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INNER-OUTER FACTORIZATION FOR RAYLEIGH FADING CHANNEL MODELING

Fernando Merchan, Flavius Turcu, Eric Grivel and Mohamed Najim

IMS- Département LAPS - UMR 5218 CNRS, ENSEIRB - Université Bordeaux 1
351 cours de la Libération 33405 cedex, TALENCE, FRANCE
Tel: + (33) 5 40 00 66 74, Fax: + (33) 5 40 00 84 06.
Email: {fernando.merchan, flavius.turcu, eric.grivel, mohamed.najim}@laps.ims-bordeaux.fr

ABSTRACT

The paper first deals with the design of Rayleigh fading channel simulators based on a Moving Average (MA) model. We present a new approach to estimate the model parameters based on the inner-outer factorization. The core of the approach consists of assimilating an infinite-order MA model to the outer spectral factor of the channel power spectral density (PSD). This outer factor, which leads to a causal minimum-phase filter, is first evaluated inside the unit disk of the z-plane. Then, we propose to compute the Taylor expansion coefficients of the outer factor since they coincide with the model parameters. This has two advantages: unlike other simulation techniques, the first p parameters remain unchanged when one increases the model order from p to p+1; in addition, our approach offers the possibility of selecting an appropriate model order for a given mean error bound. Then, we extend this approach to ARMA models to weaken the oscillatory deviations from the theoretical PSD in the case of AR models, or low peaks at the Doppler frequencies for MA models. A comparative study with existing channel simulation approaches points out the relevance of our ARMA model-based method.

1. INTRODUCTION

When designing communication systems based for instance on CDMA techniques or when conceiving new receivers and studying their performances, channel simulation is one of the steps to be carried out.

In an environment with no direct line-of-sight between transmitter and receiver, the marginal distributions of the phase and of the amplitude of the channel process are uniform and Rayleigh respectively. In addition, the theoretical power spectral density (PSD) of the real and imaginary parts of the fading channel samples is U-shaped and has two infinite peaks at the normalized maximum Doppler frequency $\pm f_d$:

$$S^{th}(f) = \begin{cases} \dfrac{1}{\pi\sqrt{f_d^2 - f^2}} & \text{if } |f| \le f_d \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

while the corresponding normalized discrete-time autocorrelation function is given by the zero-order Bessel function of the first kind:

$$R_h^{theo}(k) = J_0(2\pi f_d |k|) \ \forall \ k \in Z \tag{2}$$

Given these assumptions, various families of channel simulators have been proposed.

For instance, Jakes' fading model [5] based on a sum of sinusoids (SOS) makes it possible to generate time-correlated waveforms. In [7], Patzold *et al.* compare the statistical properties of this simulator with those of the underlying stochastic reference model. They conclude that sufficient results can be expected with 10 or more sinusoids. However, as independent channels cannot be easily simulated with this kind of model, Dent *et al.* [2] propose to weight the sinusoids by orthogonal random codes such as Hadamard ones. As an alternative, filtering-based approaches can be considered:

On the one hand, the filter can be designed in the frequency-domain. Thus, in [9], the authors combine a filtering step and an inverse discrete Fourier transform (IDFT). Although Young *et al.* [12] manage to reduce the computational cost of this approach, all samples have to be generated by using a single fast Fourier transform. Due to the IDFT, this off-line simulation requires a large memory storage.

On the other hand, the filtering can be carried out in the time-domain. This is the case of simulators based on ARMA, AutoRegressive (AR) or Moving Average (MA) models. This approach may be a priori questionable. Indeed, as the PSD of the real and imaginary parts is bandlimited, the channel process should be deterministic, according to the Kolmogorov-Szegö formula[1]. However, unlike the SOS methods, the linear stochastic models are quite simple and few parameters have to be estimated, making these approaches very popular both for channel simulator design and Kalman-filter based receiver design in mobile communication system. Thus, in [6], the transfer function associated to the ARMA model corresponds to a 3rd order Butterworth low-pass filter. Nevertheless, choosing this filter

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[1] $\sigma^2 = \exp\left[\int_{-1/2}^{1/2} \ln S(f) df\right]$ where $S(f)$ denotes the PSD of the process $s(n)$ and σ^2 is the error variance of the optimal predictor of $s(n)$.

leads to a poor approximation of the channel properties. In the method proposed in [8], a sub-sampled ARMA process followed by a multistage interpolator is considered. According to the authors, this combination makes it possible to select low orders for the ARMA process. Nevertheless, only a very high down-sampling factor can lead to a PSD which is never equal to zero and hence allows the simulated channel to match the theoretical channel properties.

In various papers including [1] and [11], the authors suggest using a p^{th} order AR process because it can generate at most p resonances in the PSD [11]. However, when selecting a 2^{nd} order AR model whose parameters are obtained by solving the Yule-Walker equations, the two peaks of the PSD are located at $\pm f_d/\sqrt{2}$ instead of $\pm f_d$ [11]. When the AR order is higher than 2, the channel autocorrelation matrix used in the Yule-Walker equations becomes ill-conditioned [1]. To overcome this problem, Baddour *et al.* [1] suggest adding a very small bias σ^2 in the main diagonal of the autocorrelation matrix of the channel (e.g., $\sigma^2 = 10^{-7}$ for $f_d = 0.01$). This leads to modeling the channel by the sum of the theoretical fading process and a zero-mean white process with variance σ^2.

In [10], the authors model the channel by a finite-order Moving Average (MA) process. The MA parameter estimation consists of designing a FIR filter by means of the window method. Indeed, the impulse response coefficients, namely the MA parameters, are estimated by first taking the inverse Fourier transform of the square root of the theoretical PSD of the channel, by windowing it and then by shifting it in time. However, the first p MA parameters change when increasing the model order from p to p+1. Another solution to estimate the MA parameters is based on Durbin's method [4]. Since this method turns the MA parameter estimation issue into a set of two of AR parameter estimation problems, the channel autocorrelation matrix used in the Yule-Walker equations may be ill conditioned. Hence, Baddour's ad-hoc solution [1] must be considered. In addition, like Verdin's approach [10], the selection of the model order has to be addressed.

In this paper, to estimate the channel model parameters and select the model order, we propose to consider the inner-outer factorization, which has been used so far in information theory, system theory, control and filter design [3]. Thus, when dealing with MA modeling, the corresponding transfer function can be expressed as the product of:

- the inner function which can be seen as an all-pass filter;
- the outer function which is causal and has its zeroes in the closed unit disk in the z-plane; hence, this corresponds to a minimum-phase filter. In addition, the square absolute value of the outer function on the unit circle in the z-plane equals the theoretical PSD of the channel. Furthermore, among all functions which have the same absolute value on the unit circle in the z-plane, the outer function is the one with the largest absolute value outside the unit disk.

In the following, we suggest using an infinite-order MA model whose transfer function coincides with its outer function. This approach is then extended to the ARMA model. Since finite-order models are rather considered in practical case, the approach we propose makes it possible to tune the upper bound of the error, in absolute value, between the theoretical PSD and the simulated one. Hence, this tuning leads to criteria for the model order selection. In addition, it should be noted that with our method, the first p parameters are unchanged when increasing the model order from p to p+1.

The paper is organized as follows: section 2 deals with the estimation of the model parameters and the selection of the model order. It also describes the numerical implementation of the estimation method. Section 3 shows how the MA approach can be adapted to the ARMA models. In the last section, a comparative study illustrates the benefits of the proposed ARMA methods with respect to the methods proposed in [4] , [10] and [1].

2. INNER-OUTER FACTORIZATION FOR MA MODEL

2.1 Mathematical overview

Let $h(n)$ be an infinite-order MA process defined as follows:

$$h(n) = \sum_{k\geq 0} b_k x(n-k) \tag{3}$$

where $\{b_k\}_{k\geq 0}$ are the real MA parameters and $x(n)$ is a zero-mean white noise sequence with unit variance. If $h(n)$ has its PSD equal to the channel's theoretical density S^{th}, the corresponding transfer function $G(z) = \sum_{k\geq 0} b_k z^{-k}$ satisfies:

$$S^{th}(f) = \left|G(z)\right|^2\Big|_{z=\exp(j2\pi f)} = G(z)G^*(z)\Big|_{z=\exp(j2\pi f)} \tag{4}$$

Given (4), the function G is a spectral factor of S^{th}.

In the following, to use the classical inner-outer factorization setting of the Hardy spaces theory over the unit disk in the z-plane instead of its exterior, we rather deal with the function $F_{MA}(z) = \sum_{k\geq 0} b_k z^k$ instead of $G(z)$. Indeed, since $\{b_k\}_{k\geq 0}$ are real, one has:

$$\left|G(z)\right|^2 = \left|F_{MA}(z^{-1})\right|^2 = \left|F_{MA}{}^*(z)\right|^2 \text{ for } z = \exp(j2\pi f) \tag{5}$$

It is known [3] that all the functions $F_{MA}(z)$ satisfying (4) are of the type:

$$F_{MA}(z) = U(z)F_0(z) \tag{6}$$

where U is any inner function and F_0 is the unique outer function satisfying (4), provided that S^{th} is log-integrable;

otherwise, (4) has no causal solutions as a consequence of Kolmogorov-Szegö formula.

For the sake of simplicity, we consider the particular case where $U(z)=1$, meaning that the spectral factor F_{MA} is outer. Selecting any other function will only affect the argument of $G(z)$.

The method we propose operates in two steps:

- the outer function F_0 is evaluated in the z-plane on a circle of radius $r<1$, by using the Poisson integral representation defined as follows:

$$F_0(re^{j2\pi\upsilon}) = \exp \int_{f=-1/2}^{1/2} \frac{1}{2(2\pi)}\left(\frac{1+re^{j2\pi(\upsilon-f)}}{1-re^{j2\pi(\upsilon-f)}}\right)\log\left|S^{th}(f)\right|df \quad (7)$$

- the MA parameters $\{b_k\}_{k\geq 0}$ are obtained by using $F_0(z)\big|_{z=r\exp(j2\pi\upsilon)}$. Indeed, the MA parameters coincide with the coefficients of the Taylor series of the outer function $F_0(z)$:

$$F_0(z) = F_0(0) + \frac{F_0^{(1)}(0)z}{1!} + \ldots + \frac{F_0^{(n)}(0)z^n}{n!} + \ldots = \sum_{k\geq 0} b_k z^k$$

Hence, $\{b_k\}_{k\geq 0}$ can be obtained through the Cauchy formula:

$$b_k = \frac{F_0^{(k)}(0)}{k!} = \frac{1}{r^k}\int_{\upsilon=0}^{2\pi} F_0(re^{j2\pi\upsilon})e^{-jk2\pi\upsilon}d\upsilon \quad \forall k \quad (8)$$

Remark 1: as suggested in [1], the channel PSD must be slightly modified to be log-integrable. Here, it corresponds to the product of two factors defined as follows:

$$S^U(f) = \begin{cases} \dfrac{1}{\pi\sqrt{f_d^2 - f^2}} & \text{if } |f| \leq f_d \\ 1 & \text{elsewhere} \end{cases} \quad (9)$$

and
$$S^{flat}(f) = \begin{cases} 1 & \text{if } |f| \leq (1+\xi)f_d \\ \varepsilon & \text{elsewhere} \end{cases} \quad (10)$$

where the parameter ε guarantees the log-integrability condition whereas ξ allows the compensation of the offset of the frequency peaks at f_d. In counterpart, it affects the decay of the PSD at f_d. In the following, F_0^{mod} denotes the outer function computed with (7) where the theoretical channel PSD S^{th} is replaced by the modified PSD, namely $S^U S^{flat}$.

Remark 2: in practical case, a truncated version of the MA model $F_p(z) = \sum_{k=0}^{p} b_k z^k$ is considered. Selecting the model order by minimizing the following criterion

$$J = \int \left| S^{th}(e^{j\theta}) - \left|F_p(e^{j\theta})\right|^2 \right| d\theta \quad \text{where } e^{j\theta} = e^{j2\pi f}$$

cannot lead to a finite value of *p*. For this reason, we suggest searching an upper bound of that criterion for a given order *p*. Hence, one has:

$$J \leq \int \left| S^{th}(e^{j\theta}) - \left|F_0^{\mathrm{mod}}(e^{j\theta})\right|^2 \right| d\theta + \int \left| \left|F_0^{\mathrm{mod}}(e^{j\theta})\right|^2 - \left|F_p(e^{j\theta})\right|^2 \right| d\theta$$

According to (1), (9) and (10), the first term in the inequality above is equal to $2\pi\varepsilon(1-2f_d)$, while, the second satisfies:

$$\int \left|F_0^{\mathrm{mod}}(e^{j\theta}) - F_p(e^{j\theta})\right| \cdot \left|F_0^{\mathrm{mod}}(e^{j\theta}) + F_p(e^{j\theta})\right| d\theta$$

$$\leq \sqrt{\int \left|F_0^{\mathrm{mod}}(e^{j\theta}) - F_p(e^{j\theta})\right|^2 d\theta} \cdot \sqrt{\int \left|F_0^{\mathrm{mod}}(e^{j\theta}) + F_p(e^{j\theta})\right|^2 d\theta}$$

$$\leq \sqrt{E^2 - \sum_{k=0}^{p}|b_k|^2} \cdot \sqrt{\sum_{k=0}^{p} 4|b_k|^2 + \sum_{k=p+1}^{+\infty}|b_k|^2} \leq 2E\sqrt{E^2 - \sum_{k=0}^{p}|b_k|^2}$$

where $E = \int \left|F_0^{\mathrm{mod}}(e^{j\theta})\right|^2 d\theta = 2\pi(1+\varepsilon(1-2f_d))$

So, one has:

$$J \leq 2\pi\varepsilon(1-2f_d) + 2E\sqrt{E^2 - \sum_{k=0}^{p}|b_k|^2} \quad (11)$$

At that stage, given a prescribed error bound $\delta_0 > 2\pi\varepsilon(1-2f_d)$, there is always a minimum value for the model order *p* that satisfies:

$$0 < E^2 - \sum_{k=0}^{p}|b_k|^2 < (\delta_0 - 2\pi\varepsilon(1-2f_d))^2(2E)^{-2} \quad (12)$$

In the next section, we present a way to estimate the Taylor coefficients of the outer function of the channel PSD, in practical case.

2.2 Implementation

Since the integrand in (7) cannot be computed analytically, we suggest estimating the values of the outer function for a finite number of points on a circle of radius $r<1$ in the z-plane.

For this purpose, let us introduce *N* values of the discrete PSD $\{S^U(m/N)\}_{m=-N/2,\ldots,N/2-1}$. Given f_d, *N* must be chosen high enough so that the discrete version of the PSD can be relevant. Then, for a given radius *r*, the outer function F^U of S^U can be computed by using the discrete version of the Poisson integral formula (7) for $-N/2 \leq n < N/2$:

$$\begin{aligned} F^U(n/N) &= \exp\left[\sum_{m=-N/2}^{N/2-1} \frac{1}{2N}\left(\frac{1+re^{j2\pi(n-m)/N}}{1-re^{j2\pi(n-m)/N}}\right)\log(S^U(m/N))\right] \\ &= \exp\left[\frac{1}{2N}\left(\frac{1+re^{j2\pi n/M}}{1-re^{j2\pi n/M}}\right) * \log(S^U(n/N))\right] \end{aligned} \quad (13)$$

where $*$ denotes the convolution. Thus, (13) can be implemented through a fast convolution algorithm. Moreover, the outer function F^{flat} of S^{flat} is expressed by:

$$F^{flat}(n/N) = \exp\int_{f=-1/2}^{1/2} \frac{1}{2(2\pi)}\left(\frac{1+re^{j2\pi(n/N-f)}}{1-re^{j2\pi(n/N-f)}}\right)\log\left|S^{flat}(f)\right|df$$

To calculate the Taylor coefficients, the following discrete version of eq. (8) is considered:

$$b(k) = \frac{1}{Nr^k} \sum_{n=0}^{N-1} F^U(n/N) \cdot F^{flat}(n/N) e^{-j2nk(n/N)} \quad (14)$$

It should be noted that (14) can be implemented as a weighted FFT of $F^U(n/N) \cdot F^{flat}(n/N)$.

Therefore, the algorithm requires $O(N \log N)$ operations.

3. INNER-OUTER FACTORIZATION FOR ARMA MODEL

Let us now consider the ARMA process $h(n)$ defined by:

$$h(n) = -\sum_{k=1}^{q} a_l h(n-k) + \sum_{k=0}^{+\infty} b_k x(n-k) \quad (15)$$

The corresponding transfer function:

$$F_{ARMA}(z) = \frac{\sum_{k=0}^{+\infty} b_k z^{-k}}{1 + \sum_{k=1}^{q} a_l z^{-k}} = \frac{B(z)}{A(z)}$$

satisfies: $\left| F_{ARMA}(z) \right|^2 \Big|_{z=\exp(j2\pi f)} = S^{th}(f)$

We choose the denominator $A(z)$ to be a second-order polynomial with roots equal to $\rho e^{\pm j2\pi f_d}$, with $0 < \rho < 1$ and close to 1. Therefore, the numerator $B(z)$ can be defined as follows:

$$\left| B(z) \right|^2 \Big|_{z=\exp(j2\pi f)} = S^{th}(f) \left| A(z) \right|^2 \Big|_{z=\exp(j2\pi f)} \quad (16)$$

To evaluate the outer function of $B(z)$, S^{th} in eq. (7) is replaced by the right side of eq. (16). Then, the parameters $\{b_k\}_{k \geq 0}$ can be estimated by using the method presented in Section 2 based on (8). Moreover, since a finite polynomial approximation of the outer factor of $B(z)$ has no roots equals to $\rho e^{\pm j2\pi f_d}$, the resulting transfer function still admits two poles equal to $\rho e^{\pm j2\pi f_d}$. This leads to a better fitting of the model PSD in a small neighborhood of Doppler frequencies.

4. COMPARATIVE STUDY

In this section, we compare the proposed simulators with:

- the AR-based simulator [1],
- the MA-based simulator given in [10] and,
- Durbin's method for MA parameter estimation [4].

For the MA and ARMA channel simulators we propose, N is set to 4096 samples. The parameter r is assigned[2] to 0.98 whereas ε and ξ are set to 0.0125 and 0.025 respectively. These values are chosen experimentally for a compromise between the maximum Doppler frequency offset and the PSD decay. For the ARMA-based model, different values of the pole modulus, namely ρ, have been heuristically chosen in each scenario.

To compare the various simulators, we introduce three criteria.

The first criterion is the mean error J_d defined as follows:

$$J_d = \frac{1}{L} \sum_{n=-L/2}^{L/2-1} \left| S^{th}(n/L) - \left| F_p(n/L) \right|^2 \right|$$

The second and third criteria are two quality measures, defined in [12], i.e. the mean power and maximum power margins G_{mean} and $G_{\max}$:

$$G_{\text{mean}} = \frac{1}{\sigma_X^2 D} \operatorname{trace}\left\{ C_X \hat{C}_X^{-1} C_X \right\},$$

$$G_{\max} = \frac{1}{\sigma_X^2} \max\left\{ \operatorname{diag}\left\{ C_X \hat{C}_X^{-1} C_X \right\} \right\},$$

where C_x and $\hat{C}_x$ are the $D \times D$ covariance matrices of respectively the theoretical channel and the simulated channel processes and σ_X^2 is set to the unity in this case.

According to various test we carried out[3] and given in table 1 and figure 1, the proposed MA-based method and Verdin simulator provide close results and outperforms Durbin-based method.

According to table 1 and figures 2 and 3, the ARMA-based approach outperforms the other solutions. More particularly, in figure 2, the PSD obtained with the ARMA(2,298) is very close to the theoretical PSD both in low frequencies and in the neighborhood of the Doppler frequency whereas Baddour's AR-based approach[4] gives important oscillations in the range $(-f_d, f_d)$ and has a maximum peak offset at Doppler frequency.

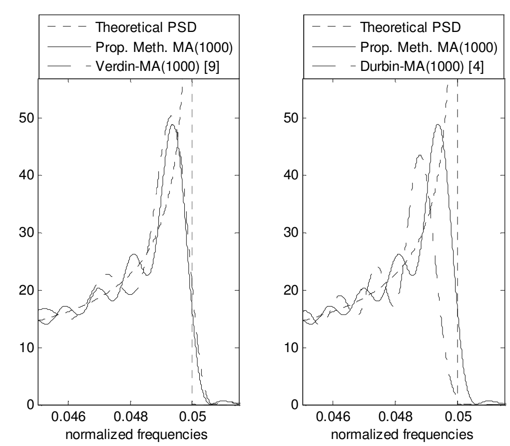

Figure 1 - PSD of MA-based simulators with $f_d = 0.05$.

[2] Simulations showed that when $r < 0.965$ and $r > 0.998$, the square of estimated MA model diverges from the theoretical PSD.

[3] For MA(p) and AR(p) models the number of parameters is the order p. For ARMA(p, q) models the number of parameters is $p+q$. For the criteria, we the set $L = 131072$ and $D = 1024$.

[4] The AR-based approach requires $O(order^2)$ operations when using Levinson recursion.

Table 1

Evaluation of error J_d and quality measures G_{mean} and G_{max} for various simulators

number of parameters		J_d		G_{mean} (dB)		G_{max} (dB)	
		$f_d = 0.01$	$f_d = 0.05$	$f_d = 0.01$	$f_d = 0.05$	$f_d = 0.01$	$f_d = 0.05$
50	Proposed ARMA	0.8829	**0.2650**	**1.0971**	**0.5078**	**1.1476**	**0.5183**
	Baddour AR [1]	**0.5570**	0.6317	1.5935	2.4487	1.7659	2.6832
	Proposed MA	0.9765	0.4271	4.0550	6.2141	4.2897	6.4728
	Verdin MA [10]	0.6040	0.3591	3.2063	3.0014	3.4206	3.1852
	Durbin MA [4]	0.9859	0.4961	4.6373	10.4704	4.8668	10.7523
120	Proposed ARMA	**0.4090**	**0.1290**	**0.7908**	**0.1765**	**0.8171**	**0.1818**
	Baddour AR [1]	0.8008	0.3382	2.1303	0.7204	2.3946	0.7940
	Proposed MA	0.6432	0.2940	7.6300	4.0548	7.9383	4.2852
	Verdin MA [10]	0.5307	0.3428	3.0998	2.0063	3.3135	2.1541
	Durbin MA [4]	0.8234	0.3762	8.4128	9.4176	8.6786	9.7032
300	Proposed ARMA	**0.2364**	**0.0772**	**0.3451**	**0.0875**	**0.3796**	**0.0927**
	Baddour AR [1]	0.5695	0.2475	0.5434	0.1830	0.7038	0.2514
	Proposed MA	0.4267	0.1670	5.3696	1.5923	5.6463	1.7150
	Verdin MA [10]	0.3831	0.1963	1.9042	0.9865	2.0559	1.0713
	Durbin MA [4]	0.5850	0.2489	11.8706	7.4949	12.1374	7.7233

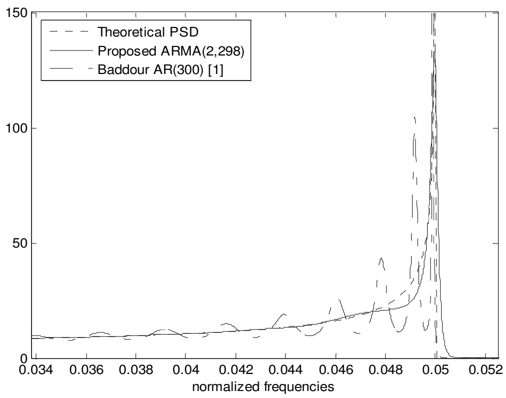

Figure 2 - PSD of simulators with $f_d = 0.05$ and 300 parameters.

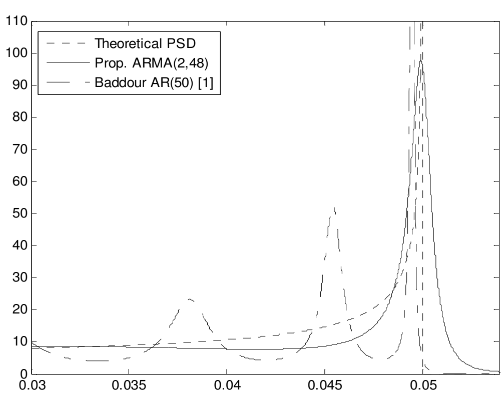

Figure 3 - PSD of simulators with $f_d = 0.05$ and 50 parameters

5. CONCLUSIONS

In this paper, we have investigated the relevance of inner-outer factorization for Rayleigh fading channel simulator. We have proposed a new method to estimate the model parameters both for MA and ARMA models. The comparative study we have carried out confirms that our ARMA-based simulator outperforms the other approaches.

REFERENCES

[1] K. E. Baddour and N. C. Beaulieu, "Autoregressive modeling for fading channel simulation," *IEEE Trans. Commun.*, vol. 4, no. 4, pp. 1650-1662, Jul. 2005.

[2] P. Dent, G. E. Bottomley and T. Croft, "Jakes' fading model revisited," *Electron. Letters*, vol. 29, no. 3, pp. 1162-1163, Jun. 1993.

[3] P. Dewilde and A. J .van der Veen, *Time-Varying Systems and Computations*, Kluwer Academic Publishers, Boston, 1998.

[4] J. Durbin, "Efficient estimation of parameters in moving average models," *Biometrika*, vol. 46, pp. 306-316, 1959.

[5] W. C. Jakes, *Microwave mobile communications*, IEEE Press Eds. Wiley, 1974.

[6] C. Loo, N. Secord, "Computer models for fading channels with applications to digital transmission," *IEEE Trans. Veh. Tech.,* vol. 40, no. 4, pp. 700-707, Nov. 1991.

[7] M. Patzold and F. Laue, "Statistical properties of Jakes' fading channel simulator," *in Proc. IEEE VTC'98*, 1998, pp. 712-718.

[8] D. Schafhuber, G. Matz, and F. Hlawatsch, "Simulation of wideband mobile radio channels using subsampled ARMA models and multistage interpolation," *IEEE-SSP*, pp. 571-574, Aug. 2001.

[9] J. I. Smith, "A computer generated multipath fading simulation for mobile radio," *IEEE Trans. Veh. Tech.*, vol. 24, pp. 39-40, Aug. 1975.

[10] D. Verdin and T.C. Tozer, "Generating a fading process for the simulation of land-mobile radio communications," *Electron. Letters*, vol. 29, no. 23, Nov. 1993.

[11] H. Wu and A. Duel-Hallen, "Multiuser detectors with disjoint Kalman channel estimators for synchronous CDMA mobile," *IEEE Trans. Commun.*, vol. 48, no. 5, pp. 752-756, May 2000.

[12] D. J. Young and N. C. Beaulieu, "The generation of correlated Rayleigh random variates by inverse Fourier transform," *IEEE Trans. Commun.*, vol. 48, no. 7, pp. 1114-1127, Jul. 2000.