BALLISTIC MISSILE PRECESSING FREQUENCY EXTRACTION BASED ON MAXIMUM LIKELIHOOD ESTIMATION

Lihua Liu[1,2], Mounir Ghogho[2], Des McLernon[2] and Weidong Hu[1]

[1]ATR Key Lab, College of Electronic Science and Engineering, National University of Defence Technology, Yanwachi Street 47th, 410073, Changsha, P.R. China.
[2]School of Electronic and Electrical Engineering, The University of Leeds, UK, LS2 9JT, Leeds, UK.
Phone: +44(0)7818402257, fax: +44(0)1133432032, email: eenlliu@leeds.ac.uk.

ABSTRACT

We establish the dynamic Radar Cross Section (RCS) signal model for a conical ballistic missile warhead with precession motion. Two Maximum Likelihood Estimation (MLE) approaches are presented for the estimation of the important missile precession frequency. In one method we approximate a log-normal multiplicative process by a Gaussian process. In the second method we assume zero additive noise. While the approximations in both methods are introduced to make the mathematics tractable, simulations show the practical usefulness of both approaches.

1. INTRODUCTION

Interception of separating ballistic missiles is particularly difficult because the warhead is a distinct object that needs to be discriminated from the nearby objects such as the booster, the attitude control module, and the debris [1], all of which are separated in mid-course flight.

Since many warheads are spin-stabilized, they will precess due to the separation disturbance, and will keep the precession motion until they re-enter the atmosphere [2, 3]. Precession motion, which is a kind of micro-Doppler motion [4], will impose a micro-Doppler modulation effect on the radar echoes, and this is a unique feature of the ballistic targets. The precession frequency is an important feature parameter in ballistic target recognition, and it can reflect kinematical characteristics as well as structural and mass distribution features.

Due to the precession motion, the radar aspect angle varies periodically. Since the RCS return signal fluctuates as a function of radar aspect angle, the precession period can be extracted by analyzing the RCS signal. The static RCS of a warhead can be predicted by approximate methods. However, due to the wide variability of RCS scintillation sources, the RCS signal is modelled statistically as a random process. Evidence from the analysis of RCS measurements has shown that the RCS distributions of ballistic targets are log-normal [5]. So taking the receiver noise into account, the signal model for the RCS signal for a ballistic missile should be in the form of the product of the deterministic signal with log-normal multiplicative noise and Gaussian additive noise. While the estimation of a deterministic signal observed in additive white Gaussian noise is a well-researched problem, not much attention has been given to the corresponding multiplicative noise problem [7, 8].

In order to estimate the parameter of precession frequency from the RCS signal, we will propose in this paper two different approaches based upon maximum likelihood estimation. Both approaches will include some simplification of the RCS signal model in order to keep the mathematics tractable.

So the structure of the paper is as follows. In Section 2, we analyze the variation of the radar aspect angle when the warhead is precessing, and then establish the model for the RCS signal of a conical warhead. Then two methods of Gaussian Maximum Likelihood (GML) estimation and Maximum Likelihood Estimation (ML∞) with infinity signal-to-noise-ratio (SNR) will be proposed in Section 3. Simulation results are presented in Section 4 and concluding remarks are given in Section 5.

2. SIGNAL MODE

Most radar systems use the RCS signal as a means of missile discrimination and so an accurate prediction of target RCS is critical in order to design and develop robust discrimination algorithms. Exact methods of RCS prediction are very complex, even for simple shaped objects. Due to the difficulties associated with exact RCS prediction, approximate methods have become the only viable alternative.

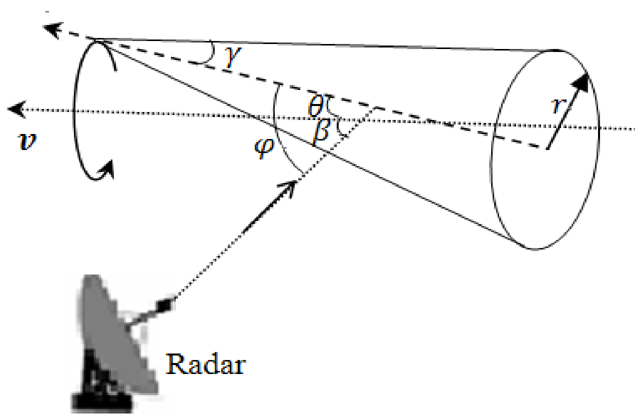

Fig. 1 Geometric model of a precessing conical warhead with velocity $\boldsymbol{v}$ m/s.

Most approximate methods can predict the RCS within few dBs of the true value and such an error is usually deemed acceptable.

Now, a conical tip is a commonly seen feature in many ballistic missiles. The RCS signal from a cone can be described as [9]

$$\sigma(\varphi) = \begin{cases} \dfrac{\lambda L \tan^2\gamma}{8c\sin\varphi}\tan^2(\varphi+\gamma), & \varphi \in (0,\pi), \varphi \neq \pi/2-\gamma \\ \dfrac{8\pi L^3}{9\lambda}\dfrac{\tan^4\gamma}{\cos^3\gamma}, & \varphi = \pi/2-\gamma \end{cases} \quad (1)$$

where φ is the radar aspect angle, λ is the wavelength, c is the speed of light, L is the length of the warhead, γ is the half cone angle of the conical warhead, ($\gamma = \tan^{-1}(r/L)$), and r is the bottom radius of the conical warhead (see Fig. 1).

When missile warheads are released, they usually spin in order to keep their orientation [10]. It is known in geostatic theory that a spinning rigid body will precess if there is latitudinal disturbance. Generally, this disturbance is unavoidable during missile release. Therefore, missile warheads will keep precessing until re-entering the atmosphere. Fig.1 illustrates the precession motion model of a conical warhead. The warhead spins around its geometrical axis and precesses along the direction of velocity $\boldsymbol{v}$ (see Fig.1).

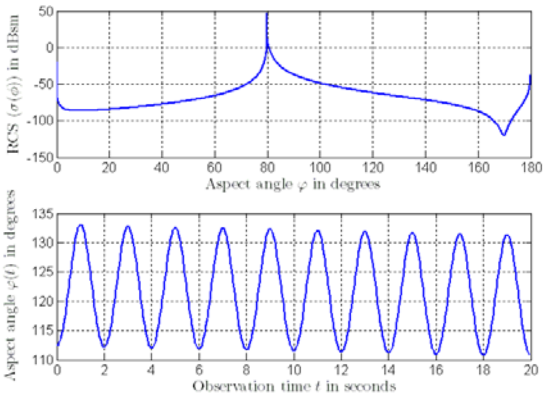

Fig. 2 RCS ($\sigma(\varphi)$) of a conical warhead versus aspect angle φ. (Note that "dBsm" refers to "dB relative to a square meter" and is commonly used for RCS signal precessing)

According to the geometry and the precession model of a rigid body object, as illustrated in Fig.1, the relationship between the aspect angle φ, the precession angle θ, the precession frequency f_p and the observation time t can be expressed by

$$\varphi(t) = \cos^{-1}\left\{\frac{\sin\theta(t)\sin\beta(t)\cos[2\pi f_p(t_0+t)+\phi_0]}{\cos^2\theta(t)} + \cos\theta(t)\cos\beta(t)\frac{\sin^3\theta(t)\sin\beta(t)}{\cos^2\theta(t)}\right\} \quad (2)$$

where ϕ_0 is the initial reference angle, t_0 is the initial reference time and β is the angle between the radar line sight (LOS) and the vector direction of the warhead velocity, $\boldsymbol{v}$.

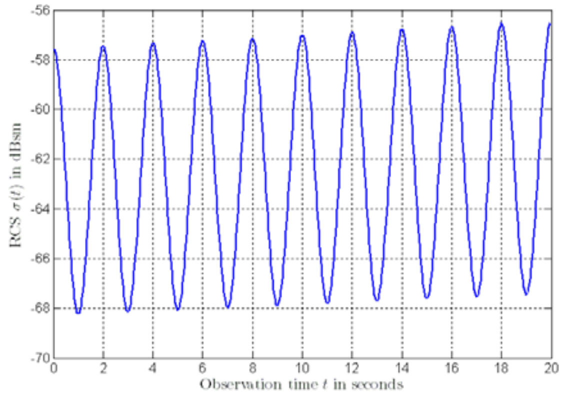

Fig. 3 Theoretical RCS signal $\sigma(t)$ versus observation time t.

It can be seen from (2) and Fig. 2 that the aspect angle $\varphi(t)$ is pseudo-periodic and the period is determined by the precession frequency f_p. If we can compensate the time-variation of parameters $\theta(t)$ and $\beta(t)$, the period of the aspect angle $\varphi(t)$ will be the same as the precession riod $T_p = 1/f_p$. In fact, compared with the aspect angle $\varphi(t)$, $\theta(t)$ and $\beta(t)$ change very slowly. So it is not complicated to compensate for the time-variation of the parameters $\theta(t)$ and $\beta(t)$ and this compensation need not be discussed in this paper.

So here we may treat the parameters $\theta(t)$ and $\beta(t)$ as constant over the observation time, and substituting (2) into (1) we can get the RCS signal versus time (i.e., $\sigma(t)$). As shown in Fig. 3, which is the plot of theoretical RCS signal ($\sigma(t)$) of a conical warhead, there is pseudo-periodicity in $\sigma(t)$, where the precession frequency is set as $f_p = 0.5$Hz..

In most practical radar systems there is relative motion between the radar and an observed target. Therefore, the RCS signal measured by the radar over a period of time fluctuates not only as a function of frequency and the target aspect angle, but also in amplitude and/or in phase. Phase fluctuation is called "glint", while amplitude fluctuation is called "scintillation" [10]. For most radar applications, glint introduces linear errors in the radar measurements and thus it is not a major concern. RCS scintillation is quite complicated and it cannot be ignored in radar measurements. It can vary slowly or rapidly depending upon the target size, shape, dynamics, and its relative motion with respect to the radar. Many of the RCS scintillation models were developed and verified by experimental measurements. Swerling [5, 6] points out that some experimental analysis conducted on RCS measurements of ships and missiles show that the fluctuation of these target types is often well modelled as a log-normal random variable.

So taking the scintillation effect and receiver noise into account, the RCS sequence model can be written as

$$\hat{\sigma}(t) = \sigma(t)\cdot g(t) + v(t). \quad (3)$$

Here we treat the scintillation effect as multiplicative log-normal $\left(g(t) \sim L(u_0, \sigma_0^2)\right)$ and where $v(t)$ is white Gaussian additive noise $\left(v(t) \sim N(u_v, \sigma_v^2)\right)$. The probability density function (pdf) of $g(t)$ is

$$f_g(\rho) = \frac{1}{\sigma_0 \sqrt{2\pi}} \frac{1}{\rho} \exp\left(-\frac{1}{2\sigma_0^2} [\ln(\rho) - u_0]^2\right) \quad (4)$$

where u_0 and σ_0 are the mean and standard deviation of the natural logarithm of g. The expected value and variance are

$$u_g = E[g] = e^{u_0 + \frac{1}{2}\sigma_0^2},$$

$$\sigma_g^2 = Var[g] = (e^{\sigma_0^2} - 1)(e^{u_0 + \frac{1}{2}\sigma_0^2})^2. \quad (5)$$

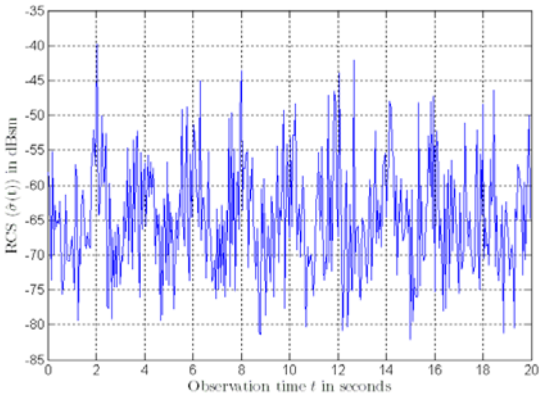

Fig. 4 RCS signal from a conical warhead with scintillation

As shown in Fig. 4, a sampled sequence of the RCS signal is quite random in appearance. In order to analyze the performance of the estimation methods, the RCS quence $\sigma(t)$ is modelled by curve fitting in the discrete-time domain as follows,

$$s(n) = a + bn + (c + dn)\cos\left(2\pi \frac{f_p}{f_s} n + \vartheta\right)$$
$$n = 0,1,2,\cdots,N-1 \quad (6)$$

where f_s is the sampling frequency, the parameters a, b, c, d and ϑ are all deterministic constants, f_p is the precession frequency, and N represents the total number of samples taken. Note that for simplicity, we write $s(n)$ instead of $s(nT_s)$, where $T_s = 1/f_s$. Thus the discrete-time observed signal model can be expressed by:

$$x(n) = s(n)g(n) + v(n), n = 0,1,\ldots,N-1 \quad (7)$$

with the following assumptions:

AS1): $g(n)$ is a real stationary, log-normal, stochastic process with mean $u_g > 0$ and variance σ_g^2;

AS2): $v(n)$ is a real stationary, white, Gaussian process with mean u_v and variance σ_v^2;

AS3): $g(n)$ and $v(n)$ are mutually independent, where $g(n)$ is a multiplicative process with a log-normal distribution, $g(n) \sim L(u_0, \sigma_0^2)$, $v(n) \sim N(u_v, \sigma_v^2)$, and where (without loss of generality) $u_v = 0$.

3. PARAMETER ESTIMATION

MLE is a popular approach in estimation theory [10]. However, if we want to estimate the precession frequency f_p via MLE then the pdf of the observed signal ($x(n)$ in (7)) must be derived. The pdf of $x(n)$ is the convolution of the pdfs of $g(n)$ and $v(n)$, which are log-normal and Gaussian distributed respectively. Even if both $g(n)$ and $v(n)$ have the same pdf, it may be hard to obtain an analytic expression for the pdf of $x(n)$ except in special cases such as Gaussianity [7].

So addressing this point, we propose two Maximum Likelihood estimators for the parameter f_p. One considers the lognormal multiplicative noise $g(n)$ as an approximately Gaussian distribution and in the other simply ignores the noise term $v(n)$. With these two approximation assumptions we can now derive the pdf of $x(n)$.

3.1 Gaussian Maximum Likelihood (GML)

Although the multiplicative noise $g(n)$ is a log-normal distributed, let us assume it is Gaussian. Let $\boldsymbol{X} = [x(0), \cdots, x(N-1)]^T$ and then the log-likelihood function of the process $\boldsymbol{X}$ can be written as

$$\ln f_x(\boldsymbol{X} | f_p, u_g, \sigma_g^2) = -\frac{1}{2}\sum_{n=0}^{N-1}[\ln \sigma_g^2 s^2(n) + \sigma_v^2] -$$
$$\frac{1}{2}\sum_{n=0}^{N-1}\frac{\left(x(n) - u_g s(n)\right)^2}{\sigma_g^2 s(n)^2 + \sigma_v^2} - \frac{N}{2}\ln(2\pi). \quad (8)$$

The estimators of the unknown parameters are:

$$\left(\widehat{f_p}, \widehat{u_g}, \widehat{\sigma_g^2}\right) = \underset{f_p, u_g, \sigma_g^2}{\operatorname{argmax}}\{\ln f_x(\boldsymbol{X} | f_p, u_g, \sigma_g^2)\}. \quad (9)$$

If parameters $\widehat{u_g}$ and $\widehat{\sigma_g^2}$ are known, we have to maximize the function with respect to just one unknown. If not, $\widehat{u_g}$ can be obtained as

$$\widehat{u_g} = \frac{\sum_{n=0}^{N-1} \frac{x(n)s(n)}{\sigma_g^2 s(n)^2 + \sigma_v^2}}{\sum_{n=0}^{N-1} \frac{s(n)^2}{\sigma_g^2 s(n)^2 + \sigma_v^2}}. \quad (10)$$

And so replacing u_g in (8) gives

$$\ln\ f_x(\boldsymbol{X} | f_p, \sigma_g^2) = -\frac{1}{2}\sum_{n=0}^{N-1}\ln\left(\sigma_g^2 s^2(n) + \sigma_v^2\right) -$$

$$\frac{1}{2}\sum_{n=0}^{N-1}\frac{\left(x(n)-\widehat{u_g}s(n)\right)^2}{\sigma_g^2 s(n)^2+\sigma_v^2}-\frac{N}{2}\ln 2\pi \quad (11)$$

and $\widehat{f_p}$ can be obtained by

$$\left(\widehat{f_p},\widehat{\sigma_g^2}\right)=\underset{f_p,\sigma_g^2}{\operatorname{argmax}}\left\{\ln f_x\left(\boldsymbol{X}\middle|f_p,\sigma_g^2\right)\right\}. \quad (12)$$

3.2 Maximum Likelihood with Infinity SNR (ML∞)

Before the derivation of the ML∞ estimator the signal-to-noise-ratio (SNR) should first be defined. The multiplicative noise $g(n)$ is deemed as a part of the signal in (7) and so let

$$y(n)=s(n)g(n), n=0,1,\dots N-1. \quad (13)$$

Now $y(n)$ is from a non-stationary, log-normal process with time-varying mean and variance:

$$u_y(n)=s(n)u_g=s(n)e^{u_0+\frac{1}{2}\sigma_0^2},$$

$$\sigma_y^2(n)=\mathrm{s}^2(n)\sigma_g^2=\mathrm{s}^2(n)(e^{\sigma_0^2}-1)\left(e^{u_0+\frac{1}{2}\sigma_0^2}\right)^2. \quad (14)$$

The SNR is then defined as

$$\mathrm{SNR}=10\log_{10}\left\{\frac{\frac{1}{N}\sum_{n=0}^{N-1}\{\sigma_y^2(n)+u_y^2(n)\}}{\sigma_v^2}\right\}. \quad (15)$$

If we assume $\sigma_v^2=0$, which means that $x(n)$ in (7) is pu rely from a multiplicative process, then the SNR defined in (15) is infinity. With this premise, we can get easily obtain th e pdf of $x(n)$, and then develop the ML estimation for the pa rameter f_p. We will refer to this as "ML∞ Estimation".

So with the infinity SNR, the signal model of the pdf of $y(n)$ is given by

$$f_y(\rho)=\frac{1}{\sqrt{2\pi}\sigma_0}\frac{1}{\rho}\exp\left\{-\frac{[\ln\rho-\ln s(n)-u_0]^2}{2\sigma_0^2}\right\} \quad (16)$$

where once again u_0 and σ_0 are respectively the mean and standard deviation of the natural logarithm of $g(n)$, and they can be derived from the mean and variance of $y(n)$ is (14). Thus

$$u_0=\ln\left(\frac{u_y^2(n)}{\sqrt{\sigma_y^2(n)+u_y^2(n)}}\right)$$

$$\sigma_0^2=\ln\left(\frac{\sigma_y^2(n)}{u_y^2(n)}+1\right). \quad (17)$$

And now we have

$$\ln f_y\left(\boldsymbol{X}\middle|f_p,u_0,\sigma_0^2\right)=-\frac{1}{2}\sum_{n=0}^{N-1}\Big[\frac{(\ln x(n)-\ln s(n)-u_0)^2}{\sigma_0^2}+\ln x^2(n)+\ln(\sigma_0^2)+\ln(2\pi)\Big]. \quad (18)$$

If u_0 and σ_0^2 are known, we can get easily estimate f_p. If not, then $\widehat{u_0}$ can be obtained as

$$\widehat{u_0}=\frac{1}{N}\sum_{n=0}^{N-1}[\ln x(n)-\ln s(n)] \quad (19)$$

and $\widehat{\sigma_0^2}$

$$\widehat{\sigma_0^2}=\frac{1}{N}\sum_{n=0}^{N-1}[\ln x(n)-\ln s(n)-\widehat{u_0}]^2. \quad (20)$$

Then, using $\widehat{u_0}$ and $\widehat{\sigma_0^2}$ instead of u_0 and σ_0^2 in (18), we get

$$\ln f_y\left(\boldsymbol{X}\middle|f_p\right)=-\frac{1}{2}\sum_{n=0}^{N-1}\Big[\frac{(\ln x(n)-\ln s(n)-\widehat{u_0})^2}{\widehat{\sigma_0^2}}+\ln x^2(n)+\ln\left(\widehat{\sigma_0^2}\right)+\ln(2\pi)\Big]. \quad (21)$$

Finally, $\widehat{f_p}$ can be obtained by

$$\widehat{f_p}=\underset{f_p}{\operatorname{argmax}}\left\{\ln f_y\left(\boldsymbol{X}\middle|f_p\right)\right\}. \quad (22)$$

4. SIMULATION AND EXPERIMENTAL RESULTS

In order to evaluate the performance of the two proposed methods, 300 independent Monte-Carlo trials were performed. The parameters were: (a) radar carrier quency $f_0=10\text{GHz}$; (b) sampling frequency $f_s=20\text{Hz}$; (c) bottom radius of warhead $r=0.329\text{m}$ (see Fig.1); (d) length of warhead $L=2.09\text{m}$.

The plots of the precession frequency estimation mean square error (MSE) versus SNR by the methods of GML and ML∞ are shown in Fig.5. We set the mean and variance of $g(n)$ as $u_g=1$ and $\sigma_g^2=0.4$. It can be seen from Fig. 5 that when the SNR is higher than 8dB, the performance of the two estimation methods are comparable. However the ML∞ approach is always superior.

Note from Fig.5 that there is an abrupt drop in the MSE curve for the GML method. This is because for low SNR, the GML does not accurately predict the MSE. This phenomenon is commonly known as the outlier or threshold effect. It is worth pointing out that ML∞ estimation exhibits a lower threshold value than GML estimation. Apparently forcing the Gaussianity assumption onto the multiplicative noise incurs a higher penalty than ignoring the additive noise. Further, ML∞ estimation not only provides better performance but also has a lower complexity. Indeed it only requires a one-dimensional search unlike the GML method which requires a two-dimensional search.

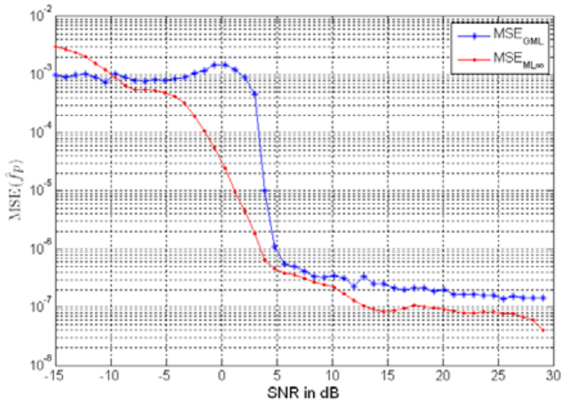

Fig.5 MSE of $\widehat{f_p}$ (versus SNR) by GML and ML∞ estimation methods.

As shown in Fig.6, if the observation duration is extended, the MSE performances (as expected) will improve. In Fig.6, we set the variance of the additive Gaussian noise as constant and let the SNR of the 20-seconds observation period be 6dB. However, in a ballistic target recognition system, the shorter the observation period the better. As we can see in Fig.6, if we set the MSE threshold approximately to 10^{-6}, then both approaches will achieve this for observation windows approximately 45 seconds or longer.

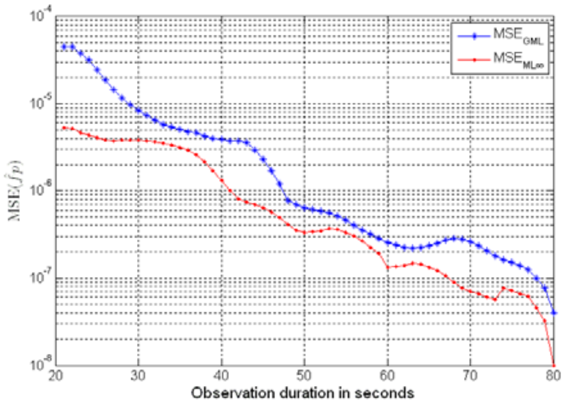

Fig. 6 MSE for $\widehat{f_p}$ versus observation duration for GML and ML∞ methods.

5. CONCLUSIONS

A ballistic missile will precess during flight, and this will cause periodicity in the RCS radar return signal. In order to extract the important precession frequency f_p, we established the model of the RCS signal from the conical warhead. The RCS signal is a deterministic signal multiplied by a log-normal process plus an additive Gaussian noise. We proposed two maximum likelihood estimators for f_p: GML and ML∞ estimation. Both of these two approaches made certain approximations about the signal model in order to make the mathematics tractable. However, even with these assumptions/approximations, both MLE methods perform well in Monte Carlo simulations, with ML∞ (i.e., assuming infinity SNR) outperforming GML (i.e., where we assumed that the log-normal multiplicative process was Gaussian).

REFERENCES

[1] Isaac Bankman, "Model of Laser Radar Signatures of Ballistic Missile Warheads," *SPIE Conference on Targets and Backgrounds: Characterization and Representation*, Orlando, Florida, April, 1999, pp.133-137

[2] K. Schultz, S. Davidson, A. Stein and J. Parker, "Range Doppler Laser Radar for Midcourse Discrimination: The Firefly Experiments," in *2nd Annual AIAA SDIO Interceptor Technology Conference*, Albuquerque, USA, Jun. 6-9. 1993, AIAA-93-2653.

[3] Chen Hang-yong, Guo Gui-rong and Jiang Bin, "Radar Feature Extraction of Micro-precession Ballistic Missile Warhead," *Journal of Electronics & Information Technology*, vol. 28, No. 4, pp. 643–646, Apr. 2006.

[4] Victor C. Chen, "Micro-Doppler Effect in Radar: Phenomenon, Model, and Simulation Study," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 42, No. 1, pp.2–21, Jan. 2006.

[5] Peter Swerling, "Radar Probability of Detection for Some Additional Fluctuating Target Cases," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 33, No. 2, pp. 698–709, Apr. 1997.

[6] A. De Maio, A. Farina and G. Foglia, " Target fluctuation Models and Their Application to Radar Performance Prediction," *IEE Proceeding Radar Sonar Navigation*, vol. 151, pp. 261–269, Oct. 2004.

[7] Ananthram Swami, "Cramer-Rao Bounds for Deterministic Signals in Additive and Multiplicative Noise," *Signal Processing*, vol. 53, pp. 231–244, Aug. 1996.

[8] Mounir Ghogho, Asoke K. Nandi and Ananthram Swami, "Cramer-Rao Bounds and Maximum Likelihood Estimation for Random Amplitude Phase-Modulated Signals," *IEEE Transactions on Signal Processing*, vol. 47 No. 112, pp. 2905–2915, Nov. 1999.

[9] B.R. Mahafza, *Radar Systems Analysis and Degign Using MATLAB*. Huntsville, Alabama: CRC Press, 2000.

[10] J. Boutelle, S. Kau and J. Marino, C.J. "ICBM Reentry Vehicle Navigation System Development at Honeywell," *Position Location and Navigation Symposium*, vol. 150, No. 4, pp. 294–302, Apr. 1998.

[11] Steven M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. New Jersey, Prentice Hall PTR, 1993.