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LATTICE IMPLEMENTATION OF SUM-SQUARED AUTO-CORRELATION MINIMIZATION (SAM) CHANNEL SHORTENER

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ABSTRACT

The Sum-squared Autocorrelation Minimization (SAM) algorithm is one technique proposed for blind adaptation of the time-domain equalizer in multicarrier systems. The SAM cost depends on the effective channel autocorrelation, which will not be changed if any TEQ zeros are flipped over the unit circle. As a consequence, the SAM cost is multimodal, and different minima may yield very different shortening performance. In order to aid SAM converge to a suitable minimum, we impose a minimum phase constraint on the TEQ by means of the lattice structure and develop a lattice version of SAM with low computational complexity.

1. INTRODUCTION

Multicarrier modulation techniques such as discrete multitone (DMT) and orthogonal frequency division multiplexing (OFDM) are adopted in broadband communications as an effective tool to compensate the time dispersiveness of the channels. In multicarrier systems, a cyclic prefix (CP) is appended to each symbol, after the IFFT operation, to ensure tone orthogonality after propagation through the channel. If the CP is longer than the channel impulse response (IR), demodulation can be implemented by means of an FFT and simple one-tap frequency domain equalization. On the other hand, when the channel IR is longer than the CP, interference between different symbols and carriers (ISI and ICI) arise.

To combat ISI and ICI, a common approach is to insert a time domain equalizer (TEQ) at the receiver, previous to the FFT operation. The TEQ is a finite impulse response (FIR) filter whose purpose is that the delay spread of the combined channel-plus-TEQ impulse response is not longer than the CP length. The TEQ design problem has been extensively studied in the literature [1] for the case in which the channel IR is available. This knowledge is usually gathered by means of training symbols inserted in the data sequence. To avoid some drawbacks of supervised channel shortening approaches, blind algorithms for TEQ adaptation have also been proposed [2, 3, 4]. As opposed to trained designs, which require periodic existence of a training signal, blind adaptive techniques can lead to reduced complexity and if there are any further variations in the channel, a blind shortener can track those variations.

In [3], it is proposed to update the shortener by minimizing the sum of the effective channel squared autocorrelation (Sum-squared Autocorrelation Minimization, SAM). SAM is a blind adaptive channel shortening algorithm which attempts to suppress the effective channel autocorrelation outside a window of length equal to that of the CP. However, the cost function serving as starting point to the development of SAM is always multimodal [2]. Certain considerations, presented in Section 3, motivate the use of a minimum phase TEQ in order to ensure convergence of the SAM algorithm to a good stationary point. This minimum phase constraint can be effectively imposed if a lattice structure is adopted.

The paper is organized as follows. Section 2 gives the system model and notation. Section 3 motivates the use of the lattice structure to implement the TEQ, whereas Section 4 develops the steepest descent implementation of the lattice-based SAM algorithm. Section 5 determines the optimal step size used to improve convergence rate of the proposed algorithm. Section 6 presents comparative simulation results for lattice-based and the original SAM algorithms. Conclusions are given in Section 7.

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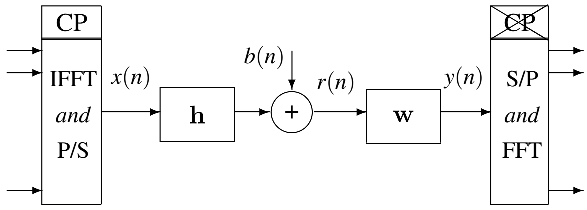

Figure 1: System model (CP: cyclic prefix, P/S: parallel to serial, S/P: serial to parallel).

2. SYSTEM MODEL

Multicarrier modulation divides the transmission bandwidth into N parallel tones by means of an inverse fast Fourier transform (IFFT). A CP is appended to each symbol to ensure tone orthogonality after propagation through the time-dispersive channel. Demodulation of the received signal is performed by an FFT.

The discrete-time system model is shown in Fig. 1. Let $x(n)$ be the source sequence to be transmitted through a linear channel with IR vector $\mathbf{h}$ having L_h+1 taps. The real-valued process $x(n)$ is modeled as zero mean, wide sense stationary with unit variance, and white (this property holds provided that sampling is done at twice the signal bandwidth). The noise $b(n)$ is a zero mean white Gaussian process with variance σ_b^2 uncorrelated with the transmitted signal $x(n)$. The discrete-time signal after sampling at the receiver is then

$$r(n) = \sum_{l=0}^{L_h} h(l)x(n-l) + b(n). \quad (1)$$

The received data will be filtered through an $L_w + 1$ taps TEQ with an IR vector $\mathbf{w}$ to obtain the output sequence $y(n)$ given by

$$y(n) = \sum_{l=0}^{L_w} w(l)r(n-l) = \mathbf{w}^T \mathbf{r}_n, \quad (2)$$

where $\mathbf{w} = [w(0), w(1), \cdots, w(L_w)]^T$ and $\mathbf{r}_n = [r(n), r(n-1), \cdots, r(n-L_w)]^T$. The effective channel is denoted by the convolution $\mathbf{c} = \mathbf{h} \star \mathbf{w}$ and has $L_c + 1$ coefficients, where $L_c = L_h + L_w$.

3. LATTICE STRUCTURE OF SAM EQUALIZER

The short autocorrelation of the effective channel is a property that is degraded by a long channel impulse response. SAM is a blind adaptive channel shortening algorithm that attempts to restore this short autocorrelation property. It performs a gradient descent of a cost function defined as the sum of the squares of autocorrelation coefficients of all lags greater than the desired channel memory. This cost function is given by

$$J(\mathbf{w}) \doteq \sum_{l=\nu}^{L_c} |R_{cc}(l)|^2, \quad (3)$$

where ν is the cyclic prefix length, and $R_{cc}(l) = \sum_{n=0}^{L_c} c(n)c(n-l)$ is the autocorrelation of the overall IR $c(n)$. A suitable constraint has to be imposed on $\mathbf{w}$ in order to avoid the trivial solution $\mathbf{w} = \mathbf{0}$. Note that if $c(n) = 0$ for $n \geq \nu$, then $R_{cc}(l) = 0$ for $l \geq \nu$ so that J will be zero. In other words, a short channel implies a short autocorrelation. The converse is not true: for example, consider the IR

$$c(n) = \begin{cases} a, & n = 0, \\ a^2 - 1, & n = 1, \\ a(a^2-1), & 2 \leq n \leq L_c \end{cases} \quad (4)$$

with $|a| < 1$. Then for sufficiently large L_c, one has $R_{cc}(l) \approx \delta(l)$, whereas the IR cannot be said to be 'short' (just take $|a|$ close to 1). This, of course, is due to the fact that $c(n)$ in (4) resembles the IR of an allpass system. Nevertheless, shortening $R_{cc}(l)$ seems to be a useful way to attempt channel shortening in practical scenarios [3].

Another important observation regarding the SAM cost (3) is its invariance to flipping any of the zeros of the overall transfer function $C(z) = \sum_{n=0}^{L_c} c(n)z^{-n}$ with respect to the unit circle, since this operation leaves $R_{cc}(l)$ unchanged. Since any zero of the TEQ transfer function $W(z) = \sum_{n=0}^{L_w} w(n)z^{-n}$ is a zero of $C(z)$, flipping the TEQ zeros leaves the cost J unaltered. As a result, for any point on the cost surface there will be 2^{L_w} points giving identical cost elsewhere, and in particular, any minimum (local or global) will be repeated 2^{L_w} times [2]. These minima may or may not yield good performance in terms of shortening the effective channel IR. For example, consider a channel with IR $h(n)$ given by (4). Then the 2-tap TEQs $\mathbf{w}_1 = [1\ -a]^T$ and $\mathbf{w}_2 = [-a\ 1]^T$ result in identical short autocorrelation. But while $\mathbf{w}_1$ yields good performance in terms of overall IR shortening, $\mathbf{w}_2$ does not.

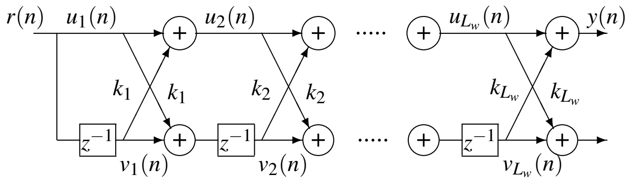

Figure 2: Lattice implementation of the TEQ.

In twisted pair lines the channel is well modeled by an IIR filter with a slowly decaying IR [5, 6]. This means that the channel transfer function presents poles inside, but very close to, the unit circle. An effective TEQ will place zeros on these critical poles to cancel them out. However, proximity to the unit circle means that small perturbations unavoidable in algorithm implementation may produce zero flipping, in such a way that the adaptive TEQ may get stuck in an undesirable minimum. To ensure SAM algorithm convergence to the good TEQ zeros cancelling poles causing the tail, we propose imposing a minimum phase constraint on the TEQ. This can be effectively implemented by using of the lattice structure [7].

The lattice implementation of the TEQ is shown in figure 2. The transfer function of lattice filter is determined by the *reflection coefficients* k_p, for $p = 1, 2, \cdots, L_w$. Stage outputs are obtained as follows: with $u_1(n) = v_1(n+1) = r(n)$,

$$v_p(n+1) = k_{p-1}u_{p-1}(n) + v_{p-1}(n), \quad (5)$$
$$u_p(n) = k_{p-1}v_{p-1}(n) + u_{p-1}(n) \quad (6)$$

and the output TEQ is given by

$$y(n) = r(n) + \sum_{p=1}^{L_w} k_p v_p(n). \quad (7)$$

A necessary and sufficient condition for this filter to be minimum phase is that all reflection coefficients have magnitude less than unity. We propose to check this condition as the algorithm progresses. The mapping from transversal to lattice form may be achieved via the Levinson-Durbin recursions [7]. Note that the lattice structure effectively imposes a *monic constraint* on the TEQ, i.e. the first tap is always 1.

To present the cost surface behavior of SAM lattice equalizer, we consider the same example treated in [3]. The channel is $\mathbf{h} = [1\ 0.3\ 0.2]^T$, the cyclic prefix is 1 sample, the lattice shortener has three taps ($L_w = 2$) and no noise is considered, with lattice implementation the TEQ first tap is already equal to 1. A 3D plot of the SAM cost function is shown in Fig. 3, in terms of the TEQ reflection coefficients k_1 and k_2. We observe that in the working range $|k_p| < 1$, the SAM cost for this example is convex and has a unique minimum. Whether this property can be extended to general channels and higher-order shorteners remains an open issue for further research.

4. ADAPTIVE ALGORITHM

Under the assumptions on $x(n)$, $b(n)$ in Section 2, the autocorrelation sequence of the TEQ output $y(n)$ satisfies

$$R_{yy}(l) = R_{cc}(l) + \sigma_b^2 R_{ww}(l), \quad (8)$$

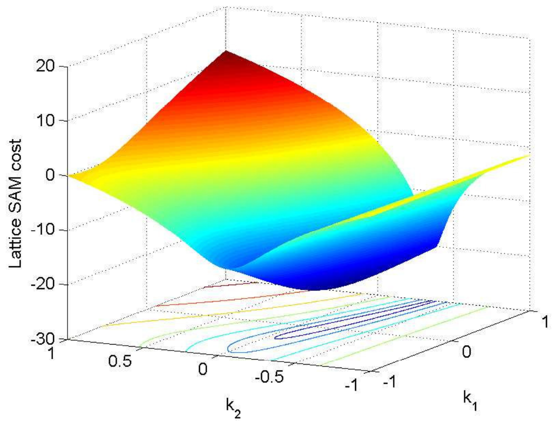

Figure 3: SAM cost function versus k_1 and k_2.

where $R_{ww}(l) = \sum_{n=0}^{L_w} w(n)w(n-l)$. For typical SNR values, the noise variance is small and the second term in (8) can be ignored [3]. Then the SAM cost can be approximated by the sum squared autocorrelation of the TEQ output sequence:

$$J(\mathbf{k}) \approx \sum_{l=\nu}^{L_c} |R_{yy}(l)|^2, \tag{9}$$

where $\mathbf{k}$ is the vector of TEQ reflection coefficients. Given the output signal $y(n)$, the task is to update $\mathbf{k}$ so as to minimize (9). This can be done by means of a gradient descent:

$$\mathbf{k}(n+1) = \mathbf{k}(n) - \mu \nabla_{\mathbf{k}} J, \tag{10}$$

where $\nabla_{\mathbf{k}} J$ is the gradient of J evaluated at $\mathbf{k} = \mathbf{k}(n)$, and μ is the step size. To implement (10), we define an instantaneous cost function by replacing the expectation operation by a moving average over a defined window of length M,

$$J(\mathbf{k}) \approx \sum_{l=\nu}^{L_c} \left| \frac{\xi(n,l)}{M} \right|^2, \text{ with } \xi(n,l) \doteq \sum_{k \in I_n} y(k)y(k-l), \tag{11}$$

where $I_n \doteq \{n, n-1, \ldots, n-M+1\}$. The partial derivative of J with respect to k_p is given by

$$\frac{\partial J}{\partial k_p} = \frac{2}{M^2} \sum_{l=\nu}^{L_c} \xi(n,l) \frac{\partial \xi(n,l)}{\partial k_p} \tag{12}$$

Now one has

$$\begin{aligned} \frac{\partial \xi(n,l)}{\partial k_p} &= \sum_{k \in I_n} \left[y(k) \frac{\partial y(k-l)}{\partial k_p} + \frac{\partial y(k)}{\partial k_p} y(k-l) \right] && (13) \\ &\approx \sum_{k \in I_n} [y(k)v_p(k-l) + v_p(k)y(k-l)] && (14) \\ &\approx v_p(n)y(n-l). && (15) \end{aligned}$$

In (14) we have neglected the dependence of the signals $v_i(n)$ with the reflection coefficients (see (7)), whereas (15) neglects all but one term in the summation in order to keep computational complexity at bay. Let us define the vectors

$$\begin{aligned} \mathbf{y}_n &\doteq [\ y(n-\nu) \quad y(n-\nu-1) \quad \cdots \quad y(n-L_c)\ ]^T && (16) \\ \Xi_n &\doteq [\ \xi(n,\nu) \quad \xi(n,\nu+1) \quad \cdots \quad \xi(n,L_c)\ ]^T && (17) \\ \mathbf{v}(n) &\doteq [\ v_1(n) \quad v_2(n) \quad \cdots \quad v_{L_w}(n)\ ]^T && (18) \end{aligned}$$

Then, after absorbing the factor $\frac{2}{M^2}$ into the stepsize μ, the proposed update rule can be written as

$$\mathbf{k}(n+1) = \mathbf{k}(n) - \mu \mathbf{v}(n)e(n), \quad \text{with} \quad e(n) = \Xi_n^T \mathbf{y}_n. \tag{19}$$

The lattice version of SAM (19) requires on the order of $L_w(L_c - \nu)$ multiplications and additions per update, which is comparable to that of the original tapped-delay-line implementation.

5. NORMALIZED SAM ALGORITHM

To improve the convergence rate of the lattice SAM algorithm, a variable stepsize can be used in the update rule:

$$k_p(n+1) = k_p(n) - \mu_n \gamma_p(n), \qquad 1 \le p \le L_w, \tag{20}$$

where $\gamma_p(n) = v_p(n)\Xi_n^T \mathbf{y}_n$.

In order to determine the optimal value of μ_n, we propose to minimize, at each n, the *a posteriori* cost function J^{pos} depending on the updated TEQ output signal $y^{\text{pos}}(n)$:

$$y^{\text{pos}}(n) = r(n) + \sum_{p=1}^{L_w} k_p(n+1) v_p^{\text{pos}}(n). \tag{21}$$

To determine $v_p^{\text{pos}}(n)$, we exploit the recursive relations (5), at *a posteriori* time. By neglecting terms proportional to μ^m for $m \ge 2$, it can be verified that $v_p^{\text{pos}}(n)$ and $u_p^{\text{pos}}(n)$ are recursively computed as follows:

$$\begin{aligned} v_p^{\text{pos}}(n) &= v_p(n) - \mu_n t_p(n), && (22) \\ u_p^{\text{pos}}(n) &= u_p(n) - \mu_n s_p(n), && (23) \end{aligned}$$

where

$$\begin{aligned} t_{p+1}(n) &= t_p(n-1) + k_p(n)s_p(n-1) + \gamma_p(n)u_p(n-1), \\ s_{p+1}(n) &= k_p(n)t_p(n) + \gamma_p(n)v_p(n) + s_p(n). \end{aligned}$$

Neglecting again terms in μ^m for $m \ge 2$, and replacing (20) and (22) into (21), the *a posteriori* TEQ output yields:

$$y^{\text{pos}}(n) = y(n) - \mu_n \left(\sum_{p=1}^{L_w} k_p(n)t_p(n) + v_p(n)\gamma_p(n) \right). \tag{24}$$

We define the *a posteriori* cost function at the n^{th} iteration as

$$J^{\text{pos}} \doteq \sum_{l=\nu}^{L_c} \left(\sum_{k=n-M+1}^{n-1} y(k)y(k-l) + y^{pos}(n)y(n-l) \right)^2. \tag{25}$$

Replacing the expression of $y^{\text{pos}}(n)$ from (24) into (25), it follows that

$$\begin{aligned} J^{\text{pos}} = & J + \mu_n^2 \sum_{l=\nu}^{L_c} y(n-l)^2 \left(\sum_{p=1}^{L_w} k_p(n)t_p(n) + v_p(n)\gamma_p(n) \right)^2 \\ & - \mu_n \Xi_n^T \mathbf{y}_n \left(\sum_{p=1}^{L_w} k_p(n)t_p(n) + v_p(n)\gamma_p(n) \right). \end{aligned}$$

The value of μ_n minimizing J^{pos} is found to be

$$\mu_n^{\text{opt}} = \frac{\Xi_n^T \mathbf{y}_n}{\sum_{l=\nu}^{L_c} y(n-l)^2 \left(\sum_{p=1}^{L_w} k_p(n)t_p(n) + v_p(n)\gamma_p(n) \right)}. \tag{26}$$

Using this optimal step size at each iteration, the updating rule for the lattice version of SAM will be given by

$$k_p(n+1) = k_p(n) - \alpha \mu_n^{\text{opt}} \gamma_p(n), \tag{27}$$

where $0 < \alpha < 1$ is a fixed stability factor.

6. SIMULATION RESULTS

We study the performance of the proposed TEQ implementation in an ADSL environment, where the transmission line is modeled as an IIR channel. Parameters were chosen to match the ADSL standard system: the cyclic prefix is 32 samples, the FFT size is 512, 4-QAM signaling is used on all of the tones, additive white gaussian noise at -140 dBm/Hz, and no crosstalk. We consider the carrier serving area (CSA) test loops, which is the standard channel model for DSL systems [8], combined with a plain old telephone service (POTS) splitter. The sampling frequency is 2.208 MHz.

The DSL channel is characterized by its long tail, caused by poles close to the unit circle. These offending poles can be cancelled by a minimum phase TEQ. First, we consider CSA loop 1, a 3-tap TEQ implemented with the lattice structure and $M = 100$ for the moving average window length. The performance metric is the mean square deviation (MSD) with respect to the optimal reflection coefficients corresponding to the minimum of the SAM cost:

$$\text{MSD} \doteq \|\mathbf{k}(n) - \mathbf{k}_{\text{opt}}\|^2. \tag{28}$$

For a 3-tap TEQ, $\mathbf{k}_{\text{opt}} = [-0.991 \quad 0.549]$. Using the update rule (27), Fig. 4 shows the MSD variation during normalized lattice SAM algorithm adaptation for different values of α, the MSD curves are obtained by averaging several independent runs. Notice that when α is increased, the adaptation noise in steady state increases, as could be expected.

After convergence of the normalized lattice SAM algorithm, the shortened channel is compared to the original one in Fig. 5. To compare convergence speed of the normalized lattice SAM algorithm to the original lattice algorithm, we have tabulated in Table 1 the MSD values achieved after 3000 iterations. For each algorithm the MSD value is determined for optimal fixed convergence factors (the values of μ and α that result in smallest MSD for each algorithm).

In the sequel we consider $\alpha = 10^{-2}$. Next we provide the performance comparison in term of frequency signal to noise ratio (SNR), using the sub-channel SNR as defined in [9] and which incorporates ISI and noise distortion :

$$\text{SNR}_i \doteq \frac{\sigma_{x,i}^2 |C_{win}^i|^2}{\sigma_{x,i}^2 |C_{wall}^i|^2 + \sigma_{b,i}^2 |W^i|^2}, \tag{29}$$

where $\sigma_{x,i}^2$, $\sigma_{b,i}^2$, C_{win}^i , W^i, and C_{wall}^i are the transmitted signal power, channel noise power (before the equalizer), effective channel gain inside the window of width ν, equalizer gain, and the ISI path gain in the i^{th} sub-channel, respectively. The equivalent path gains in sub-channel i are the i^{th} FFT coefficients of the c_{win}^i, c_{wall}^i and w^i effective channels and equalizer impulse responses. Fig. 6 presents a comparative of different algorithms in terms of SNR_i for 3-tap equalizers and after algorithm convergence.

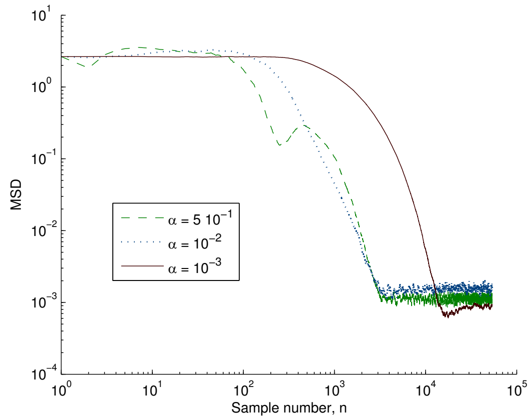

Figure 4: Mean Square Deviation (MSD) versus iteration number.

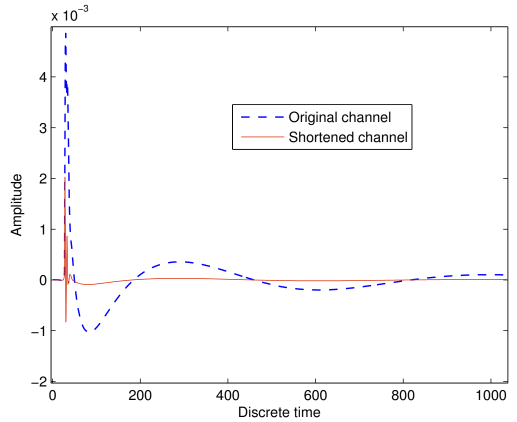

Figure 5: Original and shortened impulse responses for CSA loop 1.

Another measure of shortening performance is the achievable bit rate, approximately given by

$$R \doteq \sum_{i \in S} \log_2(1 + \frac{SNR_i}{\Gamma}), \tag{30}$$

where S is the index set of used sub-channels and Γ is the SNR gap to Shannon capacity, assumed to be constant over all sub-channels. Fig. 7 shows the bit rate evolution over time, comparing the normalized lattice and the original transversal implementations of SAM. The performance of the maximum shortening SNR (MSSNR) TEQ given in [10] is shown as well as a benchmark. Table 2 summarizes the achievable bit rate R for CSA loops $\sharp 1 - 8$ using a 6 dB margin. For all algorithms, we use the unit tap constraint; the SNR_i is computed for all possible delays and the best one is taken. From Table 2, we can see that the normalized lattice SAM equalizer outperforms the transversal SAM algorithm (which is updated with a moving average implementation as well).

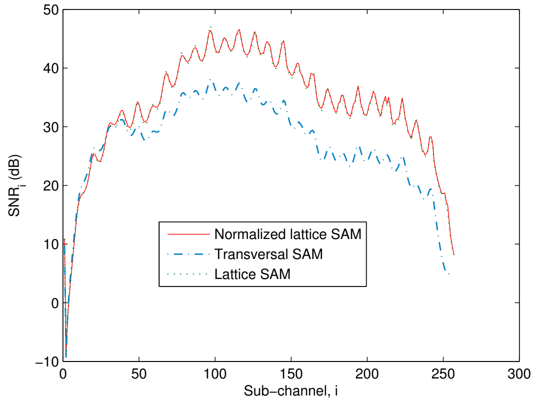

Figure 6: Sub-channel SNR, CSA loop 1.

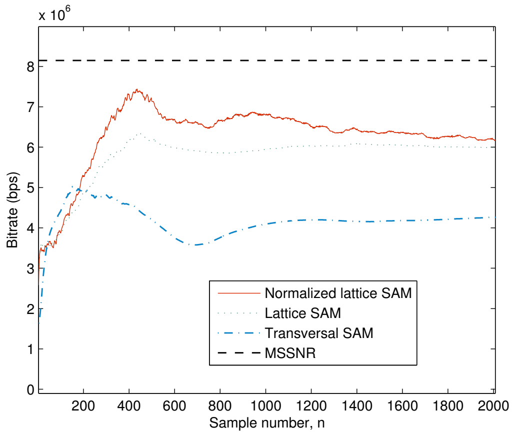

Figure 7: Achievable bit rate versus iteration number, CSA loop 1.

Adaptive algorithms	MSD
Normalized lattice SAM	$1.5\ 10^{-3}$
Lattice SAM	$5.2\ 10^{-2}$

Table 1: MSD for CSA loop 1 shortened with the unnormalized and normalized lattice SAM algorithms.

CSA loop number	Transv. SAM %	Lattice SAM %	MSSNR bit rate, Mbps
1	51.53	77.16	8.1510
2	54.67	79.05	9.7368
3	68.37	90.30	8.3605
4	52.11	89.33	8.0593
5	60.19	75.56	8.5252
6	57.42	91.31	8.3591
7	46.51	92.79	5.4010
8	40.90	77.96	6.2810

Table 2: Percentage of MSSNR achievable bit rate for the 8 CSA loops shortened with the transversal and lattice SAM algorithms.

7. CONCLUSION

In order to impose a minimum phase constraint, the lattice structure has been used for implementation of the TEQ. An implementation of the SAM adaptation algorithm has been proposed for the lattice structure, with computational complexity similar to that of the original version. Convergence of this lattice SAM algorithm has been observed, yielding comparable performance, in terms of achievable bit rate, to the optimal MSSNR TEQ.

REFERENCES

[1] R. K. Martin, K. Vanbleu, M. Ding, G. Ysebaert, M. Milosevic, B. L. Evans, M. Moonen, C. R. Johnson, Jr., ”Unification and evaluation of equalization structures and design algorithms for discrete multitone modulation systems,” *IEEE Trans. Signal Processing*, vol. 53, pp. 3880-3894, Oct. 2005.

[2] R. K. Martin and C. R. Johnson, Jr., “Adaptive equalization: transitioning from single-carrier to multicarrier systems,” *IEEE Signal Processing Mag.,* vol. 22, pp. 108-122, Nov. 2005.

[3] J. Balakrishnan, R. K. Martin and C. R. Johnson, Jr., ”Blind, Adaptive Channel Shortening by Sum-squared Autocorrelation Minimization (SAM),” *IEEE Trans. Signal Processing*, vol. 51, pp. 3086-3093, Dec. 2003.

[4] R. K. Martin, J. Balakrishnan, W. A. Sethares and C. R. Johnson, Jr. ”A blind adaptive TEQ for multicarrier systems,” *IEEE Signal Processing Lett.,* vol. 9, pp. 341-343, Nov. 2002.

[5] W. Y. Chen, *DSL Simulation Techniques and Standards Development for Digital Subscriber Line Systems*. Indianapolis, Indiana: McMillan Technical Publishing, 1998.

[6] N. Al-Dhahir, A. H. Sayed, and J. M. Cioffi, ”Stable pole-zero modeling of long FIR filters with application to the MMSE-DFE,” *IEEE J. on Selected Areas in Comm.,* vol. 45, pp. 508-513, May 1997.

[7] M. Bellanger, *Adaptive Digital Filters and Signal Processing*, Marcel Dekker Inc., New York, NY, 1987.

[8] K. Sistanizadeh, ”Loss characteristics of the proposed canonical ADSL loops with 100-Ohm termination at 70, 90, and 120 F,” ANSI T1E1.4 Committee Contribution, no. 161, Nov. 1991.

[9] G. Arslan, B. L. Evans and S. Kiaei, ”Equalization for discrete multitone receivers to maximize bit rate,” *IEEE Trans. Signal Processing*, vol. 49, pp. 3123-3135, Dec. 2001.

[10] P. J. W. Melsa, R. C. Younce and C. E. Rohrs, ”Impulse response shortening for discrete multitone transceivers,” *IEEE Trans. Commun.*, vol. 44, pp. 1662-1672, Dec. 1996.