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MULTIPATH MITIGATION IN SPECTRUM ESTIMATION USING ℓ_1 MINIMIZATION

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ABSTRACT

We consider the problem of spectrum estimation of an Auto-Regressive (AR) process in a sparse multipath environment. The presence of even a small number of delayed and attenuated replica of the source signal in the received signal may severely degrade the performance of classical AR spectrum estimation methods. Dwelling on the sparsity of the multipath reflections, we propose an approach which looks for a Finite Impulse Response (FIR) filter which, when convolved with the received signal's autocorrelation sequence, yields the sparsest sequence. We show that under certain conditions such an approach provides a consistent estimate of the source's AR parameters if the ℓ_0 norm is used as a measure of sparsity. However, To maintain computational feasibility, we use the ℓ_1 norm instead. Significant performance improvement relative to the classical Yule-Walker (or Modified Yule-Walker) based estimates is demonstrated in simulation. We also consider the expansion of the method to the case of multiple sensors.

1. INTRODUCTION

Spectrum estimation is a fundamental problem in statistical signal processing, finding use in diverse applications such as detection, identification, compression, coding, filtering and more. One of the common approaches to spectrum estimation is Auto-Regressive (AR) modeling, which is also closely related to the Maximum Entropy method and to the Prediction Error method (see. e.g., [6], ch. 12.3). AR modeling is especially suited to cases where the (sampled) signal of interest (SOI) emanates from an all-poles system driven by some white noise - a common physical assumption, e.g., when working with speech signals over short time-intervals (see, e.g., [4], ch. 5).

In some cases of practical interest, a propagation path through which the SOI arrives at the sensor(s) is possibly characterized by a finite number of reflections (see, e.g., [1]). Such reflections give rise to the presence of delayed and attenuated multipath replica of the source signal, which in turn imply the addition of spectral zeros (in the Z-plane) to the overall generating system - rendering the received signal an Auto-Regressive, Moving Average (ARMA) process, rather than an AR process.

If the multipath parameters (delays and attenuations) are known, then a multipath-cancelation equalizer may be applied to the received signal in order to recover the SOI at a preprocessing stage. However, such side-information is rarely available in practice, and so one needs to work directly with the received signal.

If classical AR estimation approaches, such as the use of Yule-Walker (Y-W) equations (e.g., [6]), are applied directly to the received signal, the resulting accuracy may be severely degraded by the presence of multipath, and the resulting estimated spectrum would generally be biased and inconsistent. A possible classical remedy in such cases can be the use of Modified Y-W equations (e.g., [6]), regarding the received signal as an ARMA process, whose AR factor coincides with the AR factor of the SOI. The Modified Y-W equations exploit estimated autocorrelations of the received signal at lags farther from the origin, centered about the presumed MA order. The resulting estimate of the AR coefficients would be consistent (under commonly met second-order ergodicity conditions) if

1. All multipath delays are multiples of the sampling period; and
2. The presumed MA order is larger or equal to the largest multipath delay (in samples).

In reality, however, the first condition above is rarely satisfied. Moreover, although the Modified Y-W approach can offer a consistent estimate (namely, can attain exact estimates when the observation interval is infinite), the use of estimated correlations at far lags usually implies poor performance with moderate sample sizes - even with respect to the ordinary Y-W equations, since the bias elimination is traded for a severe increase in the variance. This is mainly because the relative error in the estimation of far-lagged correlations is usually significantly higher than the relative error in the estimation of the short-lagged correlations, since the true correlation values usually decrease rapidly at far lags, whereas the variance of their estimates does not.

Even when the number of multipath reflections is small, the maximum delay may be relatively high. The use of the Modified Y-W equations in such cases seems rather "wasteful": The implied MA order can be very high (the maximum delay, in samples) although the effective number of MA coefficients may be very small (the number of reflections).

A key observation in this context is that in scenes of isolated reflections, the MA coefficients induced by the propagation paths are sparse. We therefore propose to recover the AR parameters in terms of a filter which, when applied to the (estimated) autocorrelation of the received signal, approximately maximizes the sparsity of the implied MA coefficients by minimizing the ℓ_1 norm of their implied autocorrelation sequence. The use of the ℓ_1-norm as a conveniently-manageable approximate measure of sparsity has been considered before, e.g., by Donoho *et al.* ([5], [7]). Exploitation of the sparsity of multipath reflections through ℓ_1-norm minimization has also been recently considered in a different context by Lin *et al.* in [3].

As we shall see in simulation results, such an approach attains significant improvement in the accuracy of the AR parameters estimation (and implied spectrum estimation) over the use of Y-W or Modified Y-W equations.

Our approach becomes even more favorable when multiple sensors are available (e.g., in a sensors-array), each receiving the same SOI, possibly undergoing different multipath propagation profiles. In such cases we may search for the filter which simultaneously minimizes the ℓ_1 norm of all auto- and cross- correlation sequences between sensors. The resulting processing gain is significantly higher than the processing gain attained by applying the (Modified) Y-W equations individually to each sensor and using the averaged result. Note that the Y-W equations cannot exploit (at least not by straightforward application) the cross-correlation sequences, as these are not even symmetric in a general multipath scenario, and the exact shift of the zero-lag autocorrelation has to be determined first. This is not a trivial matter, since the multipath delays are generally not assumed to be integer multiples of the sampling period.

2. PROBLEM FORMULATION

Let $s(t)$ be the SOI, a Wide-Sense Stationary (WSS) continuous-time random process of unknown spectrum, whose samples at sample-intervals T_s, $s[n] \triangleq s(n\cdot T_s)$ $\forall n \in \mathbb{Z}$ can be modeled as a discrete-time AR process of known order P, namely, the WSS process $s[n]$ satisfies

$$s[n] = -\sum_{k=1}^{P} a_k s[n-k] + w[n] \quad , \quad \forall n \tag{1}$$

such that $w[n]$ is a zero-mean white process ("driving-noise") of some unknown variance σ_w^2, and the parameters $a_1, a_2, \ldots, a_P$ are the unknown AR parameters. It naturally follows that the polynomial $A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_P z^{-P}$ has all its P roots inside the unit-circle (in the Z-plane).

For example, it is common practice in speech processing / coding to assume that when $s(t)$ is a speech signal sampled at a sampling interval of $T_s = 1/8000$ $[s]$, the resulting $s[n]$ is an AR process of order $P = 10$ (over short time-intervals).

Assume now that $s(t)$ propagates to some sensor (e.g., a microphone) through some isolated-multipaths environment (e.g., room acoustics [1]). The received signal $x(t)$ can be modeled as

$$x(t) = \sum_{m=0}^{M} g_m s(t-\tau_m) \tag{2}$$

where M denotes the number of multipath reflections (in addition to the direct, main path), $\{g_m\}_{m=0}^{M}$ denote the unknown path-loss coefficients and $\{\tau_m\}_{m=0}^{M}$ denote the respective unknown propagation delays. Without loss of generality we shall assume that $\tau_0 = 0$, namely that the main path has zero delay. This is merely an arbitrary determination of the time-origin, and is certainly immaterial to the statistical characterization of $s(t)$, due to its stationarity. In addition, in order to mitigate the scaling ambiguity between the direct-path loss, the sensor gain and the SOI's power, we shall assume that the SOI has unit power, namely that $E[s^2(t)] = E[s^2[n]] = 1$.

The received signal $x(t)$ is sampled at time intervals T_s, and the samples $x[n] = x(n\cdot T_s)$ are the available data, from which it is desired to estimate the AR parameters, which in turn yield the spectrum estimate of $s[n]$.

Note that we only consider the noiseless case in here. Even in the absence of multipath, additive noise is considered a significant problem in AR spectrum estimation, and special care should be taken in applying any AR estimation scheme to noisy data. Due to space limitations, the issue of additive noise remains beyond the scope of this paper.

3. AR ESTIMATION THROUGH ℓ_1 MINIMIZATION

Let $R_{ss}[\ell] \triangleq E[s[n+\ell]s[n]]$ denote the autocorrelation of $s[n]$. The Z-transform of $R_{ss}[\ell]$ is the spectrum

$$S_{ss}(z) \triangleq \frac{\sigma_w^2}{A(z)A^*(1/z^*)}. \tag{3}$$

Let $H(z) \triangleq \frac{1}{\sigma_w^2} A(z)A^*(1/z^*)$ denote a Finite Impulse Response (FIR) filter, whose impulse response is denoted $h[\ell]$. It follows immediately that the convolution of $R_{ss}[\ell]$ with $h[\ell]$ results in an impulse (Kronecker's delta function, $\delta[\ell]$):

$$(h * R_{ss})[\ell] = \sum_{k=-P}^{P} h[k] R_{ss}[\ell-k] = \delta[\ell] \quad , \quad \forall \ell. \tag{4}$$

Now consider the following constrained minimization:

$$\min_{\tilde{h}[\ell]} \sum_{\ell=-\infty}^{\infty} \left| \sum_{k=-K}^{K} \tilde{h}[k] R_{ss}[\ell-k] \right| \; s.t. \sum_{k=-K}^{K} \tilde{h}[k] R_{ss}[k] = 1, \tag{5}$$

where $K \geq P$ is some specified length parameter. This problem can be stated as: *Find the finite sequence $\{\tilde{h}[\ell]\}_{\ell=-K}^{K}$, which minimizes the ℓ_1 norm of its convolution with $R_{ss}[\ell]$, subject to the constraint that the value of the convolution at $\ell = 0$ is* 1.

In view of the preceding discussion, the minimizing solution is readily seen to be given by

$$\tilde{h}[\ell] = \begin{cases} h[\ell] & |\ell| \leq P \\ 0 & |\ell| > P \end{cases} \tag{6}$$

since $h[\ell]$ satisfies the constraint and zeros-out the convolution at all $\ell \neq 0$, which means that it attains the minimum possible ℓ_1 norm of the convolution under the constraint (this norm evidently equals 1). In fact, $h[\ell]$ would be the minimizer of *any* proper norm of the convolution under the specified constraint. However, as we shall see immediately, we take special interest in the ℓ_1 norm.

Assume now that all the multipath delays in (2) are integer multiples of the sampling interval T_s, and define a set of coefficients $\{b_q\}_{q=0}^{Q}$, where $Q \cdot T_s$ is the largest delay and

$$b_k = \begin{cases} g_m & \text{if } \exists\, m \in [0,M] \mid \tau_m = k \cdot T_s \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

The received signal $x[n]$ can now be expressed as

$$x[n] = \sum_{k=0}^{Q} b_k s[n-k]. \tag{8}$$

Subsequently, define the polynomial $B(z) \stackrel{\triangle}{=} b_0 + b_1 z^{-1} + \cdots + b_Q z^{-Q}$, and consider $R_{xx}[\ell] \stackrel{\triangle}{=} E[x[n+\ell]x[n]]$, the autocorrelation sequence of $x[n]$. The Z-transform of $R_{xx}[\ell]$ is the spectrum,

$$S_{xx}(z) = B(z)B^*(1/z^*)S_{ss}(z), \tag{9}$$

and hence the convolution of $R_{xx}[\ell]$ with $h[\ell]$ yields the sequence $\beta[\ell]$, whose Z-transform is $B(z)B^*(1/z^*)$. This sequence is given by

$$\beta[\ell] = \sum_{q=0}^{Q} b_q b_{q-\ell} \forall \ell \tag{10}$$

(under the convention that $b_k = 0$ for $k \notin [0, Q]$).

When the number of multipaths M is small, the sequence $\beta[\ell]$ is sparse, having at most $2M+1$ nonzero elements. It is then conceivable that $h[\ell]$ can be found by looking for the filter which, when convolved with $R_{xx}[\ell]$, yields the sparsest sequence (under the scaling constraint, to avoid a trivial zero solution). The most "natural" measure of sparsity is the ℓ_0 norm, which counts the number of nonzero elements. Indeed, we can establish the following theorem:

Theorem 1. *Assume the integer-delays multipath model in* (8), (7) *and let* $\tilde{\beta}[\ell] \stackrel{\triangle}{=} \sum_{k=-K}^{K} \tilde{h}[k] R_{xx}[\ell - k]$ *denote the convolution between the autocorrelation sequence of* $x[n]$ *and an FIR filter of length* $2K+1$ *with* $K \geq P$*. Let the minimum time-distance between all multipath components be given by* $D \cdot T_s$*, namely* $|\tau_m - \tau_n| \geq D \cdot T_s$ *for all* $n \neq m$, $n, m \in [0, M]$*. Then if* $K < P + D/2$*, the filter* $\{\tilde{h}[\ell]\}_{k=-K}^{K}$ *which minimizes (subject to the scaling constraint) the* ℓ_0 *norm* $\|\tilde{\beta}[\ell]\|_0$*, is given by* (6) *(namely, is equivalent to* $h[\ell]$*)).*

Proof. When $K = P$, only $\tilde{h}[\ell] = h[\ell]$ is capable of canceling all the $2P$ poles of $S_{xx}(z)$. With any other filter $\tilde{h}[\ell]$ of the same length, the resulting convolution will be infinite, due to remaining uncanceled poles, and as such would have an infinite ℓ_0 norm.

When $K > P$, the optimal $\tilde{h}[\ell]$ should still cancel all of the poles of $S_{xx}(z)$ (in order to avoid an infinite ℓ_0 norm of $\tilde{\beta}[\ell]$), but might theoretically have $2(K-P)$ extra zeros. The resulting $\tilde{\beta}[\ell]$ in such a case would be given by the convolution of $\beta[\ell]$ with a sequence of length $2(K-P)+1$, corresponding to a polynomial (in the Z-plane) representing the extra zeros. But since the minimum distance between nonzero taps in $\beta[\ell]$ is greater than $2(K-P)$ (according to the condition on K), such a convolution cannot generate any cancelation of taps, and is therefore guaranteed to at least double the number of non-zero taps in $\tilde{\beta}[\ell]$ (with respect to $\beta[\ell]$), thereby attaining at least double the ℓ_0 norm that would be attained by $h[\ell]$. This means that the optimal $\tilde{h}[\ell]$ cannot have such extra zeros, and must therefore be given by (6). □

It therefore follows, that assuming the availability of a consistent estimate of $R_{xx}[\ell]$, a consistent estimate of $h[\ell]$ can be attained by finding the ℓ_0-norm minimizer of $\tilde{\beta}[\ell]$ (under the specified length and scaling constraints). Unfortunately, however, the ℓ_0 norm has several problematic aspects as a minimization criterion: First, it is highly sensitive to numerical and/or statistical errors, as any slight deviation from zero counts as a nonzero and adds to the norm. This problem may be mitigated by thresholding; however, even in a thresholded version, ℓ_0 minimization is a non-convex optimization problem, and is NP-hard.

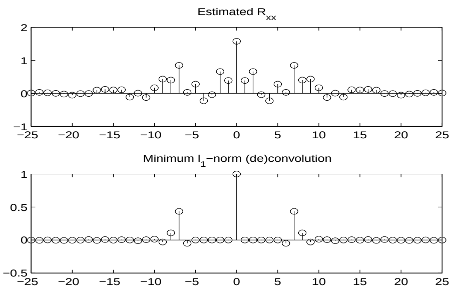

Figure 1: Top: a typical correlation sequence of the received signal in a multipath environment; Bottom: the ℓ_1 sparsified result. The multipath delay is 7.2 (non-integer)

A possible appealing alternative to ℓ_0 minimization is ℓ_1 minimization (see, e.g., [5], [7]), which, although less true to the notion of sparsity than ℓ_0, is still closely related to sparsity, yet admits convenient minimization algorithms due to its convexity.

We should emphasize that we did not prove (not claim) that the proposed ℓ_1 minimization yields a consistent estimate in our case, not even under the length and sparsity conditions of Theorem 1. Nevertheless, it is intuitively reasonable to assume that an ℓ_1-optimal filter would still have to cancel all of the poles of S_{xx}, so as to avoid an infinite-length $\tilde{\beta}[\ell]$. But this is only an intuitive notion, since in contrary to the situation with the ℓ_0 norm, in some cases an infinite sequence may have a smaller ℓ_1 norm than a finite sequence (even under our scaling constraint).

As shown above, consistency of the ℓ_1 minimizer is only guaranteed when there is no multipath ($M = 0$) - but in such cases the ordinary Y-W estimate is also consistent. But while the Y-W based estimates can suffer significant degradation in the presence of multipath, the proposed method seems better-immune to sparse multipath components. Indeed, we have observed in simulations that the proposed method significantly outperforms the competing Y-W based estimates in a sparse multipath scenario. This is also true (empirically, in our experiments) when the multipath delays are not integer multiples of the sampling intervals. In such cases Theorem 1 does not hold, and the theoretical derivations above become slightly more complicated, as the discrete coefficients b_q should be replaced with interpolation coefficients (essentially, sampled and time-shifted sinc$(\cdot)$ functions). Nevertheless, the general structure, the "essential sparseness" and the rationale behind the approach, all remain the same.

4. IMPLEMENTATION DETAILS

In order to apply the proposed approach we need to outline a minimization strategy for solving the constrained minimization problem (5). Before we do that, we would like to introduce a slight modification to this minimization problem, by imposing an additional symmetry constraint on the filter $\tilde{h}[\ell]$. This is a very reasonable constraint, since it exploits the knowledge that the true $h[\ell]$ is symmetric, and reduces the number of free parameters in the minimization by a factor of two. In addition, for practical considerations we would not consider the convolution from $-\infty$ to ∞, but only over a finite (long) lags-span, $\ell \in [-L, L]$, with $L >> K$.

Let us therefore denote by $\tilde{\mathbf{h}} \stackrel{\triangle}{=} [\tilde{h}[0]\ \tilde{h}[1]\ \cdots\ \tilde{h}[K]]^T$ the positively-indexed half of the symmetric sequence $\{\tilde{h}[\ell]\}_{\ell=-K}^{K}$. Since the convolution relation is a linear operator with respect to $\tilde{\mathbf{h}}$, we may reformulate (5) as

$$\min_{\tilde{\mathbf{h}}} \|\mathbf{R}^T\tilde{\mathbf{h}}\|_1 \quad s.t.\ \ \mathbf{c}^T\tilde{\mathbf{h}} = 1. \tag{11}$$

Here the $(2L+1)\times(K+1)$ matrix $\mathbf{R}^T$ is the sum of a partly-Hankel and a Toeplitz matrix, as follows:

$$\mathbf{R}^T = \begin{bmatrix} 0 & \hat{r}[-L+1] & \hat{r}[-L+2] & \cdots & \hat{r}[-L+K] \\ 0 & \hat{r}[-L+2] & \hat{r}[-L+3] & \cdots & \hat{r}[-L+1+K] \\ \vdots & \vdots & \iddots & \iddots & \vdots \\ 0 & \hat{r}[1] & \hat{r}[2] & \iddots & \hat{r}[K] \\ \vdots & \vdots & \iddots & \iddots & \vdots \\ 0 & \hat{r}[L+1] & \hat{r}[L+2] & \cdots & \hat{r}[L+K] \end{bmatrix} + $$
$$+ \begin{bmatrix} \hat{r}[-L] & \hat{r}[-L-1] & \cdots & \hat{r}[-L-K] \\ \hat{r}[-L+1] & \hat{r}[-L] & \cdots & \hat{r}[-L+1-K] \\ \vdots & \ddots & \ddots & \vdots \\ \hat{r}[0] & \hat{r}[-1] & \ddots & \hat{r}[-K] \\ \vdots & \ddots & \ddots & \vdots \\ \hat{r}[L] & \hat{r}[L-1] & \cdots & \hat{r}[L-K] \end{bmatrix}. \tag{12}$$

The $K+1$ vector $\mathbf{c}^T$ is the middle row of $\mathbf{R}^T$, namely (in Matlab® notation), $\mathbf{c} = \mathbf{R}(:, L+1)$. The entries $\hat{r}[\ell]$ are shorthand for the autocorrelation values $R_{xx}[\ell]$. In practice, estimated values are used,

$$\hat{r}[\ell] \stackrel{\triangle}{=} \hat{R}_{xx}[\ell] = \frac{1}{N-|\ell|}\sum_{n=0}^{N-1-|\ell|} x[n+|\ell|]x[n] \qquad \ell \in [-L-K, L+K] \tag{13}$$

(in practice, the symmetry of both $\hat{r}[\ell]$ and $\tilde{h}[\ell]$ can be readily exploited to reduce the number of rows in $\mathbf{R}^T$ by half, but we shall not pursue this option in here, due to space limitations).

Next, we can turn the constrained minimization problem (11) into a more convenient unconstrained minimization problem as follows: Set $\tilde{\mathbf{h}}_0 \stackrel{\triangle}{=} \mathbf{c}/(\mathbf{c}^T\mathbf{c})$, and denote by the $(K+1)\times K$ matrix $\mathbf{C}$ any complete basis for the K-dimensional null-space of $\mathbf{c}^T$. Thus, any vector $\mathbf{y}$ satisfying $\mathbf{c}^T\mathbf{y} = 0$ can be expressed as $\mathbf{y} = \mathbf{C}\mathbf{x}$. $\mathbf{C}$ can be easily found, e.g., using Gram-Schmidt orthogonalization. It then follows that any solution $\tilde{\mathbf{h}}$ satisfying the constraint in (11) can be expressed as $\tilde{\mathbf{h}} = \tilde{\mathbf{h}}_0 + \mathbf{C}\mathbf{x}$. Substituting back into the criterion in (11), we end up with the unconstrained minimization

$$\min_{\mathbf{x}} \|\mathbf{R}^T(\tilde{\mathbf{h}}_0 + \mathbf{C}\mathbf{x})\|_1 \ \equiv\ \min_{\mathbf{x}} \|\mathbf{A}^T\mathbf{x} - \mathbf{b}\|_1, \tag{14}$$

with $\mathbf{A} \stackrel{\triangle}{=} \mathbf{C}^T\mathbf{R}$ and $\mathbf{b} \stackrel{\triangle}{=} -\mathbf{R}^T\tilde{\mathbf{h}}_0$. This is a classical linear ℓ_1 minimization problem. Direct minimization of this convex criterion can be obtained in an iterative algorithm with guaranteed convergence to the unique (global) minimum within a finite number of iterations, see, e.g., [2] for a possible algorithm. To use more standard tools, this problem can also be cast as a linear program,

$$\min_{\mathbf{w},\mathbf{x}}\ [\mathbf{1}^T\mathbf{0}^T]\begin{bmatrix}\mathbf{w}\\ \mathbf{x}\end{bmatrix} \quad s.t.\quad \begin{bmatrix}\mathbf{I} & -\mathbf{A}^T \\ \mathbf{I} & \mathbf{A}^T\end{bmatrix}\begin{bmatrix}\mathbf{w}\\ \mathbf{x}\end{bmatrix} \geq \begin{bmatrix}-\mathbf{b}\\ \mathbf{b}\end{bmatrix}, \tag{15}$$

where $\mathbf{1}$ and $\mathbf{0}$ denote $(2L+1)\times 1$ all-ones and all-zeros vectors (resp.), $\mathbf{I}$ denotes the $(2L+1)\times(2L+1)$ identity matrix and $\mathbf{w}$ is an auxiliary $(2L+1)\times 1$ vector participating in the minimization.

Once the unconstrained minimizing solution $\hat{\mathbf{x}}$ (of (14)) is found, the desired solution of (11) is given by $\hat{\mathbf{h}} = \tilde{\mathbf{h}}_0 + \mathbf{C}\hat{\mathbf{x}}$. The symmetric filter $\hat{h}[\ell]$ is then extracted from $\hat{\mathbf{h}}$ as $\hat{h}[\ell] = \hat{\mathbf{h}}_{|\ell|}, \ell = -K, \ldots, K$ ($\hat{\mathbf{h}}_k$ denoting the $(k-1)$-th element of the vector $\hat{\mathbf{h}}$). Then, the polynomial

$$\hat{H}(z) \stackrel{\triangle}{=} \sum_{\ell=-K}^{K} \hat{h}[\ell]z^{-\ell} \tag{16}$$

is constructed and rooted, yielding $2K$ roots $\{\hat{p}_k\}_{k=1}^{2K}$. Assuming no roots on the unit-circle, they can be partitioned into two groups of K roots each: The first group of roots (indexed $1, 2, \ldots, K$) contains the roots located inside the unit-circle, and the second (indexed $K+1, K+2, \ldots, 2K$) containing their reciprocals. By forming the monic polynomial $\hat{A}(z) = \prod_{k=1}^{K}(1 - p_k z^{-1})$, one can read off the estimated AR coefficients from the resulting polynomial, or, alternatively, to obtain a direct estimate of the SOI's spectrum,

$$\hat{S}_{ss}(e^{j\omega}) = \frac{\hat{\sigma}_w^2}{|\hat{A}(e^{j\omega})|^2}, \tag{17}$$

where $\hat{\sigma}_w^2$ is determined so as to comply with the unit-power convention $\hat{R}_{ss}[0] = \frac{1}{2\pi}\int_0^{2\pi} \hat{S}_{ss}(e^{j\omega})d\omega = 1$.

5. MULTIPLE SENSORS

When more than one sensor is available, it is possible to exploit not only the autocorrelations of the individual sensors, but also the cross-correlations between sensors. Note that the Z-transforms of all of these correlation sequences share the same poles (namely, the poles of the SOI) and (possibly) differ only by their zeros. Therefore, the same filter $h[\ell]$ would sparsify all of these sequences simultaneously.

We therefore propose to apply the same ℓ_1 minimization approach to the concatenated convolutions of $\tilde{h}[\ell]$ with all of the available (estimated) auto- and cross-correlation sequences. This can be easily attained by augmenting the matrix $\mathbf{R}$ in (12) with similarly constructed matrices, in which $\hat{r}[\ell]$ denotes the respective estimated auto- or cross-correlation sequences. The scaling-constraint vector $\mathbf{c}$ would be determined from one of the autocorrelation sequences.

When several sequences are to be simultaneously sparsified by convolution with the same common sequence, the occurrence of ‘’coincidence solutions”, yielding a smaller ℓ_1 norm than the ”intended solution”, becomes more rare. As a result, the accuracy of the solutions is significantly improved - as we shall demonstrate in simulations.

6. SIMULATION RESULTS

To demonstrate the performance of the proposed method (given the acronym “SPARE”: SParsity-based AR Estimation) we simulated an AR SOI undergoing a multipath environment as follows: The SOI was generated first as a

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discrete AR process of order $P = 6$ with the AR coefficients set to correspond to six poles at $[0.8, 0.6+0.6j, 0.6-0.6j, -0.45, -0.7+0.4j, -0.7-0.4j]$ (an arbitrary choice), excited by white Gaussian random noise. This signal was then upsampled (interpolated) by a ratio of 1 : 10, so as to simulate the continuous SOI and to enable fractional delays (at a 0.1 resolution). We then simulated the propagation model (2) with $M = 1, 4$ multipath components (in addition to the main path). To apply fractional delays, each (m-th) replica of the interpolated signal was shifted by the rounded value of $10 \cdot \tau_m$ (where τ_m is the desired fractional delay, in samples), and then the combined signal was downsampled by 1 : 10 to generate the "sampled" signal $x[n]$ at the original sample rate.

We used $g_0 = 1$, and we shall show the performance versus a single parameter g, controlling the relative magnitude(s) g_m of the multipath component(s) as follows: In the single-multipath experiment the magnitude $|g_1| = g$ was held constant over repeated trials and only the sign of g_1 was drawn at random in each trial. In the 4-multipaths experiments, the magnitudes $|g_m|$ ($m = 1, 2, 3, 4$) were each drawn at random (independently) in each trial from a uniform distribution between $-g$ and g.

The random time-delays were drawn independently for each multipath component in each trial, from a uniform distribution between 1 and 5 (resp., 10) for the single multipath (resp., 4-multipaths) experiment.

As a measure of the accuracy of the spectrum estimates we used the well-known Itakura-Saito spectral distance (e.g., [4]) of AR estimates (in its frequency-domain version):

$$d(S(e^{j\omega}), \hat{S}(e^{j\omega})) \triangleq \frac{1}{2\pi} \int_0^{2\pi} \left[\frac{S(e^{j\omega})}{\hat{S}(e^{j\omega})} - 1 - \log \frac{S(e^{j\omega})}{\hat{S}(e^{j\omega})} \right] d\omega. \quad (18)$$

In addition to the single-channel experiment, we also conducted two-channels experiments, with similarly randomized, independent multipath parameters in the second channel. In addition, a random delay between the direct-paths to each channel was applied in each trial, drawn independently from a uniform distribution between -3 and 3.

In applying SPARE we used estimated correlation sequences of (one-sided) length $L = 50$ and a convolution filter of (one-sided) length $K = 7$. The data length was $N = 50000$. We compare the performance of SPARE to the performance attained by ordinary Y-W estimation in Figures 2 and 3. In the two-channels experiment the Y-W spectral estimate was constructed as the averaged Y-W spectral estimates from the two channels. Each simulation point represents the average of 1000 independent trials (in which all multipath parameters, as well as the SOI, were redrawn independently), discarding the worst 5% of the results for both algorithms, to avoid occasional outliers, occurring in both methods upon "pathological" realizations of multipath constellations. We also applied the Modified Y-W estimate, but its average performance was generally much worse (nearly by orders of magnitude) than that of the ordinary Y-W estimate, so we chose to exclude these results from the Figures.

7. CONCLUSION

We presented a new method for the mitigation of the effects of multipath on AR spectrum estimation. The method applies ℓ_1-norm minimization to the convolution of the estimated

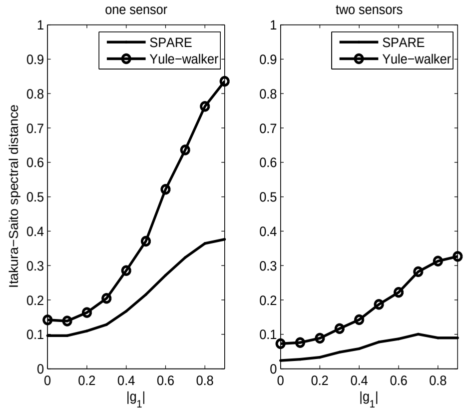

Figure 2: Single multipath: Estimation accuracy vs. multipath's relative magnitude.

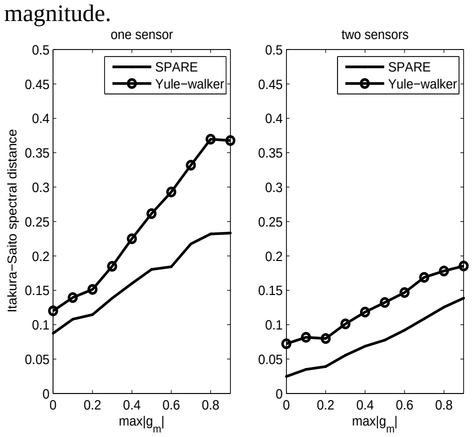

Figure 3: Four multipaths: Estimation accuracy vs. multipath's relative magnitude.

correlation sequence with an FIR filter, thereby attempting to find the filter whose zeros are the SOI's spectral poles. Although consistency is not guaranteed (due to the use of an ℓ_1 instead of an ℓ_0 norm), significant performance improvement over Y-W based methods was demonstrated in simulations. Naturally, the improvement is attained at the cost of increased computational complexity, yet the ℓ_1 minimization can be applied with guaranteed global convergence within a finite number of iterations.

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