RECONSTRUCTION OF SEQUENCES OF ARBITRARY-SHAPED PULSES FROM ITS LOW-PASS OR BAND-PASS APPROXIMATIONS USING SPECTRUM EXTRAPOLATION

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ABSTRACT

The paper discusses the problem of processing short-time pulse sequences. Since the spectrum of such signals occupies wide range, it is difficult to sample them at the Nyquist rate. Instead, the approach used for processing signals with finite rate of innovations is employed – sequence of pulses is filtered with low-pass or band-pass filter before sampling. A waveform reconstruction method is proposed, which is based on spectrum extrapolation in an iterative way. Due to spectral function of sequence of pulses is correlated, it is possible to recover sequences, which combine arbitrary-shaped pulses. In simulations three shapes of Gaussian based pulses are used as an example. Results demonstrate reconstruction of sequences of pulses from its low-pass and band-pass approximations. An application of the results can be used in ultra wideband impulse radio systems.

1. INTRODUCTION

Information in ultra wideband impulse radio (UWB-IR) systems is transmitted by generating extremely short pulses with durations less than 1 ns and thus occupying large bandwidth in frequency domain. Digital data to the analog pulses is added by means of modulation. Two possible types are pulse position and pulse shape modulations, where time instants and waveforms of pulses carry the information [1].

Different types of pulses like Gaussian pulse and its derivatives which form monocycle, doublet etc., Hermite pulses and others can be used in UWB-IR systems. The actual shape of pulses is usually driven by system and antenna designs. There can be different situations at the receiver - all pulses are with the same shape or multiple pulse types are used [2].

As UWB pulses are extremely short, it is difficult to provide the sampling rate determined by the bandwidth of transmitted signal to decode the received signal. In [3, 4] it is shown that it is possible to recover a non-bandlimited signal that has a finite rate of innovation from uniform samples of its low-pass approximation. The rate of innovation (number of degrees of freedom per unit time) can be viewed as the number of parameters per unit time required to model the signal. The reconstruction is based on the use of an annihilating filter, and the samples have to be taken at rate above the rate of innovation.

An example of a signal with finite rate of innovation is a stream of K weighted Diracs. The signal is fully specified by amplitudes and locations of the Diracs and thus the number of degrees of freedom is $2K$. Also an UWB-IR signal consisting of a stream of K pulses of fixed type can be considered as a signal with finite rate of innovation, because it can be assumed as a stream of Diracs convolved with the pulse shape. If all pulses have the same shape and duration, the number of degrees of freedom is still $2K$. In this case the sequence of pulses can be recovered by annihilating filter method if spectrum of the pulses is known [4]. The spectrum of the signal equals the product of the spectrum of stream of Diracs and the spectrum of the pulse shape, wherewith the reconstruction task reduces to finding the positions and amplitudes of Diracs. If the pulses have different shapes and durations, the number of degrees of freedom increases because, along with amplitudes and locations, the type to which each pulse belongs must also be specified. In this case the spectrum of the pulse sequence equals the sum of different products and the solution for reconstruction can not be obtained by annihilating filter method.

In this paper, an alternative method is proposed, capable of recovering the waveform of the UWB-IR signal from its low-pass or band-pass approximation even if the sequence consists of arbitrary shaped pulses. The reconstruction is based on spectrum extrapolation using signal dependent transformation kernel [5].

2. PROPERTIES OF DIFFERENT TYPES OF PULSES

In ultra-wideband impulse radio systems, a sequence of pulses is employed, where each pulse represents one information symbol. Each transmitted pulse occupies defined time interval called a ”frame”. Often, a single type of narrow pulse is used in all frames. In multi-user environment different types of pulses can be used in different frames by different users. Three of the most often used types of shapes are unilateral (bell-shaped), monocycle and doublet pulses. In the paper the Gaussian function based pulses will be employed as an example, because it is widely used model for UWB-IR transmitting circuits. However, the method proposed in Section 3 can reconstruct sequences of any arbitrary shape pulses.

Monocycle and doublet pulses are obtained as the first and the second derivatives of a Gaussian pulse. In this section temporal and spectral properties of these types of pulses are discussed.

2.1 Time-domain properties

A Gaussian pulse, monocycle and doublet are written respectively as

$$g_1(t) = A_1 e^{-(t/a)^2}, \tag{1}$$

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$$g_2(t) = A_2 \frac{-2t}{a^2} e^{-(t/a)^2}, \quad (2)$$

$$g_3(t) = A_3 \frac{-2}{a^2}(1 - \frac{2t^2}{a^2}) e^{-(t/a)^2}, \quad (3)$$

where a is the time-scaling factor, and A_1, A_2 and A_3 are constants. A Gaussian monocycle has a single zero crossing, while a doublet has two zero crossings. Further derivatives yield additional zero crossings, one additional zero crossing for each additional derivative. If the value of a is fixed, by taking an additional derivative, the fractional bandwidth decreases, while the central frequency increases [1].

Fig. 1a shows the waveforms of all three pulses. The time-scaling factor $a = 0.1$ ns is assumed, which complies with practical considerations and facilitates comparisons. The constants A_1, A_2, and A_3 are chosen to ensure the energy of pulses is equal.

The total duration of the Gaussian pulse can be estimated as six times its standard deviation, as this interval includes about 99.7 percent of the energy. In our case this parameter is about 425 ps.

2.2 Frequency-domain properties

The spectral representation of $g_1(t)$ can be obtained by taking the Fourier transforms of (1)

$$G_1(f) = A_1 a \sqrt{\pi} e^{-(\pi a f)^2} \quad (4)$$

Since $g_2(t)$ and $g_3(t)$ are proportional to the first and second derivatives of $g_1(t)$, the Fourier transforms of $g_2(t)$ and $g_3(t)$ are the Fourier transforms of $g_1(t)$ multiplied by $(A_2/A_1)(j2\pi f)$ and $(A_3/A_1)(j2\pi f)^2$, respectively.

$$G_2(f) = A_2 a \sqrt{\pi} j2\pi f e^{-(\pi a f)^2} \quad (5)$$

$$G_3(f) = A_3 a \sqrt{\pi} (j2\pi f)^2 e^{-(\pi a f)^2} \quad (6)$$

Fig. 1b shows the pulses in frequency domain. Total bandwidth of pulses is around 8 GHz and increases with the order of the derivative.

Often for the characterization of the spectrum of pulses the effective bandwidth is used. It is defined as

$$W = f_h - f_l, \quad (7)$$

where f_l and f_h are the frequencies measured at points with half of the maximum amplitude. For Gaussian pulse, monocycle and doublet the values are $W_1 = 2.65$ GHz, $W_2 = 3.61$ GHz and $W_3 = 3.68$ GHz respectively.

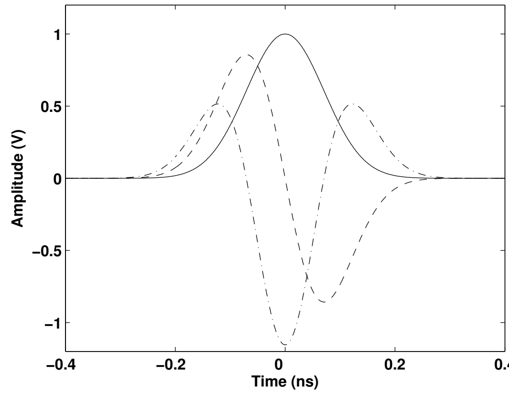
In addition the autocorrelation of the spectrum function $G(\nu)$ is investigated, defined as (8)

$$R(f) = \int_{-\infty}^{\infty} G(\nu) G^*(\nu - f) d\nu, \quad (8)$$

where $G^*(\nu)$ is the complex conjugate of $G(\nu)$. Applying (8) to $G_1(f)$, $G_2(f)$ and $G_3(f)$, autocorrelations $R_1(f)$, $R_2(f)$ and $R_3(f)$ are found:

$$R_1(f) = A_1^2 a \sqrt{\pi/2} e^{-\frac{1}{2}(\pi a f)^2} \quad (9)$$

$$R_2(f) = A_2^2 \frac{1}{a} \sqrt{\pi/2} (1 - (\pi a f)^2) e^{-\frac{1}{2}(\pi a f)^2} \quad (10)$$



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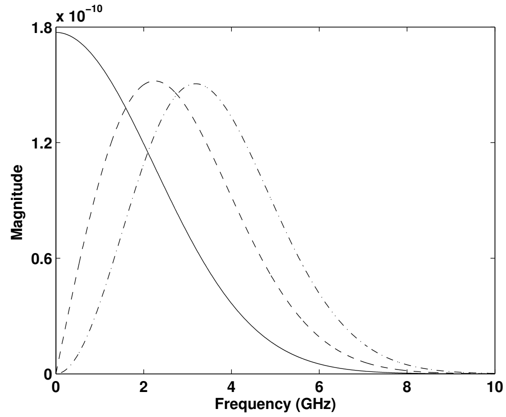

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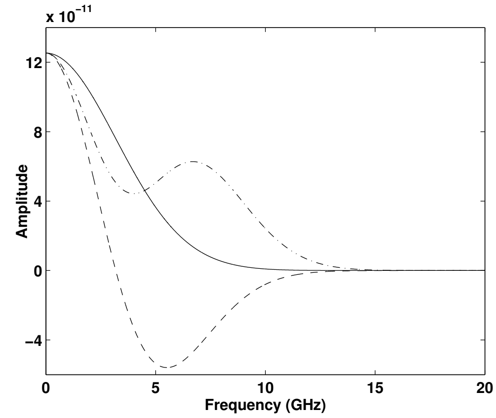
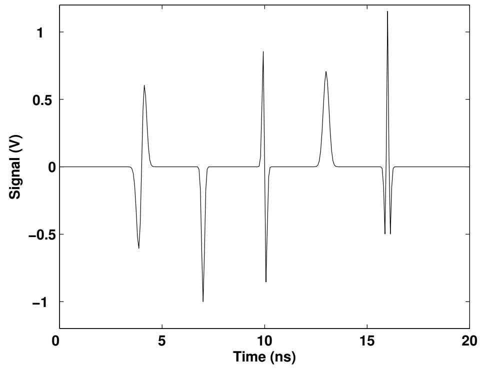


(c)

Figure 1: A Gaussian pulse (solid line), monocycle (dashed line) and doublet (dash-dotted line) in time domain (a), in frequency domain (b) and spectrum autocorrelation functions (c).

$$R_3(f) = A_3^2 a \pi^4 \sqrt{2\pi} (\frac{f^4}{2} - \frac{f^2}{a^2\pi^2} + \frac{3}{2a^4\pi^4}) e^{-\frac{1}{2}(\pi a f)^2} \quad (11)$$

Fig. 1c shows appearance of the spectrum autocorrelation even up to about 10 GHz. Thereby, it can be considered that the spectrum of the pulses is quite well extrapolatable. In the next section, the extrapolation method is proposed to re-



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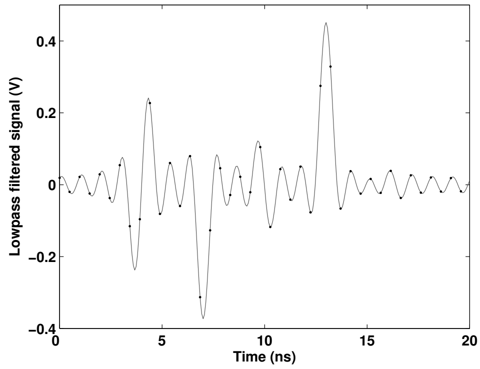

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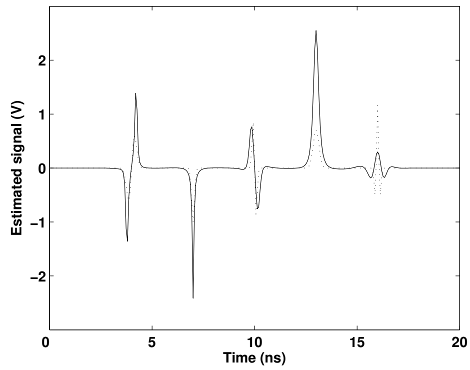

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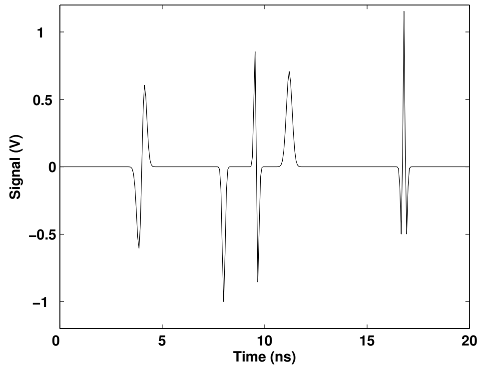
Figure 2: Original sequence of 5 uniformly spaced pulses with different shapes (a), lowpass approximation of it with signal samples (bold dots) (b) and the reconstructed waveform (solid line) of the pulse sequence (dotted line) (c).



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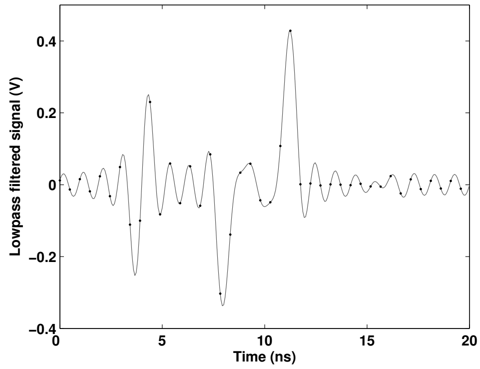

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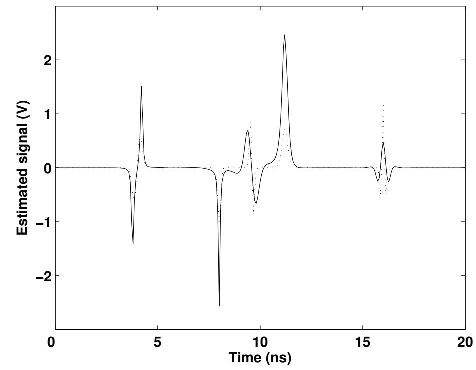


(c)

Figure 3: Original sequence of 5 non-uniformly spaced pulses with different shapes (a), lowpass approximation of it with signal samples (bold dots) (b) and the reconstructed waveform (solid line) of the pulse sequence (dotted line) (c).

cover whole spectrum of the pulse sequence from its limited region, which can be estimated using output of low-pass or band-pass filter.

3. SPECTRUM EXTRAPOLATION APPROACH

The method used for processing band unlimited signals with finite rate of innovations employs bandwidth restriction of filtering before sampling. The bandwidth of the original signal is thus decreased and the samples are taken at lower sampling rate. To reconstruct the signal it is necessary to recover the whole bandwidth in the frequency domain. It can be achieved by spectrum extrapolation provided that the spectrum autocorrelation decays slowly. Following from previous section it holds true for sequences of narrow pulses.

Let us have an UWB signal $x(t)$ consisting of stream of different pulses. The length of the shortest pulse is τ_p. Now, if the signal is sampled at rate $5/\tau_p$ providing N samples

$\mathbf{x} = [x(0), x(1), \ldots, x(N-1)]$, then each pulse is represented by at least 5 samples. The discrete Fourier transform (DFT) of $\mathbf{x}$ allows to obtain spectrum coefficients

$$\mathbf{X}(1 \times N): \; X(k) = \mathbf{x}\mathbf{W}^k, \tag{12}$$

where $\mathbf{W}^k = \left[e^{-j2\pi\frac{0k}{N}}, e^{-j2\pi\frac{1k}{N}}, \ldots, e^{-j2\pi\frac{(N-1)k}{N}}\right]^T$, and $k = 0, 1, \ldots, N-1$. To find the original signal samples, the inverse DFT of $\mathbf{X}$ is taken.

After ideal filtering, only $M < N$ spectrum coefficients $\mathbf{X}_M = [X(r), X(r+1), \ldots, X(r+M-1)]$ of the signal remain, while the rest $N-M$ coefficients become zero. In lowpass filtering case $r = 0$. Recovering of the whole set of the spectrum coefficients from $\mathbf{X}_M$ is based on Capon or minimum variance (MV) filter approach, which requires the knowledge of spectrum autocorrelation matrix [6]. In general, when signal consists of stream of different pulses, the spectrum autocorrelation is not known in advance and thus it is estimated in an iterative way [7, 8]. The algorithm is:

$$\mathbf{R}_i(M \times M): \; R(m,l) = \hat{\mathbf{P}}_{i-1}\mathbf{W}^{l-m}, \tag{13}$$

$$\hat{\mathbf{x}}_i(1 \times N): \; \hat{x}_i(n) = \frac{\mathbf{X}_M \mathbf{R}_i^{-1} \mathbf{W}_M^{-n}}{(\mathbf{W}_M^n)^T \mathbf{R}_i^{-1} \mathbf{W}_M^{-n}}, \tag{14}$$

$$\hat{\mathbf{P}}_i(1 \times N): \; \hat{P}_i(n) = |\hat{x}_i(n)|^2, \tag{15}$$

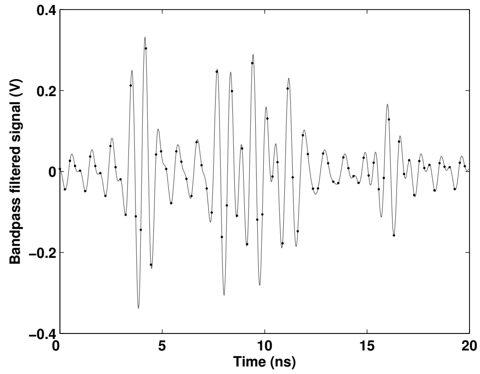
where the elements of autocorrelation matrix $\mathbf{R}_i$ are calculated from the signal power $\hat{\mathbf{P}}_{i-1}$, $\hat{\mathbf{x}}_i$ is the recovered signal (output of the MV filter) after iteration i, $\mathbf{W}_M^n = \left[e^{-j2\pi\frac{rn}{N}}, e^{-j2\pi\frac{(r+1)n}{N}}, \ldots, e^{-j2\pi\frac{(r+M-1)n}{N}}\right]^T$, and $i = 1, 2, 3, \ldots$ are iteration numbers. The estimator $\hat{x}_i(n)$ is considered as found and the iteration as completed when the power $\hat{P}_i(n)$ does not alter from iteration to iteration or the changes are small by comparison with a selected criterion. The initial conditions in the absence of a priori information are determined from the inverse DFT signal estimator

$$\hat{P}_0(n) = \left|\frac{1}{M}\mathbf{X}_M \mathbf{W}_M^{-n}\right|^2 \tag{16}$$

Extrapolation $\hat{\mathbf{X}}_i$ of $\mathbf{X}_M$ is provided by taking DFT (12) of $\hat{\mathbf{x}}_i$.

4. SIMULATION RESULTS

Consider a signal consisting of 2 Gaussian pulses, 2 Gaussian monocycles and 1 Gaussian doublet. The time-scaling factors for the second, third and fifth pulses are 0.1 ns, while for the first and fourth pulses it is 0.2 ns. The amplitudes are chosen to ensure the energy of all pulses is equal. In Fig. 2a pulses are placed equidistantly, while in Fig. 3a – non-equidistantly. The signal is ideally lowpass filtered removing frequencies higher than 1 GHz. The filtered signal (Fig. 2b and Fig. 3b) is sampled at rate 2 GHz to obtain processing data. Given 41 sampling values, the spectrum $\mathbf{X}_M$ of length $M = 21$ is found. By putting $\mathbf{X}_M$ in algorithm (13), (14) and (15) 151 spectrum coefficients up to frequency 7.5 GHz are estimated providing $N = 301$ sampling values $\hat{\mathbf{x}}_i$ (sampling rate 15 Ghz) of the recovered signal. The results after 15 iterations are shown in Fig. 2c and Fig. 3c. Time locations of reconstructed pulses exactly correspond to the original signal, the amplitudes and lengths are reconstructed with some distortions.



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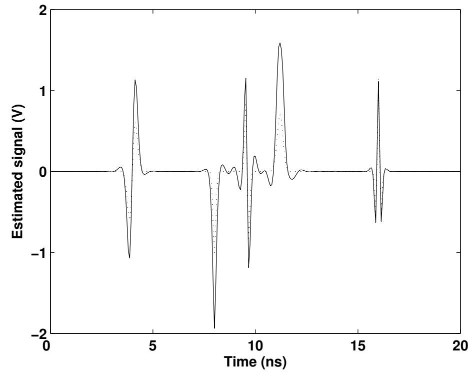

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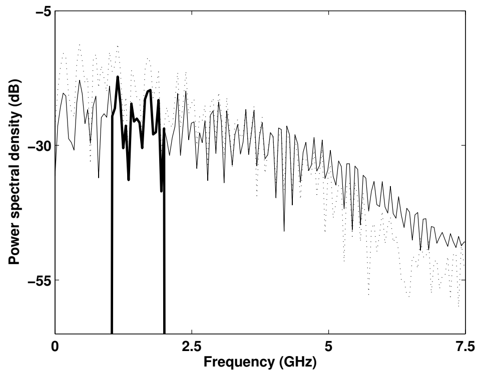

(c)

Figure 4: The bandpass filtered signal with frequencies no lower than 1 GHz and no higher than 2 GHz (a), the estimation (solid line) of the original signal (dotted line) (b), and PSD of the original signal (solid line), bandpass filtered signal (bold solid line) and after iterative extrapolation of spectrum coefficients (c).

The distortion in length becomes larger if pulses are placed closer, while the amplitude of the recovered doublet is decreased since only small part of the pulse energy remains after filtering. Therefore, it is reasonable to use the bandpass filter instead of lowpass filtering. The passband of the filter provides the same amount of spectrum coefficients $\mathbf{X}_M$, but they are placed in the frequency band where the signal energy is higher. The simulation results of this case are shown in Fig. 4. The signal with non-uniformly spaced pulses is ideally bandpass filtered removing frequencies lower than 1 GHz and higher than 2 GHz. The filtered signal (Fig. 4a) is sampled at rate 4 GHz providing 81 samples that are used to obtain $M = 21$ spectrum coefficients $\mathbf{X}_M$ in the frequency range from 1 to 2 GHz. To extrapolate spectrum, the coefficients are put in (13), (14) and (15) and $N = 301$ values of the recovered signal are calculated. Obtained results are shown in Fig. 4b and c. The power spectral densities (PSD) of the original and bandpass filtered signals are represented by a solid and a bold-solid lines, respectively, while the extrapolated PSD function is shown as a dotted line.

Reliability of spectrum extrapolation from given spectrum coefficients $\mathbf{X}_M$ can be estimated using the reliability function [9]. The simplified form of the function [5] is

$$q(f) = \mathbf{s}(f)^T \mathbf{S}^{-1} \mathbf{s}(f), \qquad (17)$$

where $\mathbf{s}(f) = s(f - f_m)$, $S_{mn} = s(f_m - f_n)$, $m, n = 0, 1, \ldots, M$, and $s(f)$ is the constructing function. In our case it is desired that $s(f)$ has to be like spectrum autocorrelation function $R(f)$. In Fig. 5 $q(f)$ is estimated if spectrum coefficients $\mathbf{X}_M$ in the frequency range from 1 to 2 GHz are provided. The grey solid line corresponds to case when $s(f)$ is estimated from spectrum coefficients of the bandpass filtered signal, while the black solid line – after iterative update of autocorrelation matrix. As it follows the update process improves the reliability of spectrum extrapolation.

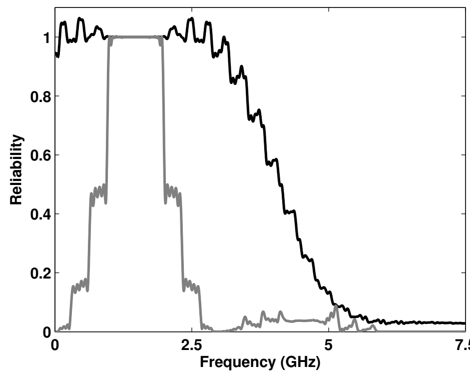

Figure 5: Reliability of spectrum extrapolation.

5. CONCLUSIONS

The method for recovering UWB-IR signals consisting of arbitrary-shaped pulses is proposed. The reconstruction uses uniform samples of low-pass or bandpass filtered signal to obtain spectrum coefficients, which are further extrapolated using iterative update of spectrum autocorrelation function. If pulse sequence contains unilateral and bilateral pulses, passband of filter has to be selected taking into account several considerations. Output of the lowpass filter can be sampled with lower sampling rate, however, pulses with spectrum maximum in higher frequency region are more suppressed and thus they are harder to recover. Instead, bandpass filtering can be used for better reconstruction, though it requires higher sampling rate. It can be decreased by the use of an analog or digital down-converting technique which increases complexity or restricts flexibility of filter bounds. An alternative can be based on non-uniform sampling with decreased density. Frequency aliasing is suppressed due to non-uniformity, and if samples come from a sufficiently frequent uniform grid, the complexity of proposed algorithm does not increase. This can be a topic for further research.

The reliability of reconstructed waveform depends on capability to extrapolate the spectrum. The paper discussed this property for particular pulse types. If sequence contains multiple types, the autocorrelation of spectrum decays faster limiting the frequency up to which it is reasonable to extrapolate. For the cases used in simulations, this frequency has been chosen 7.5 GHz. Such a limitation causes distortions in reconstructed pulses, especially in lengths of the pulses, because band of the original signal spectrum is unlimited.

As UWB is low-power communication it can be efficiently used in wireless sensor networks where data is obtained by event driven sampling techniques such as, for example, level-crossing sampling. In this case, instants of the events occurrences can be coded by the pulse position on the time axis, while type of the events (for example, upward or downward crossing) can be coded by shape of the pulse.

6. ACKNOWLEDGEMENT

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