

# BLIND SOURCE SEPARATION BASED ON CYCLIC SPECTRA: APPLICATION TO BIOMECHANICAL SIGNALS.

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## ABSTRACT

This paper introduces new frequency domain approaches for either blind source separation or MIMO system identification excited by cyclostationary inputs. The eigenvalue decomposition, the singular value decomposition, the diagonalization of a positive definite linear combination or the joint diagonalization, of the spectral correlation density matrices of the whitened measurements allows the identification of the mixing system at each frequency up to constant diagonal and frequency dependent permutation and phase ambiguity matrices. Two efficient algorithms to fix the permutation problem and to remove the phase ambiguity based on cyclostationarity are also presented. The new approaches exploit the fact that the inputs are cyclostationary with the same cyclic frequency. Simulation examples are presented to illustrate the effectiveness of this approaches. Furthermore, the AJD approach is applied to biomechanical signals for separation ends.

## 1. INTRODUCTION

In a cyclostationary context, Second-order blind Multiple-Input Multiple-Output (MIMO) system identification problem has become an intense area of research. In this study, we are concerned with a special case of inputs that are cyclostationary with unknown statistics. Such MIMO problems have been studied in [1, 2] where Antoni et al. make use of the Spectral Correlation Density (SCD) matrices for the blind identification or source separation. Also in [5] where Bradric et al. addressed the problem of blind identification of Finite Impulse Response MIMO (FIR MIMO) systems driven by cyclostationary inputs by using the correlation of short-time Discrete Fourier Transforms (DFT).

The methods offered in this paper are frequency domain techniques that exploit the cyclic spectral density matrices of the measurements to identify the mixing channel and separate convolved cyclostationary sources with the same cyclic frequency. Such MIMO problems find applicability in blind channel estimation in rotating machines -where the sources may share the same cyclic frequency- for vibratory diagnostic. The main contribution regards three respects. First, The freedom to choose either Singular Value Decomposition (SVD) or Eigenvalue Decomposition (EVD) based methods if the sources of interest have no energy on the harmonics of the cyclic frequency or else the diagonalization of a

positive definite linear combination (PDLC) or Approximate Joint Diagonalization (AJD) [3] based methods if the sources of interest have energy in the harmonics of the cyclic frequency. Second, an efficient method for addressing the permutation problem inherent with frequency domain based approaches is proposed. This method exploits the SCD matrices of the restored signals situated apart from each other at a distance equal to the cyclic frequency. Third, the phase ambiguity which is inherent in frequency domain approaches is addressed in this paper. The method exploits the correlation between the phases of the channels separated from each other with the cyclic frequency. The organization of this paper is as follows: The problem statement including the set of required assumptions is presented in Section 2. Section 3 presents the algorithms to estimate the magnitude of MIMO system as well as the methods to correct the permutation problem and to cancel the phase ambiguity. Section 4 discusses the BSS issue. Simulation experiments illustrating the effectiveness of the proposed approaches are described in section 5. We report some encouraging results on biomechanical signals in section 6. Ultimately conclusions are presented in section 7.

## 2. PROBLEM FORMULATION

### 2.1 Problem formulation

We consider  $n$ -source  $m$ -sensor MIMO linear model for the received signals for the convolutive mixing problem:

$$\mathbf{x}(t) = \sum_{k=0}^{L-1} \mathbf{H}(k) \mathbf{a}(t-k) + \mathbf{b}(t) \quad (1)$$

where  $\mathbf{x}(t) = [x_1(t), \dots, x_m(t)]^T$  is the vector of measurements,  $\mathbf{a}(t) = [a_1(t), \dots, a_n(t)]^T$  is the source vector which the components are real-valued, cyclostationary, *i.i.d.*, mutually statistically uncorrelated, centered (in the ensemble-average sense) and with the same cyclic frequency  $\beta$ .  $\mathbf{H}(t)$  is the  $m \times n$  impulse response matrix whose elements are  $\{h_{ji}(t)\}$  and  $\mathbf{b}(t) = [b_1(t), \dots, b_m(t)]^T$  is the additive noise vector which is assumed to be stationary, temporally and spatially white, zero mean and independent of the source signals. The superscript  $T$  denotes the transpose and  $L$  is the length of the real-valued channels. By taking the DFT of (1) over  $N$  lines, we obtain:

$$\mathbf{X}(f_k) = \mathbf{H}(f_k) \mathbf{A}(f_k) + \mathbf{B}(f_k) \quad (2)$$

where  $\mathbf{X}(f_k)$ ,  $\mathbf{H}(f_k)$ ,  $\mathbf{A}(f_k)$  and  $\mathbf{B}(f_k)$  are the DFT of  $\mathbf{x}(t)$ ,  $\mathbf{H}(t)$ ,  $\mathbf{a}(t)$  and  $\mathbf{b}(t)$ , respectively. From (2) it is obvious that

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at each frequency bin  $f_k$ , the estimation of  $\mathbf{A}(f_k)$  can be seen as an instantaneous complex BSS problem.

Let introduce the SCD of the source vector [7]:

$$E\{\mathbf{A}(f_k)\mathbf{A}^H(f_k - q\beta)\} = \mathbf{S}_a^{q\beta}(f_k) \\ = \text{Diag}([\sigma_1^2(q\beta), \dots, \sigma_i^2(q\beta), \dots, \sigma_n^2(q\beta)]) \quad (3)$$

where  $q \in \mathbb{Z}^*$ ,  $i = 1, \dots, n$ , the superscript  $H$  denotes the complex conjugate transpose of a matrix and  $\sigma_i^2(q\beta)$  stands for the SCD of the signal  $a_i(t)$ , at the  $q$ th multiple of  $\beta$ , which is frequency independent since  $a_i(t)$  is independent and identically distributed (*i.i.d.*). This means that the SCD matrices of the inputs at different harmonics are diagonals under the independency hypothesis. We conclude from (3) that to blindly restore the sources, we need to submit the measurements to some linear transformations so that the output signals will have the same algebraic structure as the primary sources  $\mathbf{a}(t)$ .

### 3. MIMO IDENTIFICATION

#### 3.1 Proposed approaches for system estimation

The algorithms presented hereafter aim to separate the sources, at each frequency, from the blind identification of the steering vectors. This identification requires the prewhitening of the data, aiming to orthogonalize the source steering vectors so as to search for the latter through a unitary matrix. To do so, we first start by the SVD of  $\mathbf{H}(f_k)$ , this leads to:

$$\mathbf{H}(f_k) = \mathbf{U}(f_k)\mathbf{\Sigma}(f_k)\mathbf{V}(f_k) \quad (4)$$

where  $\mathbf{U}(f_k)$  and  $\mathbf{V}(f_k)$  are  $m \times n$  and  $n \times n$  unitary matrices respectively, and  $\mathbf{\Sigma}(f_k)$  is  $n \times n$  diagonal matrix. Let us introduce the SCD matrix of the measurements [7]:

$$\mathbf{S}_x^{q\beta}(f_k) = \mathbf{H}(f_k)\mathbf{S}_a^{q\beta}(f_k)\mathbf{H}^H(f_k - q\beta) + \mathbf{S}_b^0(f_k) \quad (5)$$

where  $\mathbf{S}_x^{q\beta}(f_k)$ ,  $\mathbf{S}_a^{q\beta}(f_k)$  and  $\mathbf{S}_b^0(f_k)$  are the SCD matrices of the measurements, sources and noise respectively. The SCD of the noise is null for all nonzero cyclic frequencies since the noise is stationary, as mentioned above. The following holds for the zero cyclic frequency ( $\beta = 0$ ):

$$\mathbf{S}_x^0(f_k) = \mathbf{H}(f_k)\mathbf{H}^H(f_k) + \mathbf{S}_b^0(f_k) \quad (6)$$

$\mathbf{S}_a^0(f_k)$  is equal to the identity matrix since the sources are independent and normalized. Putting (4) into (6), the latter expression takes the following form:

$$\mathbf{S}_x^0(f_k) = \mathbf{U}(f_k)\mathbf{\Sigma}(f_k)^2\mathbf{U}^H(f_k) + \mathbf{S}_b^0(f_k) \quad (7)$$

The Eigenvalue Decomposition (EVD) of the SCD matrix of the measurements evaluated when  $\beta = 0$  allows us to compute the whitening matrix  $\mathbf{W}(f_k)$  as explained below:

$$\mathbf{W}(f_k) = \mathbf{\Sigma}^{-1}(f_k)\mathbf{U}^H(f_k) \quad (8)$$

It is important to note that the whitening reduces the dimension of the measurements to be  $n$  instead of  $m$ . So, we introduce the new quantity:

$$\mathbf{Z}(f_k) = \mathbf{W}(f_k)\mathbf{X}(f_k) = \mathbf{V}(f_k)\mathbf{S}(f_k) \quad (9)$$

where  $\mathbf{Z}(f_k)$  is a  $n \times 1$  whitened process. However, the components of  $\mathbf{Z}(f_k)$  are not yet independent. To force the

second-order independency of the whitened processes, we need therefore to look for an unitary matrix  $\mathbf{V}(f_k)$ . To this end, we shall use the SCD matrices of the whitened processes at each cyclic frequency  $q\beta$ .

$$\mathbf{S}_z^{q\beta}(f_k) = \mathbf{V}(f_k)\mathbf{S}_a^{q\beta}(f_k)\mathbf{V}^H(f_k - q\beta) \quad (10)$$

Actually, one can proceed in one of the following manners to identify  $\mathbf{V}(f_k)$ .

##### 3.1.1 SVD based approach

It is apparent from Equ. (10) that the SVD allows the identification of  $\mathbf{V}(f_k)$  for a given  $q\beta$ .

##### 3.1.2 EVD based approach

Let consider the norm of (10) in order to have a symmetric expression in the right-hand side:

$$\tilde{\mathbf{S}}_z^{q\beta}(f_k) = \mathbf{V}(f_k)\tilde{\mathbf{S}}_s^{q\beta}(f_k)\mathbf{V}^H(f_k) \quad (11)$$

The EVD of (11) makes possible the estimation of  $\mathbf{V}(f_k)$  for a given  $q\beta$ .

##### 3.1.3 Diagonalization of a PDLC

As long as  $\tilde{\mathbf{S}}_z^{q\beta}(f_k)$  is different for a set of cyclic frequencies  $\{q\beta, q \neq 0\}$ ,  $\mathbf{V}(f_k)$  can be estimated as the matrix that diagonalizes the PDLC:

$$\bar{\mathbf{S}}_z^\beta(f_k) = \sum_{q=-Q}^Q \lambda_{q\beta_i} \tilde{\mathbf{S}}_z^{q\beta}(f_k), \quad q \neq 0 \quad (12)$$

where  $2Q$  is the number of matrices and the coefficients  $\lambda_{q\beta_i}$ 's are chosen so as the matrix  $\bar{\mathbf{S}}_z^\beta(f_k)$  be positive definite.

##### 3.1.4 AJD based approach

Instead of diagonalizing one or a PDLC matrix of  $\tilde{\mathbf{S}}_z^{q\beta}(f_k)$ .

One can simultaneously diagonalize  $\tilde{\mathbf{S}}_z^{q\beta}(f_k)$  for a set of  $\{q\beta, q \neq 0\}$ . Thus, the application of the well-known AJD to (11) at different values of  $q$  will be suitable, and hence enables the estimation of  $\mathbf{V}(f_k)$ .

Once  $\mathbf{V}(f_k)$  is identified, by one of the four proposed approaches, one can estimate the mixing MIMO system  $\mathbf{H}(f_k)$  based on the Equ. (4) as shown below:

$$\hat{\mathbf{H}}(f_k) = \mathbf{H}(f_k)\mathbf{\Delta}\mathbf{P}(f_k)\mathbf{D}(f_k) \quad (13)$$

where  $\hat{\mathbf{H}}(f_k)$  is an estimation of  $\mathbf{H}(f_k)$  with constant diagonal matrix,  $\mathbf{\Delta}$ , which introduces a scaling ambiguity, frequency dependent permutation  $\mathbf{P}(f_k)$ , and phase ambiguity  $\mathbf{D}(f_k)$ , that introduces circular shifts to the sources (the  $i$ th diagonal element of  $\mathbf{D}(f_k)$  is of the form  $e^{j\phi_b^i(f_k)}$ ).

Certainly, the estimation offered by (13) can no longer provide a good estimation of the actual system  $\mathbf{H}(f_k)$  neither in the frequency domain nor in the time one by involving the Inverse DFT (IDFT). Hereafter, we propose two techniques to cancel the permutation problem and phase ambiguity, so that the mixing MIMO system will be estimated up to uniform permutation and general constant diagonal matrices, thus giving:

$$\hat{\mathbf{H}}_{opt}(f_k) = \mathbf{H}(f_k)\mathbf{\Delta}\mathbf{P} \quad (14)$$

which solves the blind identification issue.

### 3.2 Permutation correction

It is worth emphasizing that in the literature of BSS in the frequency domain, the problem of adjusting permutations received a lot of interest. Unfortunately, most of the proposed methods perform very poorly. The algorithm to correct the permutation problem must be quite robust, so that, it allows us to obtain a uniform permutation across the whole frequency range, in the sense that an error in the estimation of the correct permutation matrix for a given frequency bin will mislead the permutations of the remaining frequency bins. Besides, this will influence the accuracy of the estimation of the MIMO system and will result in very poor or no separation of the output signals.

In [12], it has been proposed a reorganization criterion to remove the permutation indeterminacy. A constraint on the size of the separating filter is made in [9] to solve the permutation problem. The paper [10] presents an algorithm that exploits the cross power spectral density matrices of nonstationary sources between adjacent frequencies. We proposed in [11] an adjusting permutation technique based on cyclic cepstra for the case where the sources have different cyclic frequencies.

An interesting property of cyclostationary signals is their spectral redundancy. In our case, the cyclic spectra of sources are frequency independent (since the sources are *i.i.d.*), different for each source and will overlap because they share the same cyclic frequency, as well. This is the property that we seek to exploit in our effort to derive a new algorithm for the correction of permutations across all the frequency bins. Let us first consider the estimating sources:

$$\hat{\mathbf{A}}(f_k) = \hat{\mathbf{H}}^\#(f_k) \mathbf{X}(f_k) = \mathbf{D}^*(f_k) \mathbf{P}^T(f_k) \Delta^{-1} \mathbf{A}(f_k) \quad (15)$$

where the superscripts # and \* denote the pseudo-inverse and the complex conjugate of a matrix, respectively. As the signals at the output of  $\hat{\mathbf{H}}^\#(f_k)$  have the same cyclic frequency as well as the actual sources, then (3) holds that:

$$|\mathbf{S}_a^{q\beta}(f_k)| = \mathbf{P}^T(f_k) |\Delta^{-1} \mathbf{S}_a^{q\beta}(f_k) \Delta^{-1}| \mathbf{P}(f_k - q\beta) \quad (16)$$

We notice that the phase effect has disappeared in (16) because of the absolute value. It is apparent therefore from (16) that performing the SVD of  $|\mathbf{S}_a^{q\beta}(f_k)|$  leads to the identification of both matrices  $\mathbf{P}(f_k)$  of and  $\mathbf{P}(f_k - q\beta)$ . This is because  $\mathbf{P}(f_k)$  and  $\mathbf{P}(f_k - q\beta)$  are an orthogonal matrices. The SVD of  $|\mathbf{S}_a^{q\beta}(f_k)|$  is given by:

$$|\mathbf{S}_a^{q\beta}(f_k)| = \Pi_1(f_k) \Gamma^{q\beta}(f_k) \Pi_2^T(f_k) \quad (17)$$

Actually, we have  $\Gamma^{q\beta}(f_k) = \mathbf{P}^T |\Delta^{-1} \mathbf{S}_a^{q\beta}(f_k) \Delta^{-1}| \mathbf{P}$ , there subsists only a single unknown permutation concerning the initial labelling of the sources because the SVD will sort the singular values of  $\Gamma^{q\beta}(f_k)$  in ascending or descending order over all frequency bins. Hence, no connection with the true order of the singular values  $\mathbf{S}_a^{q\beta}(f_k)$  is made. Therefore, the Magnitude of the MIMO system is estimated up to a uniform permutation matrix  $\mathbf{P}$  and constant diagonal matrix  $\Delta$ . As well-known in blind source separation, this indeterminacy is inevitable. At the output of the algorithm, the estimated sources will have a uniform permutation,  $\mathbf{P}$ , over all frequency bins. Moreover, the relationship (13) will be simplified to:

$$\hat{\mathbf{H}}(f_k) = \mathbf{H}(f_k) \Delta \mathbf{P} \mathbf{D}(f_k) \quad (18)$$

### 3.3 Phase retrieval

The phase retrieval of Single-Input Single-Output (SISO) systems, excited by cyclostationary inputs, has been studied in several papers [6, 8, 13, 14]. Note that, by exploiting SOCS, the identification of Non-minimum-phase channels becomes possible.

Let involve the overall mixing-unmixing system after correcting the permutation problem:

$$\hat{\mathbf{H}}^\#(f_k) \mathbf{H}(f_k) = \mathbf{D}^*(f_k) \mathbf{P}^T \Delta^{-1} \quad (19)$$

The SCD matrix of the signal, at the output of the unmixing system  $\hat{\mathbf{H}}^\#(f_k)$  of (18),  $\hat{\mathbf{a}}(t)$  for a given  $q\beta$  is as follows:

$$\mathbf{S}_a^{q\beta}(f_k) = \mathbf{D}^*(f_k) \mathbf{P}^T \Delta^{-1} \mathbf{S}_a^{q\beta}(f_k) \Delta^{-1} \mathbf{P} \mathbf{D}(f_k - q\beta) \quad (20)$$

$\mathbf{S}_a^{q\beta}(f_k)$  being diagonal. The following relation holds for the phase of the quantities involved in (20):

$$\Phi_D(f_k - q\beta) - \Phi_D(f_k) = \Psi(f_k) - \Theta(q\beta) \quad (21)$$

where  $\Theta(q\beta) = \arg\{\mathbf{P}^T \Delta^{-1} \mathbf{S}_a^{q\beta}(f_k) \Delta^{-1} \mathbf{P}\}$  and  $\Psi(f_k) = \arg\{\mathbf{S}_a^{q\beta}(f_k)\}$ . Furthermore,  $\Theta(q\beta)$  being frequency independent since the sources are *i.i.d.* and can be estimated by summing up (21) over all discrete frequencies which yield a zero right-hand side, thus we get:

$$\Theta(q\beta) = \frac{1}{N} \sum_{k=1}^N \Psi(f_k) \quad (22)$$

In this way,  $\Theta(q\beta)$  can actually be computed from  $\Psi(f_k)$ . Let take the IDFT of Equ. (21), thus giving:

$$\Phi_D(t) = \frac{\Theta(q\beta) \delta(t) - \Psi(t)}{(1 - \exp(j2\pi q\beta t))}, \quad t \neq \frac{l}{q\beta} \quad (23)$$

Actually,  $\Phi_D(t)$  cannot be completely identified whenever  $t = \frac{l}{q\beta}$ , a way to fix this is to interpolate the missing information assuming the continuity of  $\Phi_D(t)$ . The identification of  $\Phi_D(f_k)$  is possible by taking the DFT of (23). This is the key relationship that can lead to the estimation of  $\Phi_D(f_k)$ . Finally, the following relationship yields to the elimination of the phase ambiguity from (13):

$$\hat{\mathbf{H}}_{opt}(f_k) = \hat{\mathbf{H}}(f_k) e^{-j\Phi_D(f_k)} \quad (24)$$

Hence, the MIMO system is identified as in (14).

## 4. BSS ISSUE

At each frequency bin an estimate of the vector of source signals is computed by applying to the received signal  $\mathbf{X}(f_k)$  the pseudo-inverse of  $\hat{\mathbf{H}}_{opt}(f_k)$ , thus we get:

$$\hat{\mathbf{A}}(f_k) = \hat{\mathbf{H}}_{opt}^\#(f_k) \mathbf{X}(f_k) = \mathbf{P}^T \Delta^{-1} \mathbf{A}(f_k) + \hat{\mathbf{H}}^\#(f_k) \mathbf{B}(f_k) \quad (25)$$

In general, this procedure is not optimal for recovering the source signals because of the vector of noises. The Least Mean Square (LMS) algorithm which allow to get a maximal SNR at the output of the separator  $\mathbf{G}(f_k)$  can be used as a

multidimensional separator. The optimum solution for such separator is given by:

$$\mathbf{G}(f_k) = \mathbf{S}_x^0(f_k)^{-1} \mathbf{S}_{xa}^0(f_k) \quad (26)$$

where

$$\mathbf{S}_{xa}^0(f_k) = \mathbb{E}\{\mathbf{X}(f_k)\mathbf{A}^H(f_k)\} = \mathbf{H}(f_k) \quad (27)$$

Using (14) and (27) in (26), we get an estimate of  $\mathbf{G}(f_k)$ :

$$\hat{\mathbf{G}}(f_k) = \mathbf{S}_x^0(f_k)^{-1} \hat{\mathbf{H}}_{opt}(f_k) = \mathbf{G}(f_k) \Delta \mathbf{P} \quad (28)$$

Hence, the estimate of the source vector  $\mathbf{A}(f_k)$  is given by:

$$\hat{\mathbf{A}}(f_k) = \hat{\mathbf{G}}^H(f_k) \mathbf{X}(f_k) \approx \mathbf{P}^T \Delta \mathbf{A}(f_k) \quad (29)$$

As a result, in the frequency domain, the sources are recovered up to a scale factor and a permutation matrix.

## 5. SIMULATION RESULTS

The proposed methods are tested using synthetic signals in order to evaluate its effectiveness. The simulation is made with  $n = 2$  cyclostationary signals, of  $10^5$  samples each, which are actually amplitude modulated white gaussian noises:  $a_1(t) = m_1(t)(0.8\cos(\frac{2\pi}{T}t) + 0.6\cos(\frac{4\pi}{T}t))$  and  $a_2(t) = m_2(t)(\text{square}(\frac{2\pi}{T}t, 9) + 1)$  where the function *square*, which is actually a function of Matlab, generates a square wave with period  $2\pi$ . The cyclic period  $T$  is equal to 50 samples. The normalized Second-Order cyclic frequency is:  $\beta = \frac{1}{T}$ . The signals are then filtered by a,  $3 \times 2$ , ARMA-MIMO system. The transfer function of the  $(ij)$ th element of the MIMO system is given as:

$$H_{ij}(z) = \frac{1+b'_{ij}z^{-1}+b''_{ij}z^{-2}}{1+a'_{ij}z^{-1}+a''_{ij}z^{-2}} \text{ where } b' = \begin{pmatrix} -0.2 & -0.443 \\ 0.7 & 0.57 \\ 0.5 & 0.9 \end{pmatrix}, b'' = \begin{pmatrix} 0.8 & 0.6 \\ 0.6 & 0.1 \\ 0.3 & 0.2 \end{pmatrix},$$

$$a' = \begin{pmatrix} 0.64 & 0.8 \\ -0.92 & -1.64 \\ -1.2 & -1.7 \end{pmatrix} \text{ and } a'' = \begin{pmatrix} 0.1 & 0.5 \\ 0.21 & 0.74 \\ 0.740818 & 0.8 \end{pmatrix}. \text{ The length of the han-}$$

ning window and the DFT are set to 128 and 256, respectively. The number of matrices for AJD and PDLC approaches is  $2Q = 8$ .

**Comparison between the proposed methods:** We provide here a comparison between the proposed approaches. For each value of the SNR,  $M_c = 20$  Monte Carlo runs were implemented.

As a performance index to measure the identification performance and to make comparison as well, we here used the Normalized Mean-Square Error (NMSE). For  $M_c$  Monte Carlo runs, the  $\text{NMSE}_{ji}$  for the subchannel  $h_{ji}(r)$  is given by the formula:

$$\text{NMSE}_{ji} = \frac{\sum_{l=1}^{M_c} \left[ \sum_{r=1}^L (\hat{h}_{ji}^{(l)}(r) - h_{ji}(r))^2 \right]}{M_c \sum_{r=1}^L (h_{ji}(r))^2} \quad (30)$$

The overall NMSE is obtained by averaging over all subchannels:

$$\text{ONMSE}_{ji} = \frac{1}{mn} \sum_{j=1}^m \sum_{i=1}^n \text{NMSE}_{ji} \quad (31)$$

Figure (1) shows the variation of each method' ONMSE with the SNR. As can be seen from this figure, by decreasing the SNR, the ONMSE, for the proposed methods, improves (decreases). However, it shows the proposed approaches, specially the AJD based approach, perform well and provide better ONMSE even for low SNR. Moreover, EVD and PDLC based approaches have almost the same performances.

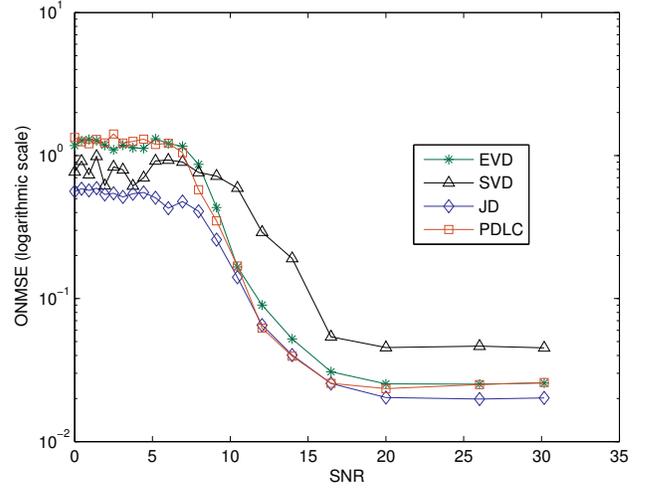


Figure 1: Performance comparison of the proposed methods: ONMSE versus SNR.

## 6. APPLICATION TO BIOMECHANICAL SIGNALS

This section concerns a real application of the proposed approaches for characterizing mechanical step variability during treadmill running. The human locomotion, in particular the walk and the running, are defined by sequences of cyclic gestures and repeated. The variability of these sequences is able to reveal the faculties or the motor failings.

The objective of this experimentation is to analyze and to characterize a runner's step from the Ground Reaction Forces (GRF) measured during a running on a treadmill. Mechanical parameters were measured for each step using treadmill dynamometer. Vertical GRF signal (see Figure 2) and belt velocity were sampled at a rate of 1000Hz.

The reason behind this application is to separate the contribution of the impact force and the propulsive GRF for the step 1 and the step 2 from the GRF signals. This means that the source number is equal to 4. The sensor number is 12. The projection of the SCD of a vertical GRF signal on the cyclic frequency axe (see Figure 3) shows that the vertical GRF signals are second-order cyclostationary with 689.6897 samples as a cyclic period. The cyclic frequency shared by the sources is 0.0014.

We apply the AJD based approach for the separation. The parameter  $2Q$  is set to 4. Figure 4 reports the separated signals on time domain. As we can see, the impact force of the step1 is alternate with the one of the step2. It is clear therefore, that we recover the impact and propulsive forces of each foot.

## 7. CONCLUSION

The spectral redundancy allowed us to apply either SVD, EVD, PDLC or AJD algorithms to the SCD matrices of the whitened processes, for each cyclic frequency and for every frequency bin as well, in order to identify the MIMO system up to certain ambiguities (namely constant diagonal and frequency dependent permutation and phase matrices). Two robust algorithms to overcome the frequency dependent permutation and phase ambiguities, based on cyclostationarity,

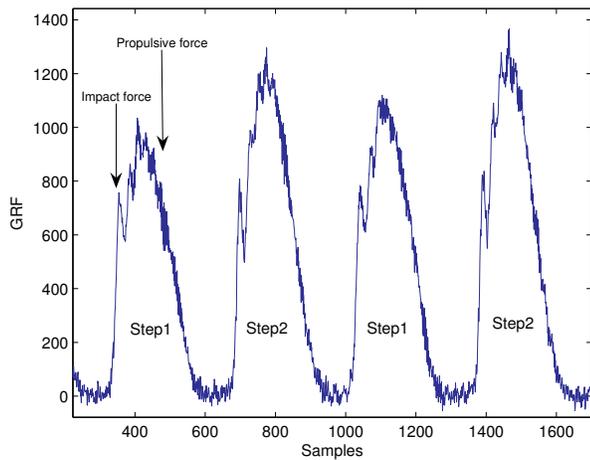


Figure 2: Vertical GRF in time domain

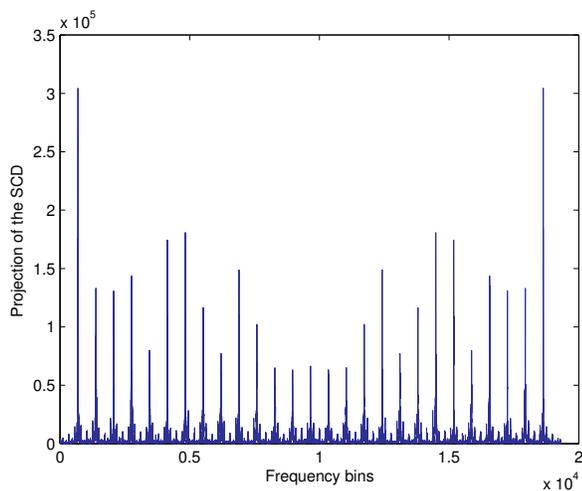


Figure 3: The SCD of the GRF signal

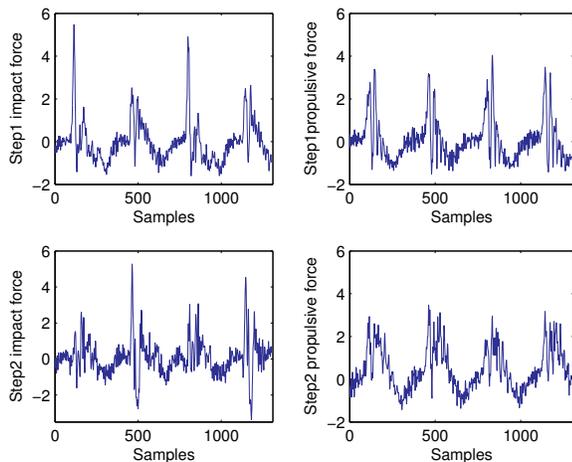


Figure 4: Separated signals

were presented. The performance of the new algorithms was demonstrated, it is apparent therefore that the proposed methods perform well specially the AJD based approach. Furthermore, the AJD based method was tested through real signals. The method is able to separate biomechanical signals that share the same cyclic frequency.

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