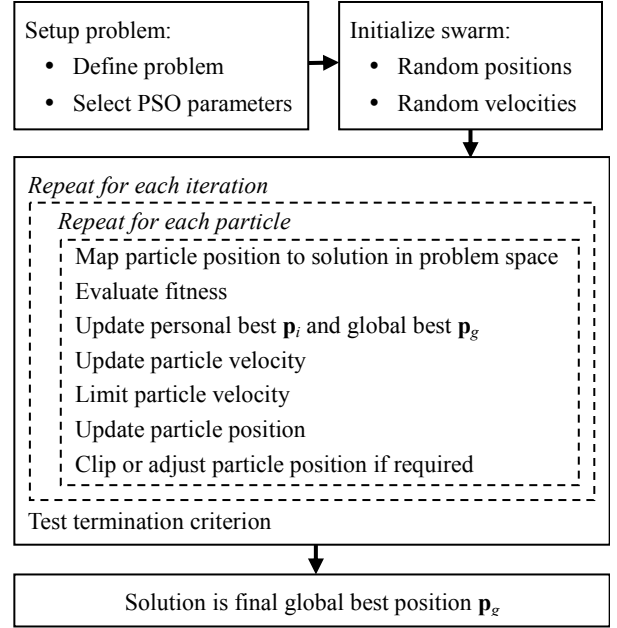


Fig. 1. Flowchart illustrating the main steps of PSO-ML technique.

parameters respectively,  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are independent random vectors.

Three components typically contribute to the new velocity. The first part refers to the inertial effect of the movement. The inertial weight  $\omega$  is considered critical for the convergence behaviour of PSO [21]. A larger  $\omega$  facilitates searching new area and global exploration while a smaller  $\omega$  tends to facilitate fine exploitation in the current search area. In this study,  $\omega$  is selected to decrease during the optimization process, thus PSO tends to have more global search ability at the beginning while having more local search ability near the end. Given a maximum value  $\omega_{\max}$  and a minimum value  $\omega_{\min}$ ,  $\omega$  is updated as follows:

$$\omega^k = \begin{cases} \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{rK} (k-1), & 1 \leq k \leq [rK] \\ \omega_{\min}, & [rK] + 1 \leq k \leq K \end{cases} \quad (12)$$

where  $[rK]$  is the number of iterations with time decreasing inertial weights,  $0 < r < 1$  is a ratio, and  $K$  is the maximum iteration number. Based on empirical practice and extensive test runs, we select  $\omega_{\max} = 0.9$ ,  $\omega_{\min} = 0.4$ , and  $r = 0.4 \sim 0.8$ . The second and third components introduce stochastic tendencies to return towards the particle’s own best historical position and the group’s best historical position. Constants  $c_1$  and  $c_2$  are used to bias the particle’s search towards the two locations. Following common practice in the literature,  $c_1 = c_2 = 2$ , although these values could be fine-tuned for the problem at hand.

Since there was no actual mechanism for controlling the velocity of a particle, it is necessary to define a maximum velocity to avoid the danger of swarm explosion and divergence [22]. The velocity limit is applied to  $\mathbf{v}_i$  along each dimension separately by

TABLE I  
THE SELECTED PSO PARAMETERS

Parameter	Value
$c_1$	2.0
$c_2$	2.0
$P$	20
$K$	200
$V_{MAX}$	0.5
$\omega_{max}$	0.9
$\omega_{min}$	0.4
$r$	0.5

$$v_{id} = \begin{cases} V_{MAX}, & v_{id} > V_{MAX} \\ -V_{MAX}, & v_{id} < -V_{MAX} \end{cases} \quad (13)$$

where  $d=1, \dots, N+J$ . Like the inertial weight, large values of  $V_{MAX}$  encourage global search while small values enhance local search. In this study,  $V_{MAX}$  is held constant at 0.5, the half dynamic range, throughout the optimization.

The new particle position is calculated using (14),

$$\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^{k+1}. \quad (14)$$

If any dimension of the new position vector is less than zero or greater than one, it is clipped to stay within this range. It should be noted that, at any time of the optimization process, two components representing AOA in a position vector are not allowed to be equal.

The final global best position  $p_g$  is taken as the ML estimates of AOA and noise parameters. Some previous works demonstrate that the performance of PSO is not significantly affected by changing the swarm size  $P$ . The typical range of  $P$  is 20 to 50, which is sufficient for most problems to achieve good results. In addition, PSO is robust to control parameters; and the convergence and stability analysis is presented in [22].

#### 4. SIMULATION RESULTS

An example is presented to evaluate PSO-ML against MUSIC [14] and the unconditional maximum likelihood (UML) method [23] in WSN specific scenarios. MUSIC is one of the most popular techniques, while UML represents the best estimator in white Gaussian noise [15]. We consider the data model (1) with the noise covariance being a linear combination of known matrices as in (6).  $J=3$ , and the noise parameters are  $\boldsymbol{\eta} = [1, 1/4, 1/9]$ .

The selected PSO parameters are summarized in Table 1. The PSO algorithm starts with random initialization, and is terminated if the maximum iteration number  $K$  is reached or the global best particle position is not updated in 20 successive iterations. We have performed 500 Monte Carlo experiments for each point of the plot.

Assume that two equal-power correlated signals with the correlation factor  $\rho=0.95$ , impinge on a four-element uniform linear array (ULA) from  $90^\circ$  and  $95^\circ$ . The number of snapshots is 80. The situation is challenging, since the separation of emitters is about 0.19 beamwidth, the conventional resolu-

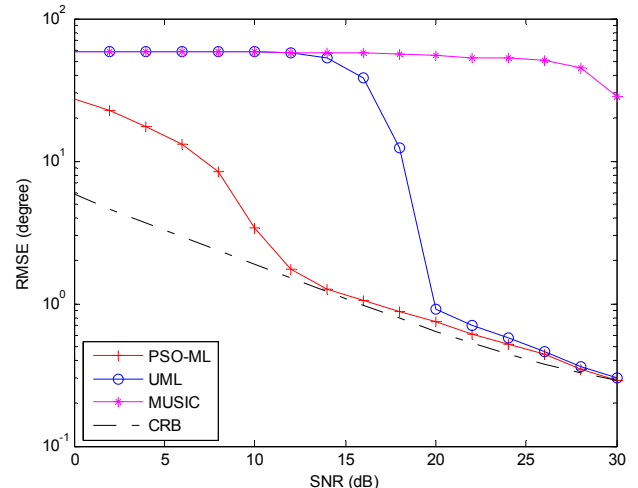


Fig. 2 AOA estimation RMSE values of PSO-ML, MUSIC and UML versus SNR.

tion limit. Fig. 2 depicts the combined AOA estimation root-mean-squared errors (RMSE) obtained using PSO-ML, MUSIC and UML as a function of SNR, and compares them with the corresponding Cramer-Rao bound (CRB) [19] (theoretically best performance). Fig. 3 shows the resolution probabilities for the same methods. Two sources are considered to be resolved in an experiment if both estimation errors are less than the half of their angular separation.

As can be seen from Fig. 2 and Fig. 3, PSO-ML yields significantly superior performance over MUSIC and UML as a whole, by demonstrating lower estimation RMSE and higher resolution probabilities. PSO-ML produces excellent estimates with RMSE approaching and asymptotically attaining the theoretic lower bound. On the other hand, as a standard high-resolution method, MUSIC fails almost in the whole SNR range. Although UML is an optimal technique in white Gaussian noise, it completely fails when SNR is lower than 15dB and only produces acceptable estimates in high SNR region. The results can be explained by the fact that existing parametric methods are sensitive to modelling errors. It is worth noting that the advantages of PSO-ML over the other methods are more prominent when SNR is low, and the benefits can be extended to other hard conditions.

#### 5. CONCLUSIONS

Arising from the requirements of sensor node and target localization, efficient communication by directional transmission and interference suppression, and exploration of angular diversity for various benefits such as location-based routing and sensor management, AOA measurement is an important technology of great practical interest in WSN. The existing high resolution algorithms may malfunction or perform poorly in sensor network specific scenarios due to the cost, energy and size constraints typical of a sensor node. In this paper, we propose an algorithm based on the maximum likelihood principle and implemented using the PSO paradigm. Simulation results demonstrate that PSO-ML significantly outperforms other popular techniques and produces

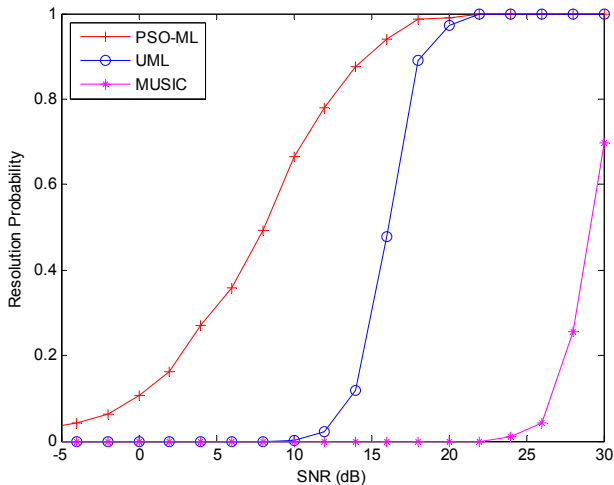


Fig. 3 Resolution probabilities of PSO-ML, MUSIC and UML versus SNR.

more accurate bearing estimates, especially in unfavourable scenarios.

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