ADAPTIVE LEVEL-CROSSING SAMPLING AND RECONSTRUCTION

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ABSTRACT

We propose an adaptive level crossing approach for the sampling and reconstruction of signals for applications where clock-free and low-power data gathering are required. Due to the lack of *a priori* information on the signal statistics, uniform levels are typically chosen in level-crossing (LC) sampling. Our approach uses as reference levels the local means obtained from an asynchronous sigma delta modulator (ASDM). This signal-dependent sampling constitutes non-uniform sampling with local means and their times of occurrence. The local means can be seen as optimal signal estimators in a mean-square sense. The reconstruction of the original signal is approached using Prolate Spheroidal Wave functions which performs well when the signal is time-limited and essentially band-limited. The optimality of the levels can be illustrated when comparing our procedure with uniform LC sampling and reconstruction. The proposed procedure is especially suited for applications where the signal occurs in bursts, and data gathering is done under clock–free, low–power and low–transmission conditions.

1. INTRODUCTION

Although non-uniform sampling is not a preferred method for data acquisition, since both samples and their corresponding sample times are needed for reconstruction, in many applications it is an alternative to conventional time-driven sampling. That is the case whenever clocks are not desirable, and high energy efficiency and low data transmission are required. In biomedical monitoring [9], for brain– or heart–computer interfaces, speech processing and networked control [1], level crossing (LC) or Lebesgue sampling have been considered as an alternative to conventional time-driven sampling. In LC sampling the signal is not sampled but directly quantized whenever the signal crosses a certain level. As indicated in [2] this approach avoids frequency aliasing and could be applied to signals that are not band-limited. LC is particularly useful in dealing with signals that deliver the information in bursts rather than in a constant stream. Such signals appear in biomedical applications [9] and in sensor network transmissions [3]. Because of their short duration and bursty nature, these signals are not necessarily band-limited. Level crossing (LC) [4] is a form of nonuniform sampling that can be used for bursty or sparse signals. Given the economical sampling, LC sampling may be used to obtain a discrete representation for sparse signals in the compressive sensing [5].

The advantages of LC sampling in both data transmission and signal reconstruction depend on the proper placement of reference levels within the dynamic range of the input. Typically, the levels have been treated as uniform quantization levels [4, 6, 7] instead of optimal level allocation [3], where the signal dictates the rate of data collection and quantization.

In this paper, we propose a novel approach to determine signal–dependent reference levels for the LC sampling. These adaptive levels are obtained by using an asynchronous sigma delta modulator (ASDM) which is a nonlinear feedback system that performs time–encoding of the signal [9]. The output of an ASDM is a binary signal that is continuous in time. The ASDM provides a clock-less data acquisition and reconstruction from the sample times [9, 10, 11]. Moreover, it enables the computation of the local means of the signal. It is these local means that we will use as the levels for the LC sampling. The local means require that more samples be taken when the signal is bursty and fewer otherwise.

Considering signals that are time limited and essentially band-limited (a high percentage of the signal energy is concentrated in a certain bandwidth) the reconstruction is approached using prolate spheroidal wave functions (PSWFs). In biomedical data monitoring, ASDM-based adaptive LC sampling can be used efficiently and implemented as in [12]. As illustrated by the simulations, the ASDM-based adaptive LC sampling and reconstruction using PSWFs show a lot of promise in the processing of time-limited and essentially band-limited signals, including bursty signals [13, 14, 15].

2. ASYNCHRONOUS SIGMA DELTA MODULATORS

An Asynchronous Sigma Delta Modulator (ASDM), Fig. (1), is a nonlinear feedback system consisting of an integrator and a non-inverting Schmitt trigger [8]. In the ASDM, amplitude information of a signal $x(t)$ is transformed into time information.

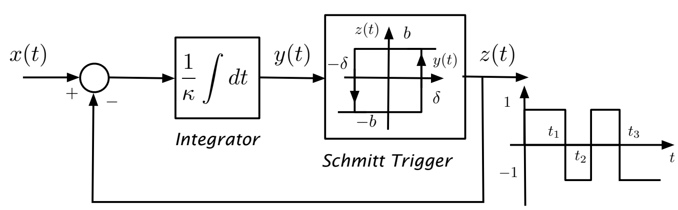

Figure 1: Asynchronous sigma delta modulator

2.1 Duty-cycle Modulation and Time-encoding

Time-encoding can be seen as duty-cycle modulation, where a sequence of binary rectangular pulses (Fig.2) is characterized by the duty-cycle defined for two consecutive pulses of duration $T_k = \alpha_k + \beta_k$, α_k being the duration of the pulse of amplitude 1 and β_k the duration of the other pulse of amplitude -1. For $x(t)$, $t_k \leq t \leq t_{k+2}$, the duty-cycle is defined as

$$0 < \frac{\alpha_k}{T_k} = \frac{1 + x(t)}{2} < 1 \qquad t_k \leq t \leq t_{k+2} \tag{1}$$

Thus, if the signal is zero for all times, $x(t) = 0$, $-\infty < t < \infty$, it is modulated into a train of square pulses with

$\alpha_k = \beta_k$ or with duty cycle of $\alpha_k/T_k = 0.5$. If $x(t) = A$, $|A| < 1$, $t_k \leq t \leq t_{k+2}$, then the two pulses for that time are rectangular where

$$\alpha_k = \frac{(1+A)T_k}{2}$$

and $\beta_k = T_k - \alpha_k$. If the signal is not constant in a time segment, the duty-cycle is not clearly defined with respect to the amplitude.

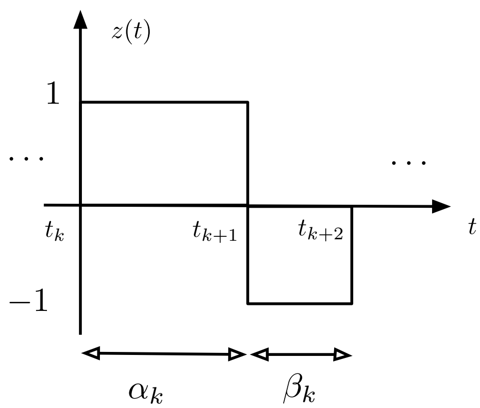

Figure 2: Duty-cycle modulation.

Suppose $\bar{x}_k$ is the local average of the signal in $t_k \leq t \leq t_{k+2}$, from the above we can see that it can be obtained from the α_k and β_k in the duty cycle modulation, i.e.,

$$\begin{aligned} \bar{x}_k &= \frac{\alpha_k - \beta_k}{\alpha_k + \beta_k} \\ \alpha_k &= t_{k+1} - t_k, \quad \beta_k = t_{k+2} - t_{k-1} \end{aligned} \quad (2)$$

Time encoding has been proposed in [9] for representation of a bandlimited signal using ASDMs. The relationship between the binary output $z(t)$ and the input $x(t)$ of the ASDM for $t_{k+1} > t_k$, and integers $k \geq 0$, is given by the integral equation

$$\int_{t_k}^{t_{k+1}} x(u)du = (-1)^k[-b(t_{k+1} - t_k) + 2\kappa\delta] \quad (3)$$

Thus from the duty-cycle modulation, or the output $z(t)$ of the ASDM, giving the time-sequence $\{t_k\}$ we are only able to recover local averages in each segment. If the signal $x(t)$ is continuous in $t_k \leq t \leq t_{k+2}$, the local average coincides with one of the values $x(\zeta)$ for $t_k \leq \zeta \leq t_{k+2}$ and one could think then of a non-uniformly sampled signal for which we would like to interpolate the rest of the signal values in that segment. Thus, equation (3) provides a way to obtain that interpolation as we will see in the next section.

The train of rectangular pulses $z(t)$ displays non-uniform zero-crossing times that depend on the input signal amplitude. The reconstruction of the signal $x(t)$ can be done by approximating the integral in (3), by the trapezoidal rule. Using an increment $\Delta = (t_{k+1} - t_k)/D$ for an integer $D > 1$ (the larger this value the better the approximation), we have that

$$\int_{t_k}^{t_{k+1}} x(\tau)d\tau \approx \Delta\left[\frac{x(t_k)}{2} + \sum_{\ell=1}^{D-1} x(t_k + \ell\Delta) + \frac{x(t_{k+1})}{2}\right]$$

We then obtain the following reconstruction algorithm:

$$\begin{aligned} &(i) \quad \mathbf{v} = \mathbf{Q}\mathbf{x} = \mathbf{Q}\mathbf{P}\gamma \\ &(ii) \quad \gamma = [\mathbf{Q}\mathbf{P}]^{\dagger}\mathbf{v} \\ &(iii) \quad \mathbf{x} = \mathbf{P}\gamma \end{aligned}$$

where $\mathbf{v}$ is the right term in (3), $\mathbf{Q}$ is the matrix for the trapezoidal approximation, $\mathbf{P}$ is either a matrix with sinc functions or PSW functions [9, 15]. The symbol † represents pseudo-inverse. Thus the signal $x(t)$ can be reconstructed from the zero crossings $\{t_k\}$ of the output of the ASDM $z(t)$. The accuracy of the reconstruction however depends on the approximation of the integral, and on the signal being band-limited. Our approach strives to avoid these two constrains.

2.2 Calculation of Local Averages

Assuming the output $x(t)$ of the ASDM is bounded as $|x(t)| \leq c < b$, the output of the integrator at time $t_{k+1} > t_k$ is

$$y(t_{k+1}) = y(t_k) + \frac{1}{\kappa}\int_{t_k}^{t_{k+1}} [x(u) - z(u)]du.$$

If the Schmitt trigger is in the state $(-b, -\delta)$ at $t = t_k$, ($y(t_k) = -\delta$ and $z(t_k) = -b$) , at some time $t_{k+1} > t_k$ we have that

$$\begin{aligned} \delta &= -\delta + \frac{1}{\kappa}\int_{t_k}^{t_{k+1}} [x(u) + b]du \\ &= -\delta + \frac{1}{\kappa}\int_{t_k}^{t_{k+1}} x(u)du + b(t_{k+1} - t_k) \end{aligned}$$

Right after t_{k+1}, the trigger switches to a (b, δ) state so that for some time $t_{k+2} > t_{k+1}$

$$\begin{aligned} -\delta &= \delta + \frac{1}{\kappa}\int_{t_{k+1}}^{t_{k+2}} [x(u) + b]du \\ &= \delta + \frac{1}{\kappa}\int_{t_{k+1}}^{t_{k+2}} x(u)du - b(t_{k+2} - t_{k+1}) \end{aligned}$$

which when added gives

$$\int_{t_k}^{t_{k+2}} x(\tau)d\tau = t_{k+2} - 2t_{k+1} + t_k = \alpha_k - \beta_k \quad (4)$$

where α_k and β_k are defined as above. To obtain the local average we need $T_k = \alpha_k + \beta_k$, which are obtained from the derivative of the binary signal $z(t)$,

$$\frac{dz(t)}{dt} = \sum_k 2(-1)^k\delta(t - t_k) \quad (5)$$

which in practice can be obtained using a time-to-digital converter [12].

The value chosen for κ is of great significance in the computation of the local averages. If the value of κ is chosen appropriately, the difference of the output of the integrator at times t_k and t_{k+1} is

$$y(t_{k+1}) - y(t_k) = 2\delta = \frac{1}{\kappa}\int_{t_k}^{t_{k+1}} [x(u) - z(u)]du$$

If we let $\delta = 0.5$,

$$\kappa = \int_{t_k}^{t_{k+1}} x(u)du + \alpha_k$$

and since $|x(t)| < c \leq 1$ we obtain the following upper bound for $\kappa > 0$

$$\kappa \leq \left| \int_{t_k}^{t_{k+1}} x(u)du \right| + \alpha_k \leq \alpha_k(c+1)$$

indicating that it depends on the local variation of the signal amplitude or local frequency.

If $x(t)$ is bandlimited, its reconstruction from the time sequence $\{t_k, \ k = 0, 1, \cdots\}$ is possible if [9]:

$$\max_k(\alpha_k) = \max_k(t_{k+1} - t_k) \leq T_N \tag{6}$$

where $T_N = \pi/\Omega_{max}$ is the Nyquist sampling period. Thus for bandlimited signals, we have

$$\kappa \leq (1 + c)T_N \tag{7}$$

Figure 3 shows the operation of ASDM on an arbitrary signal.

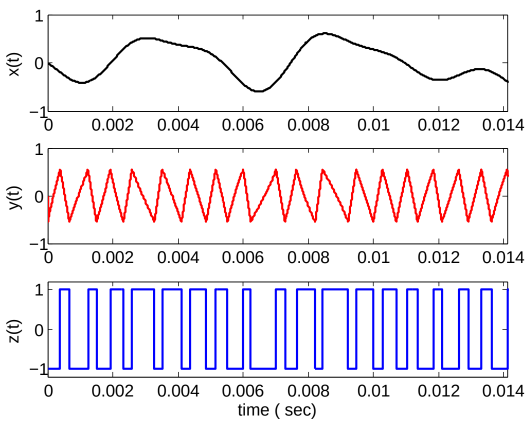

Figure 3: Operation of ASDM

3. ADAPTIVE LEVEL-CROSSING SAMPLING

Level-crossing (LC) sampling is threshold based: the signal $x(t)$ is compared with a set of reference levels and only when the signal exceeds one of the reference levels the sample is taken. Thus, the signal determines when samples are taken and what the quantization level is, i.e., it is non-uniform sampling. For a bursty signal more samples are taken during the burst and fewer otherwise. For a smooth signal the samples are randomly but uniformly distributed. The reconstruction of the signal depends on this distribution even in the case of bandlimited signals [17].

A piecewise constant reconstruction of $x(t)$, is obtained for a set of reference levels $\{q_k\}$ and non-uniform sample times $\{\zeta_k\}$ as [3]:

$$\hat{x}(t) = \sum_k q_k[u(t - \zeta_k) - u(t - \zeta_{k+1})] \tag{8}$$

where $u(t)$ is the unit-step function. Assume we wish to minimize a mean-square error

$$\varepsilon = E[x(t) - \hat{x}(t)]^2 \tag{9}$$

by choosing the levels $\{q_k\}$. It is not possible to obtain optimal levels without statistical knowledge of the signal and choosing uniform levels does not provide the optimal solution, either. In [3], the authors propose a sequential algorithm to obtain optimal levels.

To minimize the mean-square error ε with respect to the levels $\{q_k\}$, with no additional information that the local averages provided by the ASDM, the local average $\bar{x}_k$ in $[t_k, t_{k+2}]$ constitutes an optimal estimate of the signal in the interval. This is so since no second-order statistics is available. These local averages are completely determined by the time-codes $\{t_k, \ k = 0, 1, \cdots\}$.

To insure that the reconstruction is possible, we assume the signals are not only time-limited, but also essentially band-limited or that most of its energy is within a certain frequency band [15]. The essential bandwidth can be used to determine the value of κ for the ASDM.

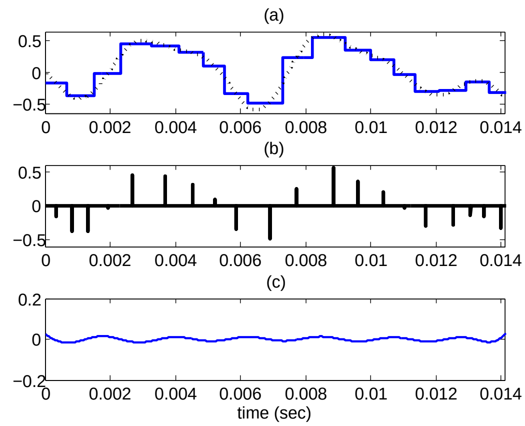

Figure 4: Adaptive LC sampling ((a) and (b)) and reconstruction error (c) of a smooth signal.

4. SIGNAL RECONSTRUCTION USING PSWF

The PSWFs, $\{\varphi_n(t)\}$, are real-valued functions with finite time support that maximize their energy in a given bandwidth [13]. These functions provide an interpolation of a continuous signal similar to the sinc interpolation in the sampling theory [14, 15]. Indeed, the sinc function $S(t)$ can be expanded in terms of the basis $\{\varphi_n(t)\}$, with T_s as the Nyquist period, as

$$S(t - kT_s) = \sum_{m=0}^{\infty} \varphi_m(kT_s)\varphi_m(t) \tag{10}$$

allowing us to write the sinc interpolation of a band-limited signal $x(t)$ as

$$\begin{aligned} x(t) &= \sum_{m=0}^{\infty} \left[\sum_k x(kT_s)\varphi_m(kT_s) \right] \varphi_m(t) \\ &= \sum_{m=0}^{\infty} \gamma_m \varphi_m(t) \end{aligned} \tag{11}$$

which is an infinite dimensional interpolation of the continuous signal in terms of PSWFs [14]. The finite reconstruction for the above at uniform sampling times t_k is:

$$\hat{x}(t_k) = \sum_{m=0}^{M-1} \gamma_{M,m}\varphi_m(t_k) \tag{12}$$

where the coefficients are

$$\gamma_{M,m} = \sum_{k=0}^{N-1} x(kT_s)\varphi_m(kT_s)$$

and the value of M is obtained when eigenvalues $\{\lambda_n\}$ associated with the length N PSWFs are approximately zero for $n \geq M$.

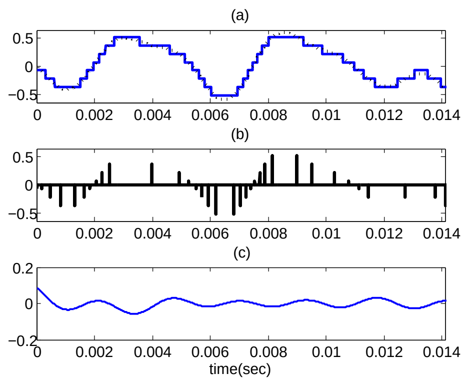

Figure 5: Uniform LC sampling ((a) and (b)) and reconstruction error (c) of a smooth signal.

The matrix form of the projected signal at the uniform times $\{t_k = kT_s\}$ is

$$\hat{\mathbf{x}}(\mathbf{t_k}) = \mathbf{\Phi}(\mathbf{t_k})\gamma_{\mathbf{M}} \tag{13}$$

where $\hat{\mathbf{x}}(\mathbf{t_k})$, $0 \leq k \leq N_n - 1$ is a length N_n vector containing samples $\{x(kT_s)\}$, $\gamma_{\mathbf{M}}$ is the vector formed by the coefficients of the projection and the matrix $\mathbf{\Phi}(\mathbf{t_k})$ is composed of PSWFs. The LC-sampling using the local means $\{\bar{x}_k\}$ gives a set of non-uniform times $t_k \leq \zeta_k \leq t_{k+2}$ (which we assume is a subset of a uniform sample values) where the continuous signal equals the local mean, i.e., $x(\zeta_k) = \bar{x}_k$ Thus, the couples

$$\{x(\zeta_k),\ \zeta_k\} \qquad k = 0, 1, ..., N_\ell - 1 \tag{14}$$

provide the necessary data for the PSWF interpolation:

$$\mathbf{x}(\zeta_{\mathbf{k}}) = \mathbf{\Phi}(\zeta_{\mathbf{k}})\gamma_{\mathbf{M}} \tag{15}$$

Since the values $\{\zeta_k\}$ occur within $[t_k\ \ t_{k+2}]$ in a non-deterministic way, the matrix $\mathbf{\Phi}(\zeta_{\mathbf{k}})$, of dimension $M \times N_\ell$, can be considered non–deterministic. Using its pseudo-inverse we obtain the coefficients

$$\gamma_{\mathbf{M}} = [\mathbf{\Phi}(\zeta_{\mathbf{k}})]^\dagger \, \mathbf{x}(\zeta_{\mathbf{k}}),$$

which can be used to obtain the reconstructed signal

$$\begin{aligned} \mathbf{x_r}(\mathbf{t}) &= \mathbf{\Phi}(\mathbf{t})\,[\mathbf{\Phi}(\zeta_{\mathbf{k}})]^\dagger\,\mathbf{x}(\zeta_{\mathbf{k}}) \\ &= \mathbf{\Theta}\ \mathbf{x}(\zeta_{\mathbf{k}}) \end{aligned} \tag{16}$$

Considering the $\{x(\zeta_k)\}$ the measurements, the reconstructed signal resembles the results obtained in compressive sensing.

Figure 4 shows adaptive LC sampling together with reconstruction error for a smooth signal, while Fig. 5 shows the results using uniform LC sampling with 8 levels and the accompanying normalized reconstruction error. We chose 8 uniform levels, because we thought it would be a fair comparison in terms of the step size of the levels. Although the uniform LC has 40 samples compared to 21 samples of the adaptive LC, the normalized reconstruction error for the uniform LC is 40×10^{-4} compared to 7×10^{-4} of the adaptive LC sampling. It is possible to obtain a lower reconstruction error for the uniform LC sampling but that would require many more samples. Notice the different distribution of the crossing times, in the adaptive case they are more evenly distributed.

5. SIMULATIONS

The significance of our procedure is highlighted when sampling and reconstructing bursty signals. In the simulations we used a bursty signal which is from the wavelet decomposition of an EEG signal with a sampling period of 5×10^{-3} provided by [16]. We used 0.64 seconds of that signal which would require 130 samples in uniform sampling case. We converted the signal into continuous form by using sinc interpolation. An ASDM is then used to find the values of the local means. For the uniform sampling the dynamic range is divided into equal levels. As shown in Figs. 6 and 8, the quantization is finer with the uniform levels compared to the adaptive one, but the distribution of the samples is more uniform for the adaptive case. There are certain intervals where the uniform sampling does not have any samples and as indicated in Fig. 9, it is in those segments where the reconstruction is the worst. Despite the fewer samples in the adaptive sampling, the reconstruction is almost perfect as in Fig.7. In fact, we found it better than using the trapezoidal approximation of the integral resulting from the ASDM processing, shown in Fig. 10.

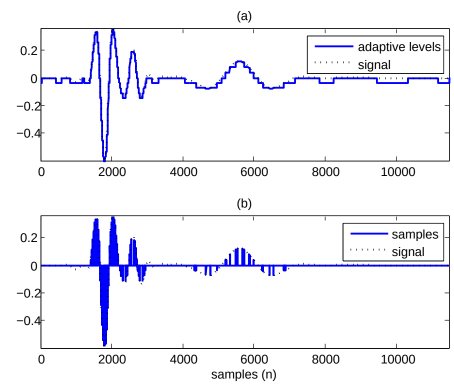

Figure 6: Adaptive LC of a bursty signal: (a) quantization with local means, (b) times of local averages.

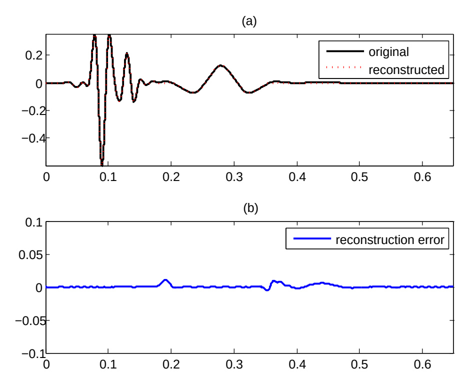

Figure 7: (a) PSWF reconstruction from local levels and their times, (b) reconstruction error.

6. CONCLUSIONS

We propose a novel approach for the sampling and reconstruction of signals using level crossing. In particular, our method is shown to perform well for bursty signals, commonly found in biomedical applications and sensor network transmission. Using local mean values as the levels for LC sampling, the signal dictates the rate of data collection. The

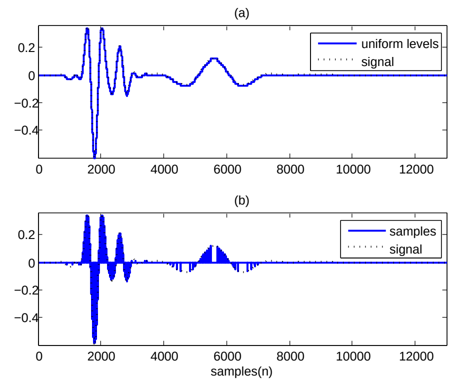

Figure 8: Uniform LC of a bursty signal: (a) quantization with uniform levels, (b) times of uniform levels.

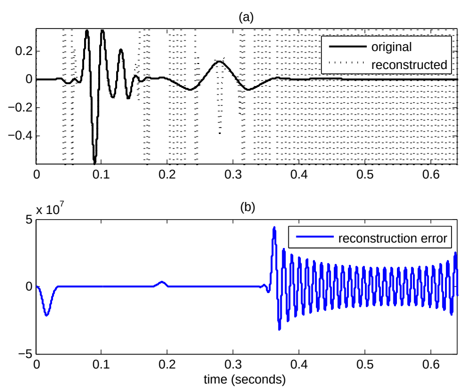

Figure 9: (a) PSWF reconstruction from uniform levels and their times, (b) reconstruction error.

reconstruction is done using PSW functions under assumptions of finite time support and essentially band-limited for the signals. The simulations indicate it is the distribution of the missing samples that determines the performance of the proposed method.

The advantage of our method over conventional uniform sampling is in demonstrated in applications where signals do not satisfy the band–limited conditions, or where high clock frequencies could cause physical complications. Using ASDM and level-crossing sampling devices provide a low-power performance and do not cause frequency aliasing. Further study is needed to establish the frequency performance of the proposed method.

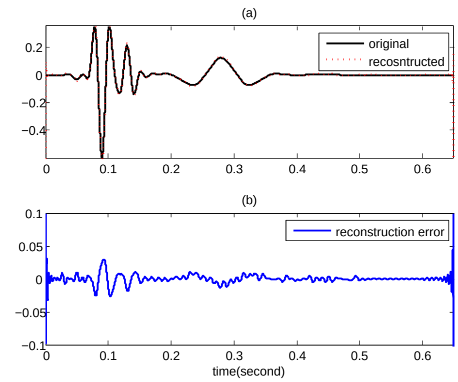

Figure 10: Time-decoding using trapezoidal approximation of the integral.

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