ITERATIVE ENHANCEMENT OF EVENT RELATED POTENTIALS THROUGH SPARSITY CONSTRAINTS

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ABSTRACT

In this paper we propose an iterative technique that enhances the average event related potential (ERP) by correcting the delay associated with the ERP in each trial. This correction is done in three steps: in the first step a sparse template function is estimated. In the second step, this template is utilized in estimating the inter–trial ERP delays. The ERPs from each trial are time-aligned using the estimated delays. In the third step, a new estimate of the ERP waveform is obtained by averaging these time–aligned signals over the trials. The algorithm iterates through these three steps until convergence. The sparse template is estimated in each iteration through the minimization of a convex objective function which compromises between the fit of the estimated ERP waveform to the template, and the sparsity of the estimated ERP waveform.

Indexing Terms: Electroencephalogram (EEG), event related potential (ERP), convex optimization.

1. INTRODUCTION

The event related potential (ERP) is a brain response to an external stimulus. The ERP has a well defined pattern whose amplitude and latency can be used in the diagnosis of possible brain injury or disorders in the central nervous system [1]. The ERP is characterized by a relatively low signal to noise ratio (SNR) with respect to the background brain EEG activity. Accordingly, the ERP is usually estimated by averaging over a large number of trials. The disadvantage of this approach is that it is based on the assumption that the ERP occurs exactly at the same time after the stimulus onset in all trials. In reality, this assumption is not realistic.

A more realistic model for the measured EEG signal in the i-th trial can be represented mathematically by [2]

$$x_i(t) = a_i s(t - d_i) + n_i(t), \quad i = 1, \ldots, N \tag{1}$$

where $x_i(t)$ is the measured EEG signal, $s(t)$ is the ERP waveform, $n_i(t)$ is the background brain activity, N is the total number of trials, a_i is a random variable representing the amplitude of the ERP, and d_i is an unknown delay. Due to the variation in the d_i's, the estimated ERP $\hat{s}(t)$ obtained due to averaging over trials will be distorted. Thus, many previous approaches have attempted to correct this problem by estimating the d_i, and time–aligning the individual responses before averaging.

There are many algorithms for solving the problem of delay estimation. These algorithms are based either on cross correlation [3], adaptive techniques [1], [2] or maximum likelihood (ML) [4]. Each of these algorithms assume different characteristics of the signal and noise; e.g. in [4], it was assumed that the background EEG is a realization of a zero-mean stationary Gaussian process, while in [2] it was assumed to be a realization of a non–Gaussian process. Clearly, invoking some assumptions on the model may result in inaccurate estimate of the delays. A comparison between some of these algorithms is presented in [5].

In this paper, our objective is to enhance the ERP, through estimation of the delays. The challenge associated with this problem is the similarity between the ERP and the background brain activity in such a way that, until now, it is not known precisely whether the ERP is a separate signal added to the background brain activity [6], or it is just a result of partial phase resetting in the background brain activity [7]. This implies that the waveforms of the ERP and the background EEG signals have similar characteristics. We invoke a realistic assumption that ERP exists only in a small window after the stimulus onset, specified by the interval $[d_{min}, d_{max}]$, referred to as the *the window of interest* (*WOI*). This assumption implies that the ERP is a *sparse* signal. A signal is sparse if it has few non zero elements. In addition, we assume also that the ERP dominates the background EEG signal inside the *WOI*.

In this paper, we make use of a sparse "template signal" $p(t)$, which is a good fit to the ERP inside the *WOI*. An optimization technique is utilized for estimating this template. Once $p(t)$ is estimated, the cross correlation between the template and each $x_i(t)$ is utilized for estimating the delays d_i.

This paper is organized as follows: in Section 2 The development of the proposed method is presented. The proposed technique is presented in Section 3. The performance of the proposed algorithm is examined in Section 4. Finally,

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Section 5 contains some concluding remarks.

2. DEVELOPMENT OF THE PROPOSED METHOD

The first attempt to correct the delays associated with the ERP was proposed by Woody [3]. Woody's method can be summarized as follows: given an initial estimate of the template $p(t)$ and a set of measured EEG signals $x_i(t), i = 1,\ldots,N$, the d_i for each trial is estimated as follows

$$\hat{d}_i = \arg \max_{d_{min} \le d \le d_{max}} \quad \frac{1}{T} \sum_{t=1}^{T} x_i(t) p(t-d) \tag{2}$$

where T is the total number of samples in each trial. The data set $\{x_i(t)\}$ can then be realigned using the $\hat{d}_i$'s, and an enhanced ERP is obtained by averaging these realigned trials. An improved iterative version of this technique was proposed in [8]. In each iteration, a new template $p(t)$ is obtained by averaging the realigned data from the previous iteration over the trials. Then this new template is used to re-estimate the delays, and the process iterates until convergence.

The template used in [3] and [8] is the average ERP, while a positive half-cycle sine wave is utilized in [9]. Each of these choices has its own relative merits. For the first approach, the advantage is the averaged ERP is a relatively good fit to $s(t)$ inside the *WOI*, specially when the maximum jitter in the delays is not large. The disadvantage is that, the tails of the averaged ERP, which exist outside the *WOI*, create false peaks in the cross correlation function of (2) and accordingly can create outliers in the delay estimates. On the other hand, the half–sinusoid template does not have these tails, but the disadvantage is that, inside the *WOI*, its waveform is not a good fit to that of $s(t)$. Accordingly, the cross–correlation peaks of (2) may not lead to a good estimate of the d_i.

In [5] it is shown that, for both cases of template, the iterative approach is subject to divergence after the first iteration. That is, peaks in (2) correspond to $p(t)$ aligning with $n_i(t)$ rather than with $s(t)$. This can be explained by noting that, estimating d_i via (2) is optimum under two conditions [10]: 1) the background noise, $n(t)$, is white, and 2) $p(t) = s(t)$, i.e. the template is exactly the hidden ERP. Clearly, neither of these two conditions is satisfied in the problem at hand; $n(t)$ is a colored noise and $s(t)$ is unknown. Accordingly, to get reliable results, a better template must be utilized, and the background noise has to be whitened.

3. PROPOSED METHOD

In this section we propose an extension to the method of [8] for estimating the ERP waveform $s(t)$. As a first step, the data is *partially whitened*, as shown in Fig. 1. For initialization of this process, the averaged $x_i(t), i = 1,\ldots,N$ are used as an estimate $\hat{s}_0(t)$ of $s(t)$. The iteration index $k := 1$. The process then iterates as follows: 1) a sparse template $p_k(t)$ is estimated from $\hat{s}_{k-1}(t)$ in a manner to be described. 2) This template function $p_k(t)$ is used for estimating the delays d_{ik} in (2). 3) The $x_i(t)$ are then realigned using the estimated delays, and an enhanced ERP signal $\hat{s}_k(t)$ is obtained by averaging these realigned $x_i(t), i = 1,\ldots,N$. The process iterates

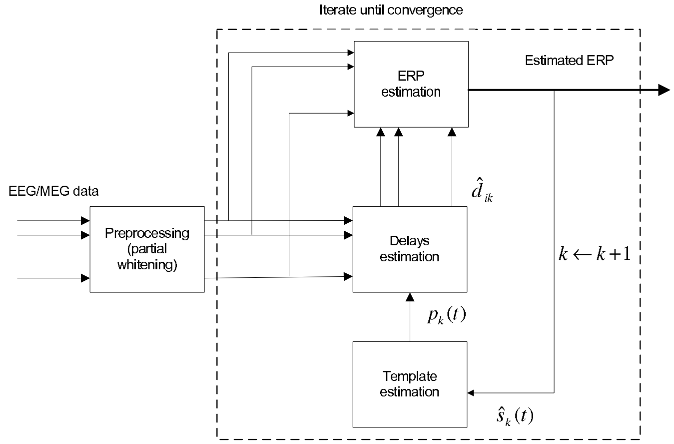


Figure 1: Block diagram of the proposed technique

until convergence. In the remaining portion of this section we give a brief description of each block shown in Fig. 1.

3.1 Partial Whitening

As explained in Section 1, the ERP and the background EEG signal almost have similar waveforms inside the *WOI*. Accordingly, whitening the background EEG signal may diminish the maximum ERP signal amplitude relative to the background EEG. As an alternative approach, we propose partial whitening instead of complete whitening.The idea of partial whitening is presented in [11]. This method is applied in the frequency domain. The following quantity is evaluated:

$$\tilde{X}_i(f) = \frac{X_i(f)}{|X_i(f)|^{\alpha}} \tag{3}$$

where $X_i(f)$ is the Fourier transform of $x_i(t)$ and $0 \le \alpha \le 1$. The partially whitened signal $\tilde{x}_i(t)$ is the inverse Fourier transform of $\tilde{X}_i(f)$. Here $\alpha = 0$ corresponds to no whitening, while $\alpha = 1$ corresponds to complete whitening. Empirically, good results are obtained in our context when $0.5 \le \alpha \le 0.7$.

3.2 Template Estimation

From the discussion in Section 2, it is clear that a good template function $p(t)$ is the one which possesses these properties: i) it fits the average ERP inside the *WOI* and ii) it is a sparse signal, i.e. it has no tails outside the *WOI*. In this subsection we propose a convex objective function for determining such a template.

For convenience, we represent a signal $y_k(t), t = 1,\ldots,T$ as a vector $\mathbf{y_k} \in \Re^{\mathbf{T}}$. At iteration k, the template is estimated by minimizing the following objective function

$$\mathbf{p_k} = \arg\min_{\mathbf{p}} \quad \mu \|\hat{\mathbf{s}}_{\mathbf{k-1}} - \mathbf{p}\|_{\mathbf{2}} + \|\mathbf{p}\|_{\mathbf{1}} \tag{4}$$

where $\hat{s}_{k-1}(t)$ is the averaged ERP waveform at the $k-1$ iteration.

This objective function consists of two different terms, one corresponding to each condition. The first term, $\|\hat{\mathbf{s}}_{k-1} - \mathbf{p}\|_2$, measures the fit between $\hat{\mathbf{s}}_{k-1}$ and $\mathbf{p}$. The second term, $\|\mathbf{p}\|_1$, suppresses the tails of $p(t)$ and hence encourages sparseness. Here, the l_1–norm is used to enhance the sparsity of the template function [12]. Since these two terms are both convex, the overall objective function is also convex. As a result, for a given μ, the algorithm always converges to the global minimum which is a compromise between sparsity and fit. The cvx software[1] is utilized for solving the objective function.

The tuning parameter μ is chosen to give a balance between decreasing the error between $\mathbf{p}$ and $\hat{\mathbf{s}}_{k-1}$ on the one hand, and increasing the sparsity of $\mathbf{p}$ on the other hand. In the simulation results, good results are obtained when $5 \leq \mu \leq 10$.

3.3 Delay Estimation

The estimated template is used to estimate the delays $d_{ik}, i = 1, 2, \ldots, N$. At the kth iteration, for the i-th trial, $\tilde{d}_{ik}$, the estimate of d_{ik}, is the delay at which the cross correlation between $\tilde{x}_i(t)$ and $p_k(t)$ is maximum, i.e.

$$\begin{aligned} \tilde{d}_{ik} &= \arg \max_{d_{min} \leq d \leq d_{max}} \frac{1}{T} \sum_{t=1}^{T} \tilde{x}_i(t) p_k(t-d) \\ &= \arg \max_{d_{min} \leq d \leq d_{max}} \frac{1}{T} \sum_{t=1}^{T} [s(t-d_i) p_k(t-d)] \\ &\quad + \frac{1}{T} \sum_{t=1}^{T} [n_i(t) p_k(t-d)] \end{aligned} \quad (5)$$

where $\tilde{x}_i(t)$ is the partially whitened version of $x_i(t)$ from (3). Note that, utilizing cross correlation for estimating the delays relies on the fact that the background EEG noise and the ERP, represented by the template in (5), are uncorrelated [1, 2, 4]. Regardless, the second term in (5) is generally non–zero for finite T.

Assuming that the ERP dominates the noise inside the *WOI*, then the contribution of the second term is alleviated by utilizing a sparse template, which has negligible values where the background noise dominates. Further, due to the fit between the waveforms $p_k(t)$ and $s(t)$ within the *WOI*, the cross correlation between $\tilde{x}_i(t)$ and $p_k(t-d)$ will be maximized when $d \simeq d_i$, thus reducing the probability of occurrence of outliers in the delay estimates (in contrast to the case where, e.g., $p(t)$ is a half-sine wave function). Thus, the difficulties with the previous approaches are alleviated with the proposed approach.

3.4 ERP Estimation

After estimating the delays, each trial is time–aligned and $\hat{s}_k(t)$, the estimate of the ERP at the k-th iteration, is calculated by averaging all the time-aligned trials, i.e., $\hat{s}_k(t) = \frac{1}{N} \sum_{i=1}^{N} x'_{ik}(t)$ where $x'_{ik}(t) = \tilde{x}_i(t + \tilde{d}_{ik})$. Then $\hat{s}_k(t)$ is used for estimating a new template $p_{k+1}(t)$ in (4), and the process iterates until there is no significant change in the estimated delays.

[1] This is a free software available at http://www.stanford.edu/˜boyd/cvx/

4. EXPERIMENTAL RESULTS

In this section we present two examples in order to quantify the performance of the proposed algorithm.

Example 1

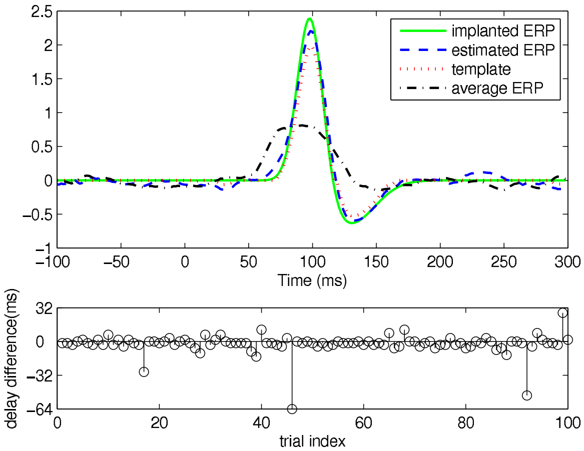
In this example the ability of the proposed algorithm in estimating the ERP waveform is examined. In doing this, 100 trials of synthesized EEG data are generated. The duration of each trial is 400 ms and the sampling frequency is 625 Hz. The first 100 ms are considered as a pre-stimulus period. Then a synthesized ERP signal is implanted in each trial according to (1), where $\{a_i\}$ are adjusted such that the $SNR = -7dB$. The delays are sampled from a uniformly distributed random variable. The mean of the delays is 100 ms after the stimulus onset, and the maximum delay jitter is ± 32 ms. The *WOI* is therefore specified by $[68, 132]$ ms.

To examine the performance of the proposed algorithm in estimating the ERP waveform, two different waveforms are used in simulations. These waveforms are: $s_1(t) = g(100, 10) - 0.5g(130, 20)$, and $s_2(t) = g(100, 10) - 0.5g(130, 20) - 0.5g(80, 10)$, where $g(m, \sigma)$ is a Gaussian pulse with mean m and standard deviation σ. In addition, the result of applying Woody's method [3], for the case when $s(t) = s_2(t)$, is also presented. The results are shown in Fig. 2. As we can see in this figure, for a varity of shapes of implanted ERP, the proposed algorithm can estimate the ERP correctly. From the figures, the sparsity of the template function (shown as red dots) outside the *WOI* is apparent. Sparsity is the key behind this improved level of performance. The lower panel in each sub-figure contains the difference between the exact and estimated delays. From these sub-figures it is clear that the estimated delays show a reasonable level of accuracy. Particularly, it is apparent that there are relatively few outliers in the delay estimates. In contrast, the results for the original implementation of Woody's method are shown in Fig. 2c, where it is seen that the relative occurrence of outliers in the delay estimates is significantly higher, and consequently the waveform estimates are degraded.

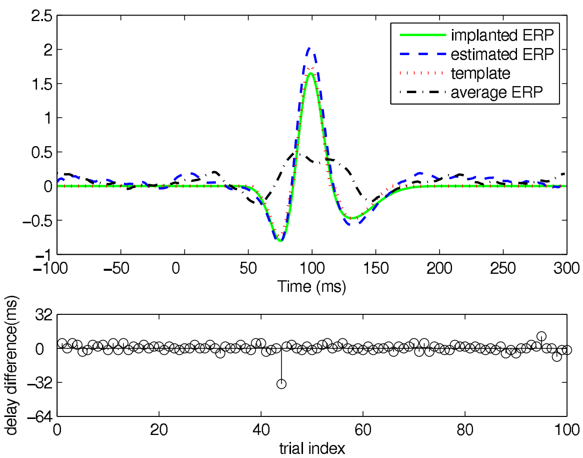
Example 2

In this example, the performance of the proposed algorithm at different values of the SNR and jitter is examined. In doing this, $s_2(t)$ is used as an ERP signal and added to 100 different synthesized realizations of background EEG signal. The range of the SNR is from -12 to 0 dB, and the delays are sampled from a uniformly distributed random variable with maximum delay jitter in the range from ± 8 to ± 40 ms.

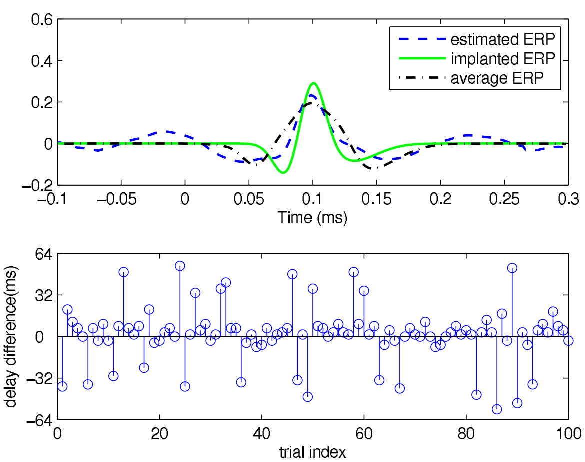
As measures of the performance, we propose the following two parameters 1) the number of *outlier delays*, which we define as the delays at which the difference between the exact delay and the estimated delay is greater than the range



(a)



(b)



(c)

Figure 2: Results for Example 1. In each sub-figure, the upper panel indicates how accurately the implanted ERP can be estimated, while the lower panel represents the difference between the exact and estimated delays versus the trial index, i. The "average ERP" represents the conventionally estimated ERP without correcting the delays. (a) and (b) show results for the proposed method for different $s(t)$ waveforms, whereas (c) corresponds to Woody's method.

of the jitter at which the ERP is implanted. The ratio between this number and the total number of trials can be considered as a measure of the probability that the algorithm converges to the background activity rather than the implanted ERP, 2) the variance of the difference, after removing the outlier delays, between the exact and estimated delays. The smaller the value of the variance, the better the performance of the algorithm.

For each value of the SNR and jitter, each one of these two parameters is calculated as the average over 100 different runs, and in each run the SNR is kept unchanged while the delays are randomly changed in the pre-specified jitter range.

The effect of partial whitening on the performance of the algorithm is also shown. The results are shown in Fig. 3a. By comparing the left (with no partial whitening) and right (with partial whitening) panels, we can see that this process is effective in reducing the level of outliers. This figure shows that the probability that the proposed algorithm converges to the background EEG noise is small, e.g. for partially whitened signals at -8 dB SNR, and for all jitter values, the percentage number of outlier delays is less than 4% .

The variance of the difference between the exact and estimated delays is shown in Fig.3b for Woody's method and for the proposed method. As we can see in this figure, the variances for the proposed method are very close to the Crammer–Rao Lower Bound (CRLB), for the case of Gaussian noise [10]. Although this bound may not be suitable in our case since the background noise may not be Gaussian, it nevertheless gives an indication of the best possible performance. It may be observed that the variances for Woody's method, shown on the left-hand side, are degraded relative to those of the proposed method (right).

5. SUMMARY AND CONCLUSION

In this paper we propose an iterative method for enhancing an ERP waveform embedded in a background EEG signal. Sparsity is used to alleviate the contribution of the background EEG signal in the calculated cross–correlation coefficient between the measured EEG signal and the template. The proposed process is based on estimating the delay associated with the ERP in each trial in two steps per iteration. The first step estimates a sparse template which has a waveform similar to the average ERP. This is achieved by minimizing a convex objective function. The second step uses this template to estimate the delays between the template and the ERP signals in each trial. The estimated ERP waveform is the average of the time–aligned EEG signals over the trials. Iterations continue until convergence.

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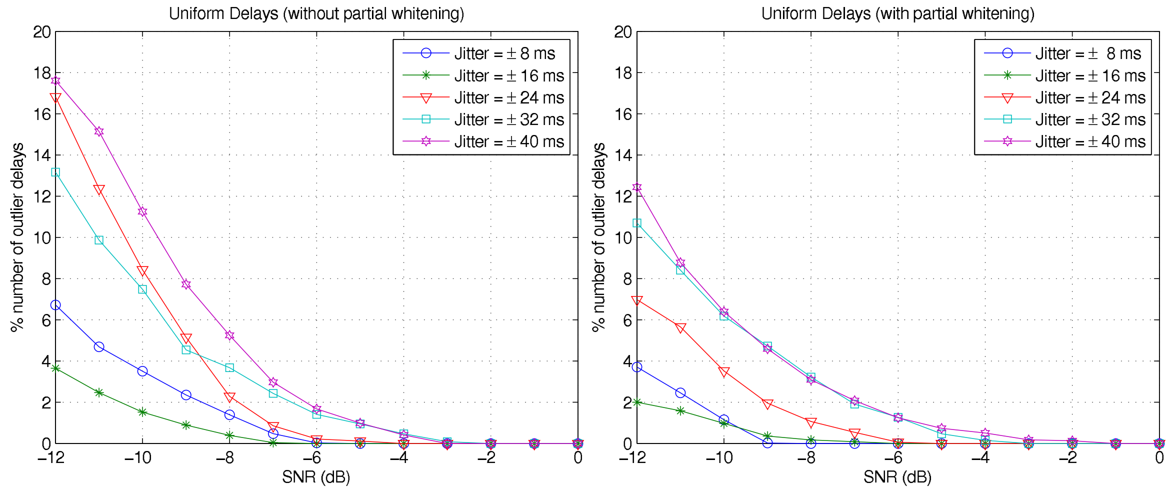
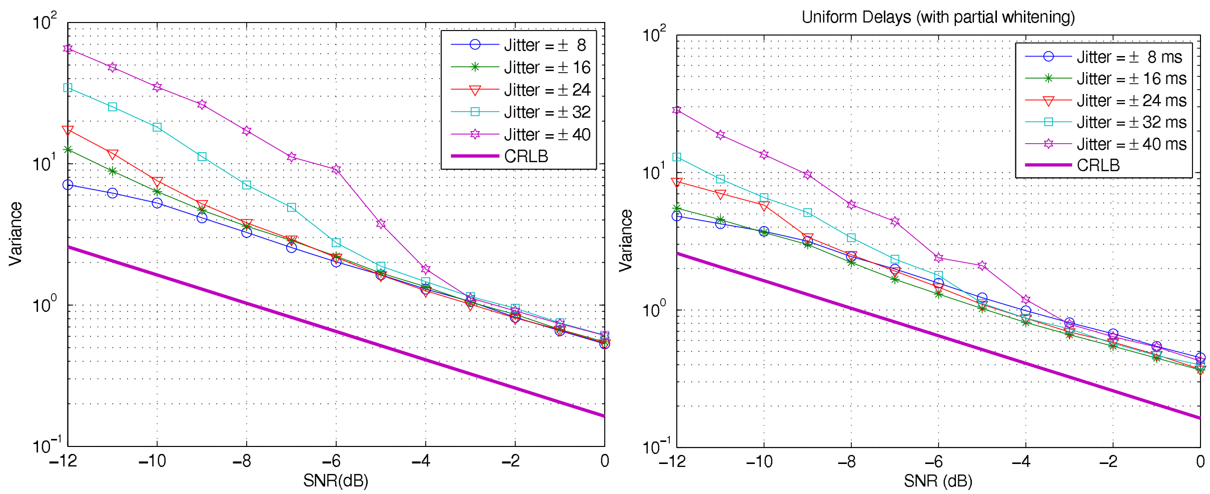


(a)



(b)

Figure 3: Results of Example 2. (a) % number of outlier delays for the proposed method when no partial whitening is applied (left) and and for partial whitening (right). (b) variance of the delay difference between the exact and estimated delays, after removal of outliers, for Woody's method (left) and the proposed method (right).

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