ACHIEVABLE MAXIMUM-DIRECTIVITY BEAMFORMING FOR SPHERICAL MICROPHONE ARRAYS WITH RANDOM ARRAY ERRORS

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ABSTRACT

The maximum-directivity beamformers, or the so-called plane wave decomposition (PWD) beamformers have been widely studied and used for spherical microphone arrays, but they are known to be sensitive to random array errors that exist in practical applications. In this paper, a robust maximum-directivity beamforming approach based on the spherical harmonics framework and worst-case performance optimization techniques is proposed. The approach is developed for both the space domain and the spherical harmonics domain. Given the known maximum level of array errors, it can automatically find and yield the achievable maximum-directivity for spherical microphone array beamformers, and does not require manual tuning of robustness parameters, which is the major advantage over the white noise gain constrained robustness control approaches that have been developed earlier for spherical microphone arrays.

1. INTRODUCTION

Spherical microphone array beamforming technology has recently become an important research subject in three-dimensional (3-D) source reception, wavefield analysis, source localization, and noise suppression applications [1-8]. More flexible three-dimensional beampattern synthesis can be realized than with other standard array geometries, and array processing can be developed and analyzed by using the elegant spherical harmonics framework.

Among all kinds of spherical microphone array beamforming approaches, the plane wave decomposition (PWD) beamforming, which can provide closed-form solutions for array weights and maximum-directivity beampatterns [3], [4], [8], has possibly become the most widely used technique. However, it is also well known that such a beamformer has low white noise gain at low frequencies [1], [3], [4], and may lead to degraded and non-consistent beamforming performance in the presence of random array errors (sensor sensitivity and phase variations, sensor self noise, positioning errors, etc.). Therefore, in practical applications and mass productions, it is important to have a robust beamforming algorithm that can yield the achievable optimum performance, according to the knowledge of array error levels, which could be well controlled and specified by microphone manufactures.

Up to now, several robust beamforming algorithms have been proposed for spherical microphone arrays in both the spherical harmonics domain [5], [7], [15] and the space domain [8]. They commonly impose a white noise gain constraint to their optimal beamforming formulations, which can effectively improve the robustness against random array errors [3]. However, the major shortcoming of the white noise gain constrained approach is that it is not clear how to accurately choose the robustness control parameters, based on the *a priori* knowledge of the error levels.

In this paper, a new robust spherical microphone array beamforming approach that can yield achievable maximum-directivity based on the known level of array errors is developed. It is based on the worst-case performance optimization (WCPO) methods in [10], [11, and the references therein], and can work for both the space domain and the spherical harmonics domain processing. In the proposed approach, all actual manifold vectors and spherically isotropic noise covariance matrixes are assumed to belong to two different uncertainty sets. Then the optimal performance is obtained by maximizing the minimum directivity within the covariance matrix uncertainty set, while forcing the minimum response in the look direction within the manifold vector uncertainty set to be not smaller than unity. The robust maximum directivity beamforming problem can be rewritten in a form of tractable convex optimization and solved by second-order cone programming (SOCP).

2. SPHERICAL ARRAY PROCESSING

The reader is referred to [3] for a comprehensive analysis of spherical microphone array beamforming technique. The standard spherical coordinate system [6] is used. Consider a unit magnitude plane wave from direction $\Omega_0 = (\theta_0, \phi_0)$, with wave number k, impinging on a spherical microphone array with M microphones and radius a, using the frequency-domain model, the space domain sound pressure at a microphone position $\Omega_s = (\theta_s, \phi_s)$, $s = 1, \cdots, M$, and the according expression in the spherical harmonics domain can be written as [3]

$$p(ka, \Omega_0, \Omega_s) = \sum_{n=0}^{\infty} b_n(ka) \sum_{m=-n}^{n} [Y_n^m(\Omega_0)]^* Y_n^m(\Omega_s), \qquad (1)$$

$$p_{nm}(ka, \Omega_0) = b_n(ka) Y_n^{m*}(\Omega_0), \qquad (2)$$

where Y_n^m is the spherical harmonic of order n and degree m, superscript * denotes complex conjugation, and $b_n(ka)$ depends on the sphere configuration, e.g. rigid sphere, open sphere, etc., as given by [1], [3]. Typically, $(N+1)^2 \leq M$.

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The $M\times1$ space domain manifold vector for the direction Ω_0, and the $(N+1)^2\times1$ spherical harmonics domain manifold vector can be written as

$$\mathbf{p} = [p(ka,\Omega_0,\Omega_1), p(ka,\Omega_0,\Omega_2),\cdots, p(ka,\Omega_0,\Omega_M)]^T ,$$

$$\mathbf{p}_{nm} = [p_{00}, p_{1(-1)}, p_{10}, p_{11},..., p_{NN}]^T = \text{vec}(\{[p_{nm}]_{m=-n}^{n}\}_{n=0}^{N}) ,$$

where $(.)^T$ denotes the transpose.

Define the $M\times1$ space domain array weight vector and the $(N+1)^2\times1$ spherical harmonics domain array weight vector by

$$\mathbf{w} = [w_1, w_2,\cdots, w_M]^T ,$$

$$\mathbf{w}_{nm} = \text{vec}(\{[w_{nm}]_{m=-n}^{n}\}_{n=0}^{N}) .$$

Array processing can be performed in either the space domain or the spherical harmonics domain, and the array output is given as

$$y(ka) = \frac{4\pi}{M}\mathbf{w}^H(k)\mathbf{p}(ka) = \mathbf{w}_{nm}^H(k)\mathbf{p}_{nm}(ka) , \quad (3)$$

where $(.)^H$ denotes the Hermitian transpose. For simplification, we assume a uniform sampling, where the output amplitude for the spherical harmonics domain array processing is a factor of $4\pi/M$ higher than the space domain [9].

3. MAXIMUM-DIRECTIVITY SPHERICAL ARRAY BEAMFORMING

Directivity is a common measure of beamforming performance. In the space domain, the directivity factor $D(k)$ of a spherical array beamformer can be interpreted as the array gain against spherically isotropic noise.

$$D(k) = \frac{|\mathbf{w}^H\mathbf{p}(\Omega_l)|^2}{\mathbf{w}^H\mathbf{Q}\mathbf{w}} ,$$

where Ω_l denotes the look direction, and $\mathbf{Q}$ is the covariance matrix of a spherically isotropic noise field with unit power spectral density.

From the above definition, it is seen that the maximum-directivity beamforming problem can be written as

$$\min_{\mathbf{w}} \ \mathbf{w}^H\mathbf{Q}\mathbf{w} \text{, subject to } \mathbf{w}^H\mathbf{p}(\Omega_l) = 1 , \quad (4)$$

and the solution to (4) is given by

$$\mathbf{w}(k) = \frac{\mathbf{Q}^{-1}\mathbf{p}(\Omega_l)}{\mathbf{p}^H(\Omega_l)\mathbf{Q}^{-1}\mathbf{p}(\Omega_l)} . \quad (5)$$

In the space domain, the spherically isotropic noise covariance matrix $\mathbf{Q}$, with power spectral density $\sigma^2(\omega)$, can be calculated as [8]

$$\mathbf{Q} = \frac{1}{4\pi}E[\int_{\Omega\in S^2}\sigma^2(\omega)\mathbf{p}(ka,\Omega)\mathbf{p}^H(ka,\Omega)d\Omega] .$$

where the $[s,s']$ th entry of $\mathbf{Q}(\omega)$ is

$$[\mathbf{Q}]_{s,s'} == \frac{\sigma^2(\omega)}{4\pi}\sum_{n=0}^{\infty}|b_n(ka)|^2\sum_{m=-n}^{n}[Y_n^m(\Omega_s)]^*Y_n^m(\Omega_{s'}) . \quad (6)$$

In the spherical harmonics domain, the spherically isotropic noise covariance matrix is given by [7]

$$\mathbf{Q}_{nm} = \frac{1}{4\pi}E[\int_{\Omega\in S^2}\sigma^2(\omega)\mathbf{p}_{nm}(ka,\Omega)\mathbf{p}_{nm}^H(ka,\Omega)d\Omega]$$

$$= \frac{\sigma^2(\omega)}{4\pi}\text{diag}\{\mathbf{b}(ka)\circ\mathbf{b}^*(ka)\} , \quad (7)$$

where $\mathbf{b} = \text{vec}(\{[b_n]_{m=-n}^{n}\}_{n=0}^{N})$.

It is seen that since the noise covariance matrix (7) has been successfully diagonalized by the spherical Fourier transform, it can be used in the spherical harmonics domain version of (5), leading to a more elegant closed-form solution for array weight vectors in the spherical harmonics domain

$$\mathbf{w}_{nm} = \frac{(4\pi)^2}{M(N+1)^2}\mathbf{Y}^*(\Omega_0)\circ/\mathbf{b}^*(ka) , \quad (8)$$

which is identical to the weight vector of a plane wave decomposition (PWD) beamformer [3]. Thus, the PWD beamformer can be interpreted as a maximum-directivity beamformer for spherical arrays. However, it is also known that, at low frequencies, the inversion of $\mathbf{b}^*$ in (8) will lead to a very low robustness against random array errors [4], and the nominal maximum-directivity can hardly be achieved in reality, when the frequency is low and random array errors exist. Therefore, it is desired to design a beamformer that can automatically provide the achievable maximum directivity for spherical arrays based on the known error range, and be robust against all of the random array errors within this error range.

4. ACHIEVABLE MAXIMUM-DIRECTIVITY IN THE PRESENCE OF ARRAY ERRORS

4.1 Space domain processing

We assume that array errors are additive and uncorrelated with manifold vectors (in [3], it has been shown that microphone self noise, sensitivity and phase variations, positioning errors, etc. can be modelled as additive errors). When random errors are present, the actual manifold vector in the space domain $\tilde{\mathbf{p}}$ with array errors can be expressed as

$$\tilde{\mathbf{p}} = \mathbf{p} + \mathbf{e} ,$$

where $\mathbf{e}$ is an unknown complex vector that describes the random array error. Assume that the array error is independent from sensor to sensor, and that the vector $\mathbf{e}$ is norm-bounded by the maximum error level $\varepsilon > 0$, $\|\mathbf{e}\| \le \varepsilon$. Then the actual manifold vector $\tilde{\mathbf{p}}$ belongs to the following uncertainty set A, which is an ellipsoid that can cover all possible actual manifold vectors

$$A = \{\tilde{\mathbf{p}} \mid \tilde{\mathbf{p}} = \mathbf{p} + \mathbf{e}, \ \|\mathbf{e}\| \le \varepsilon\} ,$$

Similarly, the actual spherically isotropic noise covariance matrix is written as

$$\widetilde{\mathbf{Q}} = \frac{1}{4\pi} E[\int_{\Omega \in S^2} \widetilde{\mathbf{p}}(ka,\Omega)\widetilde{\mathbf{p}}^H(ka,\Omega)d\Omega]$$

$$= \mathbf{Q} + E[\mathbf{e}\mathbf{e}^H], \quad \|\mathbf{e}\| \le \varepsilon .$$

In [3], it has been shown that the power of sensor self noise can be an effective measure for random array errors (sensitivity and phase variations, positioning errors, etc.), therefore, for simplification, we approximate the covariance matrix set of $\mathbf{e}$ as a set of white noise covariance matrix, which may not be obvious. For more details see reference [12]. The uncertainty set of the noise covariance matrix is

$$B = \{\widetilde{\mathbf{Q}} \mid \widetilde{\mathbf{Q}} = \mathbf{Q} + \sigma_{\mathbf{e}}^2 \mathbf{I},\ \sigma_{\mathbf{e}}^2 \le \varepsilon^2\} .$$

Using the worst-case performance optimization method, the achievable maximum-directivity beamformer in the presence of random array errors can be written as the following constrained minimax optimization problem:

$$\min_{\mathbf{w}} \max_{\widetilde{\mathbf{Q}} \in B} \mathbf{w}^H \widetilde{\mathbf{Q}} \mathbf{w} \text{ , subject to } \min_{\widetilde{\mathbf{p}} \in A} \left| \mathbf{w}^H \widetilde{\mathbf{p}}(\Omega_l) \right| \ge 1 . \quad (9)$$

With the definition of $\widetilde{\mathbf{p}}$ and $\widetilde{\mathbf{Q}}$, and by applying the triangle and Cauchy-Schwartz inequalities, we have

$$\left|\mathbf{w}^H \widetilde{\mathbf{p}}(\Omega_l)\right| = \left|\mathbf{w}^H [\mathbf{p}(\Omega_l) + \mathbf{e}]\right| \ge \left|\mathbf{w}^H \mathbf{p}(\Omega_l)\right| - \left|\mathbf{w}^H \mathbf{e}\right|$$

$$\ge \left|\mathbf{w}^H \mathbf{p}(\Omega_l)\right| - \varepsilon \|\mathbf{w}\| = \min_{\widetilde{\mathbf{p}} \in A} \left|\mathbf{w}^H \widetilde{\mathbf{p}}(\Omega_l)\right| , \quad (10)$$

and similarly,

$$\mathbf{w}^H \widetilde{\mathbf{Q}} \mathbf{w} = \mathbf{w}^H (\mathbf{Q} + \sigma_{\mathbf{e}}^2 \mathbf{I}) \mathbf{w}$$

$$\le \mathbf{w}^H (\mathbf{Q} + \varepsilon^2 \mathbf{I}) \mathbf{w} = \max_{\widetilde{\mathbf{Q}} \in B} \mathbf{w}^H \widetilde{\mathbf{Q}} \mathbf{w} . \quad (11)$$

Substituting (10) and (11) into (9) gives

$$\min_{\mathbf{w}} \mathbf{w}^H (\mathbf{Q} + \varepsilon^2 \mathbf{I}) \mathbf{w} \text{ , subject to } \left|\mathbf{w}^H \mathbf{p}(\Omega_l)\right| - \varepsilon \|\mathbf{w}\| \ge 1 . \ (12)$$

Using the fact that the cost function in (12) is unchanged when $\mathbf{w}$ undergoes an arbitrary phase rotation, the non-convex problem (12) can now be written as a tractable convex optimization by letting the imaginary part of $\mathbf{w}^H \mathbf{p}(\Omega_l)$ to be zero

$$\min_{\mathbf{w}} \mathbf{w}^H (\mathbf{Q} + \varepsilon^2 \mathbf{I}) \mathbf{w} \text{ , subject to}$$

$$\varepsilon \|\mathbf{w}\| \le \mathbf{w}^H \mathbf{p}(\Omega_l) - 1 , \ \mathrm{Im}[\mathbf{w}^H \mathbf{p}(\Omega_l)] = 0 , \quad (13)$$

where Im[.] denotes the imaginary part.

The optimization problem of (13) is a convex problem, which can be easily solved by SOCP solvers [10]. The overall optimization complexity is around $O(M^3)$.

4.2 Spherical harmonics domain processing

We can write the spherical Fourier transform [3] in the form of a matrix with dimension $M \times (N+1)^2$

$$\mathbf{T} = [\mathbf{Y}_0^0, \mathbf{Y}_1^{-1}, \mathbf{Y}_1^0, \mathbf{Y}_1^1, \cdots, \mathbf{Y}_n^m, \cdots, \mathbf{Y}_N^N] ,$$

where $\mathbf{Y}_n^m = (4\pi)/M \cdot [Y_n^m(\Omega_1), \cdots, Y_n^m(\Omega_s), \cdots, Y_n^m(\Omega_M)]^T$.

The actual spherically isotropic noise covariance matrix in the spherical harmonics domain is

$$\widetilde{\mathbf{Q}}_{nm} = \mathbf{T}^H \widetilde{\mathbf{Q}} \mathbf{T} .$$

Then we have

$$\max_{\widetilde{\mathbf{Q}}_{nm} \in B_{nm}} \mathbf{w}_{nm}^H \widetilde{\mathbf{Q}}_{nm} \mathbf{w}_{nm} = \max_{\widetilde{\mathbf{Q}} \in B} \mathbf{w}_{nm}^H (\mathbf{T}^H \widetilde{\mathbf{Q}} \mathbf{T}) \mathbf{w}_{nm}$$

$$= \mathbf{w}_{nm}^H [\mathbf{T}^H (\mathbf{Q} + \varepsilon^2 \mathbf{I}) \mathbf{T}] \mathbf{w}_{nm}$$

$$= \mathbf{w}_{nm}^H (\mathbf{Q}_{nm} + \frac{4\pi}{M} \varepsilon^2 \mathbf{I}_{nm}) \mathbf{w}_{nm} \quad (12)$$

The discrete spherical harmonics orthonormal property (13) has been employed in the above derivation.

$$\mathbf{T}^H \mathbf{T} = \frac{4\pi}{M} \mathbf{I}_{nm} \quad (13)$$

Regarding the actual spherical harmonics domain manifold vector in the look direction, we have

$$\min_{\widetilde{\mathbf{p}}_{nm} \in A_{nm}} \left|\mathbf{w}_{nm}^H \widetilde{\mathbf{p}}_{nm}(\Omega_l)\right| = \frac{4\pi}{M} \min_{\widetilde{\mathbf{p}} \in A} \left|\mathbf{w}^H \widetilde{\mathbf{p}}(\Omega_l)\right| ,$$

$$= \frac{4\pi}{M} [\left|\mathbf{w}^H \mathbf{p}(\Omega_l)\right| - \varepsilon \|\mathbf{w}\|] .$$

$$= \left|\mathbf{w}_{nm}^H \mathbf{p}_{nm}(\Omega_l)\right| - \sqrt{\frac{4\pi}{M}} \varepsilon \|\mathbf{w}_{nm}\| \quad (14)$$

Note that the Parseval's relation for the spherical Fourier transform to array weights (15), has been applied in the above derivation.

$$\frac{4\pi}{M} \|\mathbf{w}\|^2 = \|\mathbf{w}_{nm}\|^2 . \quad (15)$$

Using (12) and (14) leads to the worst-case performance optimization (16) for the maximum-directivity beamformer in the spherical harmonics domain

$$\min_{\mathbf{w}} \mathbf{w}_{nm}^H (\mathbf{Q}_{nm} + \frac{4\pi}{M} \varepsilon^2 \mathbf{I}_{nm}) \mathbf{w}_{nm} \text{ , subject to}$$

$$\sqrt{\frac{4\pi}{M}} \varepsilon \|\mathbf{w}_{nm}\| \le \mathbf{w}_{nm}^H \mathbf{p}_{nm}(\Omega_l) - \frac{4\pi}{M} ,$$

$$\mathrm{Im}[\mathbf{w}_{nm}^H \mathbf{p}_{nm}(\Omega_l)] = 0 , \quad (16)$$

which can also be easily solved by SOCP solvers. The overall optimization complexity is around $O((N+1)^6)$, which could be lower than that of the space domain, since spatial over sampling is typically employed for spherical arrays.

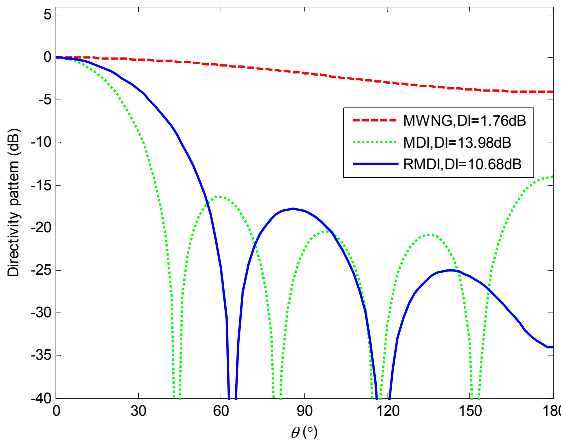

Figure 1 – Directivity pattern comparison between MWNG, MDI, and RMDI (ε =0.01), without any error added

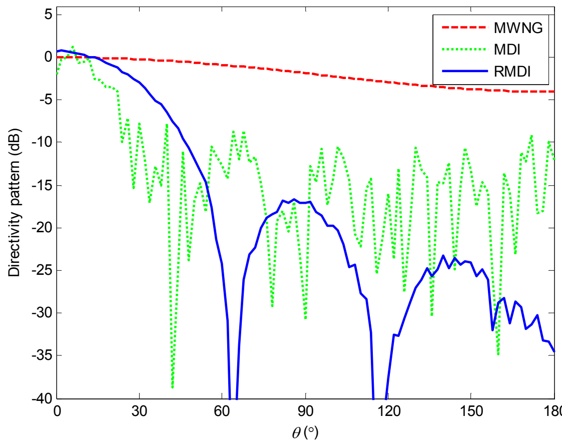

Figure 2 – Directivity pattern comparison between MWNG, MDI, and RMDI (ε =0.01), which are perturbed with complex Gaussian random noise with maximum amplitude 0.01

5. SIMULATION EXAMPLES

In the following numerical examples, we consider a spherical array with 32 microphones embedded in a rigid spherical baffle of radius 4.2 cm and the same sampling scheme as in [1], which can decompose the sound field for up to 4th order spherical harmonics. For the interest of brevity, only the results of spherical harmonics domain processing are provided, since the space domain processing can give the similar performance. The SeDuMi MATLAB Toolbox is used to solve the SOCP problem (16).

Figs. 1-4 show the performance comparison among three kinds of beamformers, the maximum white noise gain beamformer (MWNG), or delay-and-sum beamformer, in [4], the nominal maximum-directivity beamformer (MDI) obtained by (7), and the proposed robust maximum-directivity approach (RMDI) that is obtained by solving the SOCP problem (16). ka =0.5 is used in the examples. The maximum error level ε is set to be 0.01 in Figs. 1 and 2, and 0.1 in Figs. 3 and 4. The beampatterns obtained by the three approaches when there is no array error are given in Figs. 1 and 3. It seen that, as expected, the MDI approach can provide the highest directivity indices (DI), while the MWNG gives the lowest DIs. The proposed RMDI approach provides the actually achievable solutions with DI values between MWNG and MDI. Then the cases with additive array errors are considered. We assume that each microphone is perturbed with two zero-mean circularly symmetric complex Gaussian random variables with maximum amplitude 0.01 and 0.1, respectively. The resulting beampatterns are shown in Figs. 2 and 4, which are the averages of Monte Carlo simulations with 100 repetitions. It is seen that the MWNG still has the lowest directivity but it is very robust against array errors. MDI degrades significantly with large stochastic variations, which means that it is very sensitive to array errors. The performance degradation in the RMDI beampattern is much less than for MDI, which shows both acceptable directivity and robustness.

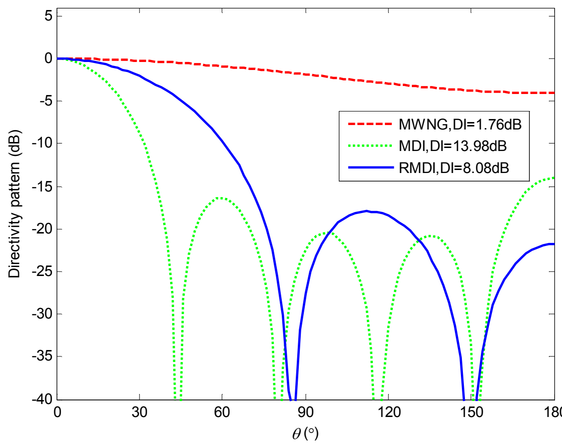

Figure 3 – Directivity pattern comparison between MWNG, MDI, and RMDI (ε =0.1), without any error added

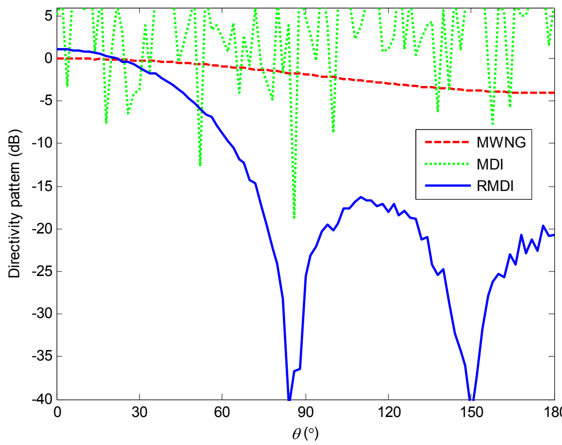

Figure 4 – Directivity pattern comparison between MWNG, MDI, and RMDI (ε =0.1), which are perturbed with complex Gaussian random noise with maximum amplitude 0.1

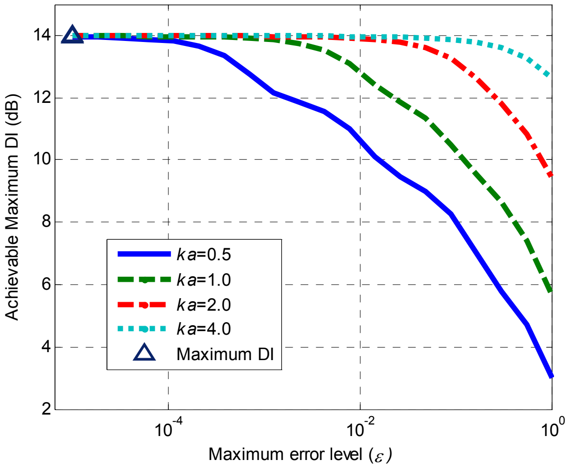

Figure 5 – The achievable maximum directivity index as function of the know error level ε, at different frequencies, using the proposed RMDI approach

Fig. 5 shows the achievable DI of the proposed RMDI, as function of different error levels, at frequencies corresponding to $ka = 0.5$, 1, 2 and 4. It is seen that at higher frequencies, the beamformer can provide a good balance between the maximum DI and robustness against the random array errors, while at lower frequencies, it is difficult to obtain a very high DI with acceptable robustness. This is consistent with the results in Figs. 1-4.When the error level is very low and negligible (e.g. <=0.0001), we can also find that the DI of RMDI is identical to that of MDI (denoted by the "Δ" mark). These results demonstrate that our approach can achieve a suitable compromise among those conflicting beampattern synthesis objectives.

Therefore it is shown that the proposed RMDI provides a good tradeoff between the achievable directivity and the robustness against array errors for spherical microphone array beamformers. Moreover, it does not need any complicated manual tuning for robust parameters (as in [5], [7], [8] [15]), and can automatically yield the achievable optimum performance based on the known range of array errors.

It is also worth noting that, by using the white noise gain constrained methods [5], [7], [8], [15], we can also get the similar robustness control performance as the proposed method (the results are not shown here in the interest of brevity). However, for those white noise gain constrained methods, manual tuning of robustness parameters is needed, which could be more time-consuming and complicated than the proposed method.

6. CONCLUSIONS

A new robust spherical microphone array beamformer based on the worst-case performance optimization methods is proposed. Compared with the conventional white noise gain constrained robustness control approaches, it can automatically find the optimal array weights, and yield the achievable maximum-directivity based on the known level of array errors. More convex beamforming constraints can be further added to the proposed optimization formulations.

7. ACKNOWLEDGEMENT

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