CUMULANT-BASED ESTIMATION OF QUADRATIC MIXTURE PARAMETERS FOR BLIND SOURCE SEPARATION

Chahinez Chaouchi, Yannick Deville, and Shahram Hosseini

Laboratoire d'Astrophysique de Toulouse-Tarbes, Université de Toulouse, CNRS
14, Avenue Edouard Belin, 31400 Toulouse, France
email:{chahinez.chaouchi, yannick.deville, shahram.hosseini}@ast.obs-mip.fr

ABSTRACT

In this paper, we consider a quadratic model in the blind source separation problem, and we propose a method to estimate the mixing coefficients using cumulants, by solving a nonlinear system of equations. This system is derived from the cumulants of the observations and depends on the mixing parameters and the source moments. We solve it using optimization algorithms, i.e. Levenberg-Marquardt and Gauss-Newton. The numerical results thus obtained confirm the effectiveness of our method.

1. PROBLEM STATEMENT

Blind source separation consists in retrieving a vector $\underline{S}$ of source signals from an observation vector $\underline{X}$ which is a mixture of these sources, i.e.

$$\underline{X} = \mathscr{F}(\underline{S}), \tag{1}$$

where $\mathscr{F}$ is the mixing function, which is unknown. When $\mathscr{F}$ is an arbitrary nonlinear function, it has been shown that the hypothesis of statistical independence of the sources is not sufficient to retrieve them [1], [2]. In order to simplify the problem we can constrain the mixing model [3], [4], [5], [6]. In this paper, we consider the 'quadratic mixture model' defined in [3], [7]. It is an instantaneous model including only quadratic terms of sources (auto-terms and cross-terms):

$$\begin{cases} x_i(n) = \left[a_{ii}s_i^{'}(n)\right]^2 + \left[a_{ij}s_j^{'}(n)\right]^2 + b_i s_i^{'}(n)s_j^{'}(n), \\ (i,j) \in U,\ U = \{(1,2),(2,1)\} \end{cases} \tag{2}$$

where $(s_1^{'}(n), s_2^{'}(n))$ are the sources, and $(x_1(n), x_2(n))$ the observations.

In this work, we focus on the *normalized* version of this model, assuming that $a_{ii,i\in\{1,2\}}$ are non zero:

$$x_i(n) = s_i^2(n) - L_{ij}s_j^2(n) - Q_i s_i(n)s_j(n),\ (i,j) \in U \tag{3}$$

$$\text{where} \quad \begin{cases} s_i(n) = a_{ii}s_i^{'}(n),\ i \in \{1,2\}, \\ L_{ij} = -\frac{a_{ij}^2}{a_{jj}^2}\ ,\ L_{ij} < 0 \\ Q_i = -\frac{b_i}{a_{ii}a_{jj}}, \quad (i,j) \in U \end{cases} \tag{4}$$

The mixing parameters $(L_{12}, L_{21}, Q_1, Q_2)$ are supposed unknown. Our objective in this paper is to estimate them from the observations. We assume that the mixing parameters and the normalized sources are real-valued. We propose a direct estimation of the mixing coefficients in the source separation problem, using a system of nonlinear equations involving cumulants. This kind of approach has been already used in the linear case [8],[9],[10], and we show that its extension to nonlinear mixtures is more complex. We first derive the system of equations of the observation cumulants, then solve it using different optimization algorithms. After estimating the coefficients, the sources can be retrieved using a separating structure such as the recurrent one proposed in [7].

The paper is organized as follows: in Section 2 we provide the details of cumulant calculation from which we derive our system of equations. Section 3 shows how the observation cumulants may be estimated and how we propose to solve the system of nonlinear equations to estimate the mixing coefficients. Section 4 presents some simulation results and we finally conclude in Section 5.

2. PROPOSED APPROACH AND CUMULANT EXPRESSIONS

The observations considered in (3) are linear combinations of power functions of the sources. Therefore, by using the multilinearity properties of cumulants, the cumulants of the observations may be expressed with respect to cumulants of power functions of the source signals and to the mixing coefficients. We here aim at deriving these expressions for observation cumulants of order one to four. Moreover, some of the cumulants associated to the sources are zero, because the sources are assumed to be statistically independent with symmetric probability density functions (pdf), and therefore centered. To determine which of the considered source cumulants are zero we derive their expressions with respect to the associated moments using the classical formula which is provided hereafter. This permits us to obtain a nonlinear equation system depending on the mixing parameters and the source moments.

2.1 Cumulant properties and Notations

We remind that the r-th order cumulant is related to the moments of order p, $p \leq r$, by the Leonov and Shiryayev formula [11]

$$C[X_1, X_2, ..., X_r] = \sum_{\pi}(-1)^{|\pi|-1}(|\pi|-1)! \prod_{\beta\in\pi} E[\prod_{i\in\beta} X_i] \tag{5}$$

where π runs through the list of all partitions of $\{1,...,r\}$, β runs through the list of all blocks of the partition π and $|\pi|$ is the number of parts in the partition. E is the mathematical expectation. We also remind some properties of cumulants [12]:

$CP1$: If $\lambda_i,\ i = 1,...,n$ are constants, and $x_i,\ i = 1,...,n$ are

random variables, then

$$C(\lambda_1 x_1, ..., \lambda_n x_n) = \prod_{i=1}^{n} \lambda_i C(x_1, ..., x_n)$$

*CP*2: Cumulants are symmetric in their arguments, i.e.,

$$C(x_1, ..., x_n) = C(x_{i_1}, ..., x_{i_n})$$

where $(i_1, ..., i_n)$ is a permutation of $(1, ...n)$.
*CP*3: Cumulants are additive in their arguments, i.e., cumulants of sums equal sums of cumulants

$$C(x_0 + y_0, z_1, ..., z_n) = C(x_0, z_1, ..., z_n) + C(y_0, z_1, ..., z_n)$$

*CP*4: If a subset of k random variables $\{x_i\}$ is independent of the rest then

$$C(x_1, .., x_k, .., x_n) = 0$$

The following type of shortened notations is used in this paper:
$C[(s_1^2)_2] = C[s_1^2, s_1^2]$ represents the second-order cumulant of $\{s_1^2\}$.
$C[(x_1)_2, x_2] = C[x_1, x_1, x_2]$ is the third-order cross-cumulant involving $\{x_1\}$ twice and $\{x_2\}$ once.

2.2 First-Order Cumulants

It can be easily derived from (3), (5) and from the independence of the sources that the first-order cumulant of the first observation is

$$\begin{aligned} C[x_1] &= E[x_1] && (6) \\ &= E[s_1^2] - L_{12}E[s_2^2] - Q_1 E[s_1]E[s_2]. && (7) \end{aligned}$$

Using the fact that the sources are centered we have

$$C[x_1] = E[s_1^2] - L_{12}E[s_2^2]. \quad (8)$$

By the same way we obtain

$$C[x_2] = E[x_2] = -L_{21}E[s_1^2] + E[s_2^2]. \quad (9)$$

2.3 Second-Order Cumulants

2.3.1 Observation Cumulants

We detail the calculation for $C[(x_1)_2]$. Using (3) and the above cumulant properties we have

$$\begin{aligned} C[(x_1)_2] &= C[x_1, x_1] \\ &= C[s_1^2 - L_{12}s_2^2 - Q_1 s_1 s_2, s_1^2 \\ &\quad - L_{12}s_2^2 - Q_1 s_1 s_2], \\ &= C[s_1^2, s_1^2] - L_{12}C[s_1^2, s_2^2] - Q_1 C[s_1^2, s_1 s_2] \\ &\quad - L_{12}C[s_2^2, s_1^2] + L_{12}^2 C[s_2^2, s_2^2] + L_{12}Q_1 C[s_2^2, s_1 s_2] \\ &\quad - Q_1 C[s_1 s_2, s_1^2] + L_{12}Q_1 C[s_1 s_2, s_2^2] + Q_1^2 C[s_1 s_2, s_1 s_2]. \end{aligned}$$

Thanks to the properties of cumulants and by using Equations (14) mentioned hereafter in Subsection 2.3.2, and corresponding to the cancellation of some "source cumulants", we get finally

$$\begin{cases} C[(x_1)_2] = C[(s_1^2)_2] + L_{12}^2 C[(s_2^2)_2] + Q_1^2 C[(s_1 s_2)_2], \\ C[(x_2)_2] = L_{21}^2 C[(s_1^2)_2] + C[(s_2^2)_2] + Q_2^2 C[(s_1 s_2)_2], \\ C[x_1, x_2] = -L_{21}C[(s_1^2)_2] - L_{12}C[(s_2^2)_2] + Q_1 Q_2 C[(s_1 s_2)_2]. \end{cases} \quad (10)$$

2.3.2 "Source Cumulants"

We begin the calculation with $C[(s_1^2)_2]$. From (5), we have two possibilities for the partitions: $\{s_1^2, s_1^2\}$ and $\{\{s_1^2\}, \{s_1^2\}\}$, which respectively have 1 and 2 elements, so using the formula (5)

$$\begin{aligned} C[(s_1^2)_2] &= (-1)^{(1-1)}(1-1)!E[s_1^2 s_1^2] + \\ &\quad (-1)^{(2-1)}(2-1)!E[s_1^2]E[s_1^2] \\ &= E[s_1^4] - E[s_1^2]^2. && (11) \end{aligned}$$

We get also

$$C[(s_2^2)_2] = E[s_2^4] - E[s_2^2]^2, \quad (12)$$

by similar calculations, just replacing s_1^2 by s_2^2.
For $C[(s_1 s_2)_2]$, we also have the two partitions, $\{s_1 s_2, s_1 s_2\}$ and $\{\{s_1 s_2\}, \{s_1 s_2\}\}$, so

$$\begin{aligned} C[(s_1 s_2)_2] &= (-1)^0 0! E[s_1 s_2 s_1 s_2] + (-1)^1 1! E[s_1 s_2]E[s_1 s_2] \\ &= E[s_1^2]E[s_2^2] - E[s_1]^2 E[s_2]^2 \\ &= E[s_1^2]E[s_2^2] && (13) \end{aligned}$$

where we have first used the property of statistical independence and then the fact that the sources are centered. The other "source cumulants" are zero

$$C[s_1^2, s_2^2] = C[s_1^2, s_1 s_2] = C[s_2^2, s_1 s_2] = 0. \quad (14)$$

This may be shown as follows:

$$\begin{aligned} C[s_1^2, s_2^2] &= E[s_1^2 s_2^2] - E[s_1^2]E[s_2^2] \\ &= E[s_1^2]E[s_2^2] - E[s_1^2]E[s_2^2] = 0, && (15) \end{aligned}$$

using the independence property, and

$$\begin{aligned} C[s_1^2, s_1 s_2] &= E[s_1^2 s_1 s_2] - E[s_1^2]E[s_1 s_2] \\ &= E[s_1^3]E[s_2] - E[s_1^2]E[s_1]E[s_2] = 0, && (16) \end{aligned}$$

where we have used the independence property and the fact that the sources have symmetric pdf. A similar derivation can be made for $C[s_2^2, s_1 s_2]$, and we find $C[s_2^2, s_1 s_2] = 0$.
Due to space limitations, we will not detail the calculations for the third- and the fourth-order. They are based on the same approach as above, but they are much more tedious because they involve higher-order statistics. We keep a similar scheme hereafter: firstly we give the observation cumulants with respect to the "source cumulants" and secondly we give the latter with respect to the source moments.

2.4 Third-Order Cumulants

The third-order observation cumulants are

$$\begin{cases} C[(x_1)_3] = C[(s_1^2)_3] - L_{12}^3 C[(s_2^2)_3] \\ \qquad + 3Q_1^2 C[s_1^2, (s_1 s_2)_2] - 3L_{12}Q_1^2 C[s_2^2, (s_1 s_2)_2], \\ C[(x_2)_3] = C[(s_2^2)_3] - L_{21}^3 C[(s_1^2)_3] \\ \qquad - 3L_{21}Q_2^2 C[s_1^2, (s_1 s_2)_2] + 3Q_2^2 C[s_2^2, (s_1 s_2)_2], \\ C[(x_1)_2, x_2] = -L_{21}C[(s_1^2)_3] + L_{12}^2 C[(s_2^2)_3] \\ \qquad + (2Q_1 Q_2 - L_{21}Q_1^2)C[s_1^2, (s_1 s_2)_2] \\ \qquad + (Q_1^2 - 2L_{12}Q_1 Q_2)C[s_2^2, (s_1 s_2)_2], \\ C[x_1, (x_2)_2] = L_{21}^2 C[(s_1^2)_3] - L_{12}C[(s_2^2)_3] \\ \qquad + (Q_2^2 - 2L_{21}Q_1 Q_2)C[s_1^2, (s_1 s_2)_2] \\ \qquad + (2Q_1 Q_2 - L_{12}Q_2^2)C[s_2^2, (s_1 s_2)_2]. \end{cases} \quad (17)$$

and the third-order "source cumulants" can be expressed as

$$\begin{cases} C[(s_1^2)_3] = E[s_1^6] - 3E[s_1^4]E[s_1^2] + 2E[s_1^2]^3, \\ C[(s_2^2)_3] = E[s_2^6] - 3E[s_2^4]E[s_2^2] + 2E[s_2^2]^3, \\ C[s_1^2,(s_1s_2)_2] = E[s_2^2]E[s_1^4] - E[s_2^2]E[s_1^2]^2 \\ C[s_2^2,(s_1s_2)_2] = E[s_1^2]E[s_2^4] - E[s_1^2]E[s_2^2]^2. \end{cases} \tag{18}$$

$$\begin{cases} C[(s_1s_2)_3] = C[(s_1^2)_2, s_1s_2] = C[(s_2^2)_2, s_1s_2] = 0, \\ C[s_1^2,(s_2^2)_2] = C[s_2^2,(s_1^2)_2] = C[s_1^2, s_2^2, s_1s_2] = 0. \end{cases} \tag{19}$$

2.5 Fourth-Order Cumulants

The fourth-order observation cumulants are

$$\begin{cases} C[(x_1)_4] = C[(s_1^2)_4] + L_{12}^4 C[(s_2^2)_4] + Q_1^4 C[(s_1s_2)_4] \\ \quad +6Q_1^2 C[(s_1^2)_2,(s_1s_2)_2] + 6L_{12}^2 Q_1^2 C[(s_2^2)_2,(s_1s_2)_2] \\ \quad -12L_{12}Q_1^2 C[s_1^2, s_2^2, (s_1s_2)_2], \\ C[(x_2)_4] = L_{21}^4 C[(s_1^2)_4] + C[(s_2^2)_4] + Q_2^4 C[(s_1s_2)_4] \\ \quad +6L_{21}^2 Q_2^2 C[(s_1^2)_2,(s_1s_2)_2] + 6Q_2^2 C[(s_2^2)_2,(s_1s_2)_2] \\ \quad -12L_{21}Q_2^2 C[s_1^2, s_2^2, (s_1s_2)_2], \\ C[(x_1),(x_2)_3] = -L_{21}^3 C[(s_1^2)_4] - L_{12}C[(s_2^2)_4] + Q_1Q_2^3 C[(s_1s_2)_4] \\ \quad +(-3Q_2^2L_{21} + 3Q_1Q_2L_{21}^2)C[(s_1^2)_2,(s_1s_2)_2] \\ \quad +(-3L_{12}Q_2^2 + 3Q_1Q_2)C[(s_2^2)_2,(s_1s_2)_2] \\ \quad +(3Q_2^2 - 6L_{21}Q_2Q_1 + 3L_{12}L_{21}Q_2^2)C[s_1^2,s_2^2,(s_1s_2)_2], \\ C[(x_2),(x_1)_3] = -L_{21}C[(s_1^2)_4] - L_{12}^3 C[(s_2^2)_4] + Q_2Q_1^3 C[(s_1s_2)_4] \\ \quad +(-3L_{21}Q_1^2 + 3Q_2Q_1)C[(s_1^2)_2,(s_1s_2)_2] \\ \quad +(-3L_{12}Q_1^2 + 3Q_2Q_1L_{12}^2)C[(s_2^2)_2,(s_1s_2)_2] \\ \quad +(3Q_1^2 - 6L_{12}Q_2Q_1 + 3L_{12}L_{21}Q_1^2)C[s_1^2,s_2^2,(s_1s_2)_2], \\ C[(x_1)_2,(x_2)_2] = L_{21}^2 C[(s_1^2)_4] + L_{12}^2 C[(s_2^2)_4] + Q_2^2Q_1^2 C[(s_1s_2)_4] \\ +(L_{21}^2Q_1^2 + Q_2^2 - 4Q_1Q_2L_{21})C[(s_1^2)_2,(s_1s_2)_2] \\ +(L_{12}^2Q_2^2 + Q_1^2 - 4L_{12}Q_1Q_2)C[(s_2^2)_2,(s_1s_2)_2] \\ +(4Q_1Q_2 - 2L_{12}Q_2^2 - 2L_{21}Q_1^2 \\ \quad +4Q_1Q_2L_{12}L_{21})C[s_1^2,s_2^2,(s_1s_2)_2]. \end{cases} \tag{20}$$

and the fourth-order "source cumulants" are

$$\begin{cases} C[(s_1^2)_4] = E[s_1^8] - 4E[s_1^6]E[s_1^2] - 6E[s_1^2]^4 \\ \quad -3E[s_1^4]^2 + 12E[s_1^4]E[s_1^2]^2, \\ C[(s_2^2)_4] = E[s_2^8] - 4E[s_2^6]E[s_2^2] - 6E[s_2^2]^4 \\ \quad -3E[s_2^4]^2 + 12E[s_2^4]E[s_2^2]^2, \\ C[(s_1s_2)_4] = E[s_1^4]E[s_2^4] - 3E[s_1^2]^2E[s_2^2]^2, \\ C[(s_1^2)_2,(s_1s_2)_2] = E[s_1^6]E[s_2^2] - E[s_1^2]E[s_2^2](3E[s_1^4] - 2E[s_1^2]^2), \\ C[(s_2^2)_2,(s_1s_2)_2] = E[s_2^6]E[s_1^2] - E[s_2^2]E[s_1^2](3E[s_2^4] - 2E[s_2^2]^2), \\ C[s_1^2,s_2^2,(s_1s_2)_2] = E[s_1^4]E[s_2^4] - E[s_1^2]^2E[s_2^4] \\ \quad -E[s_2^2]^2E[s_1^4] + E[s_1^2]^2E[s_2^2]^2. \end{cases} \tag{21}$$

$$\begin{cases} C[(s_1^2)_3, s_2^2] = C[(s_2^2)_3, s_1^2] = C[(s_1s_2)_3, s_1^2] = 0. \\ C[(s_1s_2)_3, s_2^2] = C[(s_1^2)_3, s_1s_2] = C[(s_2^2)_3, s_1s_2] = 0. \\ C[(s_1^2), s_1s_2, (s_2^2)_2] = C[s_2^2, s_1s_2, (s_1^2)_2] = C[(s_1^2)_2, (s_2^2)_2] = 0. \end{cases} \tag{22}$$

The systems (8), (9), (10), (17) and (20) altogether form the final system of equations to be solved, which is highly nonlinear. We thus obtain a system of 14 equations depending on 12 unknowns $\{L_{12}, L_{21}, Q_1, Q_2, E[s_1^2], E[s_1^4], E[s_1^6], E[s_1^8], E[s_2^2], E[s_2^4], E[s_2^6], E[s_2^8],\}$ but we will eventually be interested in the mixing parameters $\{L_{12}, L_{21}, Q_1, Q_2\}$. If we do not use the fourth-order cumulants, we obtain a system of 9 equations with 10 unknowns (see Table 1) which cannot be solved in general. We can also use higher than fourth order cumulants to obtain more equations and more unknowns. We propose to solve the above system by using numerical optimization methods defined hereafter.

Order	Observation Cumulants (number)	Added Unknowns
1	$C[x_1], C[x_2]$ (2)	L_{12}, L_{21}, $E[s_1^2], E[s_2^2]$
2	$C[(x_1)_2], C[(x_2)_2]$, $C[x_1, x_2]$ (3)	Q_1, Q_2 $E[s_1^4], E[s_2^4]$
3	$C[(x_1)_3], C[(x_2)_3]$, $C[(x_1)_2, x_2], C[x_1, (x_2)_2]$ (4)	$E[s_1^6], E[s_2^6]$
4	$C[(x_1)_4], C[(x_2)_4], C[(x_1),(x_2)_3]$, $C[(x_2),(x_1)_3], C[(x_1)_2,(x_2)_2]$ (5)	$E[s_1^8], E[s_2^8]$

Table 1: Numbers of equations and unknowns for our system depending on the cumulant order.

3. MIXING PARAMETER ESTIMATION

Once the system of equations has been obtained, we use alternative optimization algorithms to solve it. The goal is to retrieve the mixing parameters $(L_{12}, L_{21}, Q_1, Q_2)$. We need first to estimate the cumulants of the observations at the different orders before solving the system, using numerical methods.

3.1 Estimation of the Observation Cumulants

There exist several estimators of the auto-cumulants and the cross-cumulants in the literature [13], [14]. Unfortunately these estimators consider zero-mean signals, which is not the case in this paper (see Equations (8) and (9)). We then use the functions proposed in the HOSA Matlab toolbox [15], i.e. *cumest*, to estimate the auto-cumulants and *cum2x*, *cum3x* and *cum4x* to estimate the cross-cumulants of the observations.

3.2 System Solving

In this subsection, we present the alternative optimization algorithms used to solve the nonlinear equation system. We use the *fsolve* function, provided by the Matlab optimization toolbox [16], which attempts to solve nonlinear equation systems of the form

$$F(x) = 0,$$

$$\text{where} \quad \begin{cases} x = [x_1, ..., x_m]^T \\ F(x) = [f_1(x), ..., f_n(x)]^T, \end{cases}$$

$x_i,\ i = 1..m$ are the variables (i.e. the 12 unknowns in our case) and $f_i,\ i = 1..n$ are nonlinear functions. So to obtain this form, with a right-hand term of the equations equal to zero, we rearrange our system by moving all its right-hand terms to the left-hand side. The *fsolve* function is used with two configurations: the first one involves the Levenberg-Marquardt algorithm and the second one the Gauss-Newton algorithm.

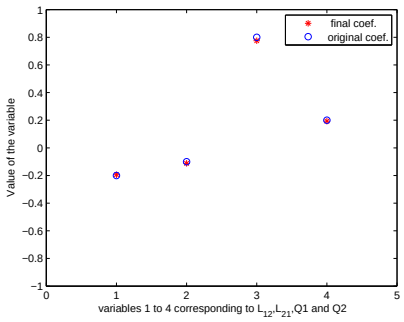

Figure 1: Convergence towards "S" for the parameters $\{L_{12}, L_{21}, Q_1, Q2\}$

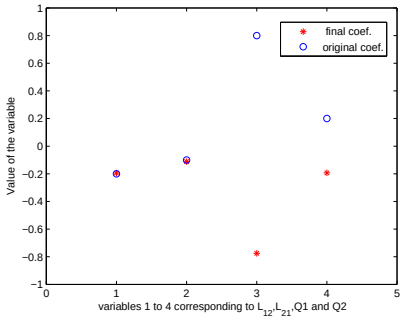

Figure 2: Phenomenon of sign inversion for the parameters Q_1 and Q_2

4. SIMULATION RESULTS

4.1 First Scenario

In a first scenario we consider the set of parameters $\{L_{12}, L_{21}, Q_1, Q_2\} = \{-0.2, -0.1, 0.8, 0.2\}$, and the sources of length $N = 10000$ samples are uniformly distributed over $[-1, 1]$. The *fsolve* function returns an EXITFLAG output that describes if the algorithm converges or not. For each configuration of the *fsolve* function, we perform 450 tests. For each test, we use the EXITFLAG output to reset the variable x of the *fsolve* function (with a different initialization generated randomly) if the algorithm does not converge and this is done 150 times if it is necessary. For the Levenberg-Marquardt configuration the algorithm converges in 95 per cent of the tests (otherwise the algorithm diverges), and 33 per cent (161 tests) towards the point of interest, called S, the rest corresponding to other solution points of our equation system. We observed a sign inversion phenomenon for the estimated parameters Q_1 and Q_2 as illustrated by Figure 1 and Figure 2. This is due to the form in which they appear in our system, i.e. if (Q_1, Q_2) is a solution of the system, then $(-Q_1, -Q_2)$ is a solution too. For the Gauss-Newton configuration we obtain 91 per cent of convergence in general and 37 per cent (167 tests) of convergence towards S. Better results of convergence towards S are obtained in this case because less spurious points are reached in a significant number of tests (see Table 2). For this configuration we also observe the phenomenon of sign inversion. Therefore, for the two configurations in this scenario, the algorithm converges most often towards the point of interest compared to

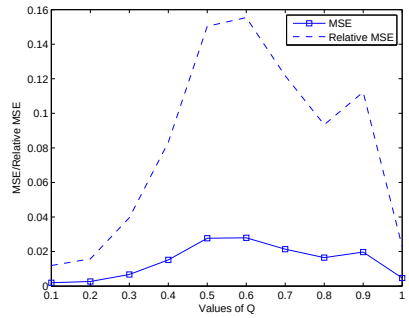

Figure 3: Mean Value of the MSE vs Q

the other points of convergence. Table 2 shows that these spurious points can be eliminated by considering the fact that $L_{12}, L_{21} < 0$ from (4) because they do not meet this condition.

Algo.	Convergence Point $\{L_{12}, L_{21}, Q_1, Q_2\}$		Nb.
L. M.	S	$-0.1829, -0.0897, -0.7579, -0.1972$	161
	Pt 1	$1.6839, -0.2104, -0.2173, -0.7231$	62
	Pt 2	$0.1618, 0.0992, 1.3733, 0.0914$	53
	Pt 3	$1.7598, -0.2066, -0.2283, 0.7147$	47
	Pt 4	$-0.1918, 0.0918, -1.4870, -0.0764$	43
G. N.	S	$-0.1747, -0.1097, 0.7779, 0.2086$	167
	Pt 1	$-0.3184, 0.4684, 0.8806, 0.6188$	58
	Pt 2	$-0.1298, 0.0704, -1.5460, -0.0944$	49
	Pt 3	$6.3623, -0.3529, 0.1305, 1.1516$	25

Table 2: Point of interest S and spurious points for the Levenberg-Marquardt and Gauss-Newton algorithms.

4.2 Second Scenario

For this scenario we consider the set of parameters $\{L_{12}, L_{21}, Q_1, Q_2\} = \{-0.9, -0.75, 0.2, 0.1\}$, and the sources of length $N = 10000$ samples are respectively uniformly distributed over $[-1, 1]$ and normally distributed $N(0, 0.2)$. For each configuration of the *fsolve* function we consider 450 tests with the same conditions as mentioned previously. For the Levenberg-Marquardt configuration the algorithm converges in 100 per cent of the tests, and 10.5 per cent (43 tests) towards the point of interest, called S', the rest corresponding to other solution points of our equation system. For the Gauss-Newton configuration we obtain 100 per cent of convergence in general and approximately 21 per cent of convergence towards S' (94 tests). In this scenario, we also observe the phenomenon of sign inversion.

4.3 Third Scenario

We generate Monte Carlo runs using 100 different random realizations of the sources s_1 and s_2 of length N=10000 and uniformly distributed over $[-1, 1]$. The realizations are independent. The parameters L_{12}, L_{21}, are respectively set to -0.8 and -0.9 and $Q_1 = Q_2 = Q$ where Q is varied between 0.1 and 1 with a step size of 0.1. Then, for each realization of the mixtures corresponding to one value of the parameter

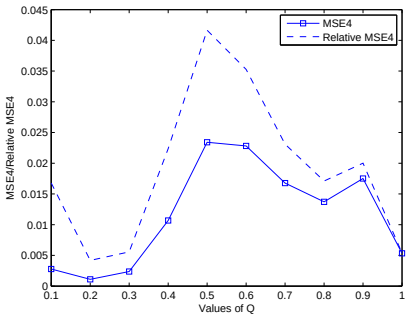

Figure 4: Mean Value of the MSE vs Q calculated for the parameters $\{L_{12}, L_{21}, Q_1, Q2\}$

Q, we carry out the experiments for all sources using the procedure described in Section 4.1 and the *fsolve* function only with the Levenberg-Marquardt algorithm configuration. Due to the sign inversion phenomenon, the error and the relative error are calculated as follows:

$$\varepsilon_i = |X_i| - |S_i|, \quad i = 1..12 \tag{23}$$

$$\varepsilon_{rel,i} = \frac{|X_i| - |S_i|}{|S_i|}, \quad i = 1..12 \tag{24}$$

where i is the index of the variable, X contains the variables at convergence and S is our point of interest.
The associated *MSE* and *relative MSE* are respectively:

$$MSE_n = \frac{1}{n}\sum_{i=1}^{n}(|X_i| - |S_i|)^2 = \frac{1}{n}\sum_{i=1}^{n}\varepsilon_i^2, \tag{25}$$

$$MSE_{rel,n} = \frac{1}{n}\sum_{i=1}^{n}(\frac{|X_i| - |S_i|}{|S_i|})^2 = \frac{1}{n}\sum_{i=1}^{n}\varepsilon_{rel,i}^2, \tag{26}$$

where the index n is set to 12 and 4 when we calculate these criteria respectively for the whole set of parameters and only for the 4 parameters L_{12}, L_{21}, Q_1, Q_2. We resort to the relative quantities because they are more appropriate when adding the different variables. Figures 3 and 4 show the mean values of MSE_n and $MSE_{rel,n}$ over all 100 sets of sources for the different values of the parameter Q. The graph in Figure 4 shows that the mean relative MSE is lower or equal to 4 per cent when we consider only the 4 parameters of interest, which proves the effectiveness of our method.

5. CONCLUSION AND PERSPECTIVES

In this paper we presented a method for estimating the parameters of a quadratic mixture for blind source separation. This method is based on the resolution of a nonlinear equation system provided by the observation cumulants. Numerical optimization algorithms permit us to estimate the coefficients of the mixture. Through simulations, we showed the effectiveness of the method. The use of constrained optimization algorithms which take into account the negativity of the parameters $\{L_{12}, L_{21}\}$ and the positivity of the source moments will be studied in future work. A theoretical analysis is needed to complete this work, i.e. on the one hand, try to solve analytically the nonlinear equation system, and on the other hand try to analyze the simulations theoretically by replacing the source moments by their expressions for given source pdf.

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