COLOUR IMAGE EDGE DETECTION USING QUATERNION QUANTIZED LOCALIZED PHASE

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ABSTRACT

Recently, a novel edge detection method for gray-level image using quantized localized phase was proposed in [1]. The rationale of this method for edge detection is that we can obtain more edge and contour information of interested image from its quantized localized phase rather than from its magnitude. This is because the phase information is of greater importance on edge areas than on smooth areas. In this paper, we generalize the method in [1] to deal with colour image edge detection using quaternion polar form and two dimensional quaternion short-term Fourier transform (2-D QSTFT). By applying 2-D QSTFT to colour image, locally quantizing the phase part of quaternion polar form of the transformed image, and then reconstructing the resulted image using 2-D QISTFT and quantized phase, we can preserve the edge information and therefore achieve the goal of colour image edge detection.

1. INTRODUCTION

Edge detection is a technique of fundamental importance in the image analysis and computer vision areas, especially the area of feature extraction and feature detection. By applying edge detection to images, we can identify points with sharp brightness changes or discontinuities in images. Edges represent object boundaries in typical images. Therefore, applying a good edge detector to pre-process the images can be useful for the following segmentation, registration, and identification of objects in an image scene analysis. There are already many edge detection methods for gray-level image proposed in the image processing area, like quantized localized phase method [1], Canny edge detector [2], and phase congruency method [3], etc. For colour image, the colour Canny method [5] is a useful technique for colour image edge detection. Nikolay Skarbnik et.al. [1] proposed a novel edge detection and skeletonization method using quantized localized phase. The authors demonstrated the importance of phase in image reconstruction process [1][4] and constructed a simple algorithm for edge detection based on localized phase of STFT. They compared its performance with [2][3] and showed its efficiency in edge detection. However, the method can only be used to process gray-level images and therefore the breadth of its capability is limited. The concept of quaternions was first proposed by Hamilton in 1843 [6]. It is a four-dimensional, non-commutative algebra which has found many applications in many research fields such as computer science, mathematics, signal processing and image processing. The fundamental theorems are well developed, and mathematical operations like Fourier transform, Wavelet transform, convolution of this four-dimensional, non-commutative algebra have been constructed maturely [7]-[14]. The usefulness and efficacy of quaternions in dealing with multidimensional computations are doubtless. Since a colour image has three components (RGB), we can encode its pixels to pure quaternions and regard the whole image as a two dimensional quaternion image. Many tasks of colour image processing, such as three dimensional rotation and many other geometrical transformations can be done more easily in quaternion domain rather than in RGB domain. Inspired by the quantized localized phase method [1], the mathematical theory of the quaternions, and the type one polar form [15] of the quaternions, we propose a novel colour image edge detection method based on quaternion quantized localized phase. In section 2, the fundamentals of the quaternions and quaternion polar form are reviewed. The spatial transform of the quaternions, such as quaternion short-term Fourier transform is presented in section 3 and the review of the concepts of quantized localized phase and the proposed colour image edge detection method are presented in section 4. The experimental results of the proposed algorithm are summarized in section 5. Finally, section 6 concludes this work.

2. REVIEW OF THE QUATERNIONS

The quaternions, which were usually denoted as **H**, in honour of its inventor, Sir William Rowan Hamilton, can be viewed as a four-dimensional vector space defined over real numbers. The quaternions were also generalizations of conventional complex numbers. Contrary to a complex number, which has two components, i.e. the real part and the imaginary part, on the other hand, a quaternion consists of four components, i.e. one real part and three imaginary parts. A quaternion is often represented as the following form :

$$q = q_r + q_i i + q_j j + q_k k \tag{1}$$

where q_r, q_i, q_i, q_i are all real numbers, i.e. q is defined over R^4, and the elements $\{1, i, j, k\}$ form the basis of the quaternion vector space. The member of this vector space can be uniquely represented as a linear combination of these basis elements, and the 3-tuple $\{i, j, k\}$ obeys the follo-

wing multiplication rules :

$$i^2 = j^2 = k^2 = -1 \quad , \quad ij = -ji = k \quad , \quad jk = -kj = i \quad , \quad ki = -ik = j \tag{2}$$

The number q_r is called the real part of q and $q_i i + q_j j + q_k k$ is called the vector part. Denote the real part as S(q) and the vector (imaginary) part as V(q), a quaternion can be represented as follows :

$$q = S(q) + V(q) \tag{3}$$

The conjugate of a quaternion is defined as :

$$q^c = q_r - q_i i - q_j j - q_k k \tag{4}$$

The norm of a quaternion can be written as :

$$|q| = \sqrt{qq^c} = (q_r^2 + q_i^2 + q_j^2 + q_k^2)^{1/2} \tag{5}$$

If the real part of q is zero, we call q a pure quaternion. When the norm of q is one and the real part of q is zero, we call q a unit pure quaternion.

In order to facilitate the discussion of the proposed method in section 4, we briefly review the polar form of quaternions. In general, there are two polar forms of quaternions. Each of them represents different geometric meaning of quaternions. We only discuss the type one polar form [15], and the information about the other type can be found in [16]. The type one polar form of quaternions is similar to that of complex numbers, and we can denote it as follows :

$$q = q_r + q_i i + q_j j + q_k k = re^{u\theta} = r(\cos\theta + u\sin\theta)$$

where $r = |q| = (q_r^2 + q_i^2 + q_j^2 + q_k^2)^{1/2}$ and

$$\cos\theta = \frac{q_r}{r}, \ \sin\theta = \pm\frac{\sqrt{q_i^2 + q_j^2 + q_k^2}}{r} \tag{6}$$

If $q_i^2 + q_j^2 + q_k^2 \neq 0$, u is the unit pure quaternion, i.e. $u = \pm\frac{q_i i + q_j j + q_k k}{\sqrt{q_i^2 + q_j^2 + q_k^2}}$. In some special case, when q is a complex number $(q_j = q_k = 0)$, $u = \pm i$. When q is a real number $(q_i = q_j = q_k = 0)$, u can be any unit pure quaternion.

3. QUATERNION STFT

Assume we have a quaternion signal $s(x)$ and define the transformation axis as $u = (i + j + k)/\sqrt{3}$, its 1-D QSTFT of axis u appears as :

$$S(\tau, v) = 1DIQSTFT_u[s(x)] = \int_{-\infty}^{+\infty} s(x)w(x-\tau)e^{-uvx}dx \tag{7}$$

where $w(x)$ is the overlapped window function, commonly a rectangular window or Gaussian window, and $s(x)$ is the signal to be transformed. $S(\tau, v)$ is the short term quaternion Fourier Transform of axis u for $s(x)w(x-\tau)$. It is a complex function representing the phase and magnitude of the signal over space and frequency. The 1-D QSTFT is invertible and the original signal can be recovered by applying the Inverse 1-D QSTFT. The inverse 1-D QSTFT of axis u appears as:

$$s(x) = 1DIQSTFT_u[S(\tau, v)] = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} S(\tau, v)e^{+uvx}d\tau dv \tag{8}$$

In case of 2-D QSTFT and 2-D IQSTFT, we can apply the 2D rectangular or Gaussian window to the quaternion image. The equation of 2-D QSTFT and 2-D IQSTFT can be written as following form:

$$S(\tau_1, \tau_2, v_1, v_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} s(x, y)w(x-\tau_1, y-\tau_2)e^{-uv_1x}e^{-uv_2y}dxdy \tag{9}$$

$$s(x, y) = \frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} S(\tau_1, \tau_2, v_1, v_2)e^{+uv_1x}e^{+uv_2y}d\tau_1 d\tau_2 dv_1 dv_2 \tag{10}$$

where $s(x, y)$ is quaternion image and $S(\tau_1, \tau_2, v_1, v_2)$ is the 2-D QSTFT of $s(x, y)$.

4. QUANTIZED LOCALIZED PHASE AND PROPOSED METHOD

The global phase of a spatial transform, such as the well known Fourier transform, can be used to reconstruct important features of an image. However, the lack of localization in position spaces [17] is a severe problem. Therefore, the space-frequency analysis methods, such as STFT, Wavelet Transform, and Gabor transform, are proposed to address the localization problem. Take STFT for example, the aim of localization is accomplished because of the introduction of window function in the integral. The localized phase of QSTFT (take 2D case for example) can be obtained using (6)

$$\measuredangle S(\tau_1, \tau_2, v_1, v_2) = \arg\left\{\left\|S(\tau_1, \tau_2, v_1, v_2)\right\| e^{u\theta}\right\} \tag{11}$$

We can take advantage of the localized phase of STFT and the magnitude of the transformed result to reconstruct an image. The next question to be addressed is how to use the localized phase to achieve the goal of colour image edge detection. Similar to the procedure of gray-level image edge detection mentioned in [1], we first quantize the localized phase of the quaternion polar form of QSTFT result to Q levels and use the nearest quantization level to approximate the interested phase, as the following equation shows:

$$\tilde{\theta} = \frac{\pi}{Q}\left[\!\left[\frac{\theta}{\pi/Q}\right]\!\right] \tag{12}$$

where $\tilde{\theta}$ is the quantized localized phase, θ is the localized phase, the values of θ are distributed between $[0, \pi]$, and the $[\![\bullet]\!]$ operator denotes the round operation. The localized phase is related to the edge information of an image and we can define a colour image edge detector by using the error of reconstructed quaternion image with quantized localized phase, compared to the original quaternion image.

$$e(x,y) = error = s(x,y) - 2DIQSTFT[\tilde{S}(\tau_1,\tau_2,v_1,v_2)]$$

$$\tilde{S}(\tau_1,\tau_2,v_1,v_2) = \|S(\tau_1,\tau_2,v_1,v_2)\| e^{u\tilde{\theta}}$$

$$\tilde{\theta} = \frac{\pi}{Q}\left[\!\left[\frac{\theta}{\pi/Q}\right]\!\right], \tilde{\theta} = \measuredangle \tilde{S}(\tau_1,\tau_2,v_1,v_2) \qquad (13)$$

$$u = (i+j+k)/\sqrt{3}$$

where $e(x,y)$ is the edge map of the colour image.

We summarize the algorithm of the proposed color image edge detection method as follows:

(1) Encode the RGB components of color image using pure quaternion and obtain the quaternion image, i.e. $s(x,y) = R(x,y)i + G(x,y)j + B(x,y)k$

(2) Transform encoded quaternion image $s(x,y)$ using 2DQSTFT of axis u, the space-frequency spectrum is: $S(\tau_1,\tau_2,v_1,v_2) = 2DQSTFT_u[s(x,y)]$, where the window function of QSTFT is rectangular or Gaussian window and we choose the size of the window as 5x5, the transformation axis as $u = (i+j+k)/\sqrt{3}$.

(3) Represent the transformed spectrum using type one quaternion polar form [15], i.e. $S(\tau_1,\tau_2,v_1,v_2) = \|S(\tau_1,\tau_2,v_1,v_2)\| e^{u\theta}, \theta = [0,\pi]$.

(4) Quantize the phase of the polar form using Q levels: $\tilde{\theta} = \frac{\pi}{Q}\left[\!\left[\frac{\theta}{\pi/Q}\right]\!\right], \tilde{\theta} = \measuredangle \tilde{S}(\tau_1,\tau_2,v_1,v_2)$.

(5) Reconstruct the quaternion image using the quantized phase of step (4) and 2DIQSTFT of axis u. The quantized quaternion STFT spectrum is: $\tilde{S}(\tau_1,\tau_2,v_1,v_2) = \|S(\tau_1,\tau_2,v_1,v_2)\| e^{u\tilde{\theta}}$.

The reconstructed quaternion image is: $2DIQSTFT[\tilde{S}(\tau_1,\tau_2,v_1,v_2)]$ and the colour image edge map is the difference of the original quaternion image and the reconstructed quaternion image. i.e.

$$e(x,y) = s(x,y) - 2DIQSTFT[\tilde{S}(\tau_1,\tau_2,v_1,v_2)].$$

5. EXPERIMENTAL RESULTS

In what follows, we perform several experiments to demonstrate the results of proposed colour image edge detection method and compare the performance with the colour Canny method [5]. First, we take the first column of colour Lena image as input signal. We can observe from fig. 1 that when the quantization level Q is small (ex: fig.1(a),Q=2), the edge detection result is obvious and good (fig.1 (b)). On the other hand, when quantization level Q is large (ex: fig.1(c),Q=16), the edge detection result is inferior (fig.1 (d)). We can see that we obtain stronger edge detection result when Q is smaller. Thus, we choose Q=2 in the following experiments. The minimum value of Q is 2. When Q is 1, no edge detection can be done, the resulting image is gray-level image.

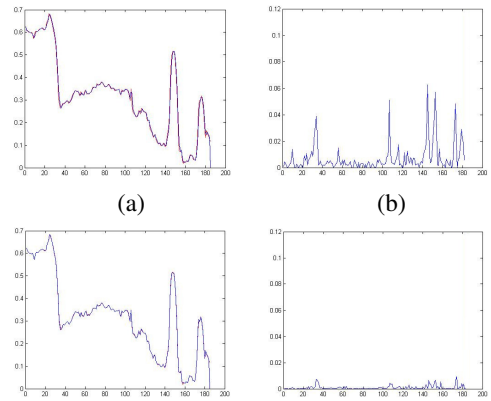

Figure 1. (a) Red line:original quaternion image of 1st column of colour Lena. Blue line:reconstructed quaternion image (Q=2). (b) error signal of (a) (Red line-Blue line). (c) Q=16 case of (a). (d) error signal of (c) (Red line-Blue line).

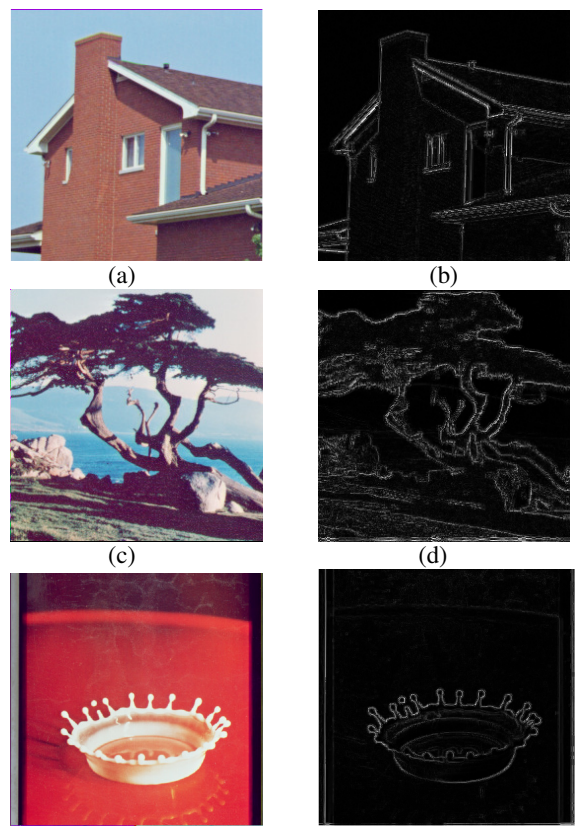

Figure 2. Colour image edge detection results of the proposed quaternion quantized localized phase method. (Q=2, 5x5 rectangular window).

colour image edge detection results. From fig.2, we can see that the edge detection performance is successful and satisfactory. The horizontal edges, vertical edges, and the remaining directions of edges are all clearly detected by applying our method to these colour images. Next, we compare our

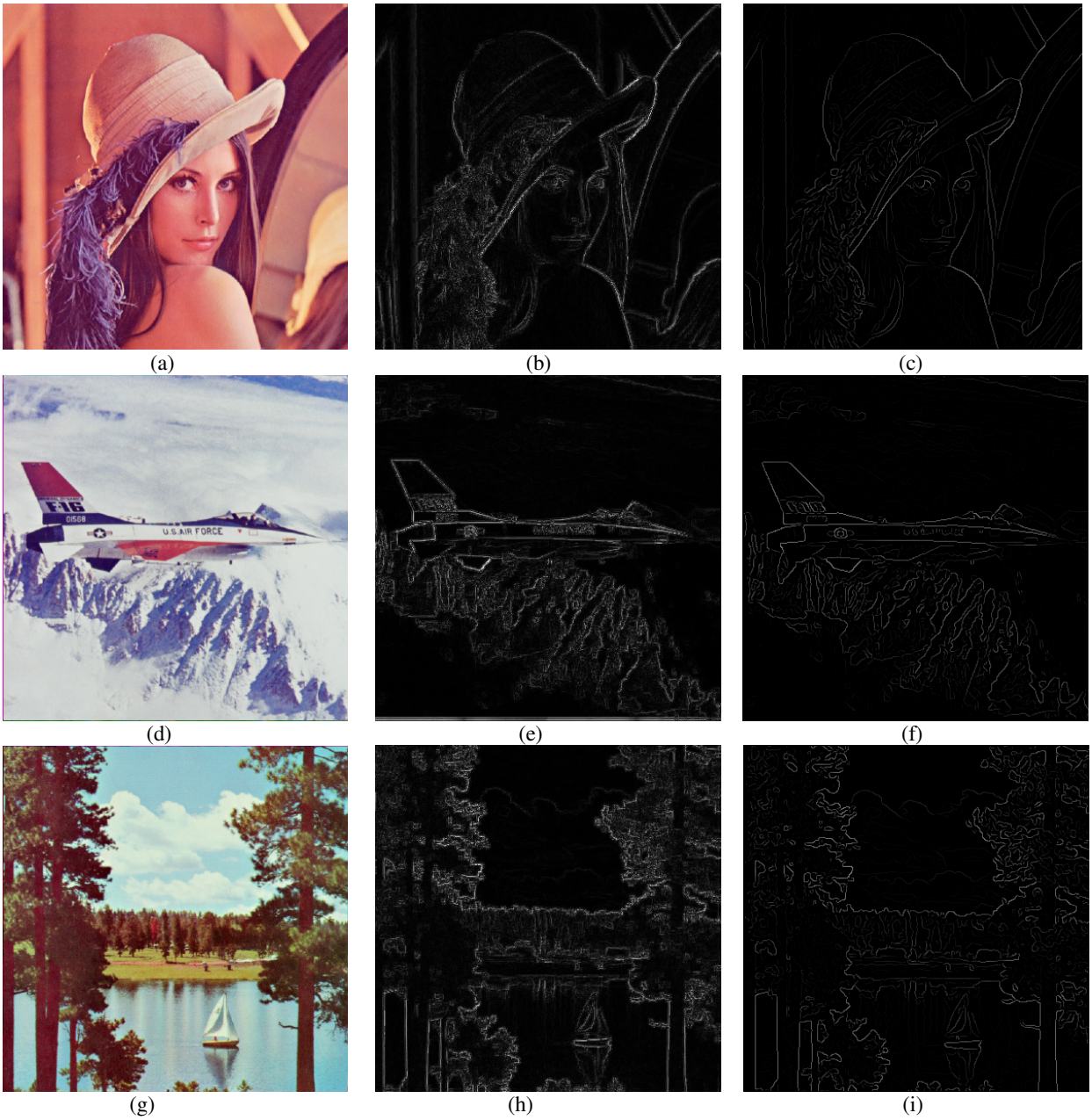

Figure 3. Comparison of edge detection using the quaternion quantized localized method and colour Canny method. (a)(d)(g) original colour images. (b)(e)(h) edge detection results of (a)(d)(g) using the proposed quaternion Fourier transform method. (c)(f)(i) edge detection results of (a)(d)(g) using colour Canny method.

methods with the color Canny method [5]. Color Canny edge detector is a gradient-based method and is often used to pre-process color images for the sake of following image processing operations. As can be seen from fig.3, we can obtain more details of the edge detection results by using our methods than using the color Canny method [5]. Some edges that are not so obvious can be detected by our method but can not be detected by color Canny method clearly. Besides, the algorithm of our method (see the bottom part of sec.4) is very simple and easy to be implemented using computer. The color Canny algorithm is somewhat complicated and increases the difficulty of implementation. However, the execution time of the color Canny method is faster than our method (about a few seconds, under the same experimental environment). This is because of the high computational complexity of the 2-D QSTFT/2-D QISTFT. The 2-D QSTFT/2-D QISTFT is realized using quaternion fast Fourier transform algorithm. We use MATLAB and the toolbox developed by Steve Sangwine and Nicolas Le Bihan to conduct all the experiments presented in this paper. Interested readers can get the quaternion toolbox from the website [18].

5. CONCLUSION

In this paper, we combine the concepts of quantized localized phase, quaternion polar form, QSTFT, and propose a colour image edge detection method. The edge detection results are demonstrated and compared with the results of well known colour Canny method. The performance of our method is good and satisfactory while the algorithm is simple and easy to be implemented. The future work is combining other quaternion space-frequency transforms with quantized localized phase to explore more possible colour image edge detection methods.

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