

# TRANSCEIVER DESIGN FOR NON-REGENERATIVE MIMO RELAY SYSTEMS WITH DECISION FEEDBACK DETECTION

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## ABSTRACT

In this paper we consider the design of zero forcing (ZF) and minimum mean square error (MMSE) transceivers for non-regenerative multiple input multiple output (MIMO) relay networks. Our designs utilise linear processors at each stage of the network along with a decision feedback detection device at the receiver. Under the assumption of full channel state information (CSI) across the entire link the processors are jointly optimised to minimise the system arithmetic mean square error (MSE) whilst meeting average power constraints at both the source and the relay terminals. We compare the presented methods to linear designs available in the literature and show the advantages of the proposed transceivers through simulation results.

## 1. INTRODUCTION

MIMO relay networks have gained significant attention from researchers lately due to the fact that they can extend communication range and network coverage [1] as well as provide other benefits such as increased data throughput and link reliability [2]. Due to the various advantages offered by multi-antenna relay systems they are considered an integral component in the design of future generation wireless networks [2].

Relay networks are generally classed as either regenerative or non-regenerative depending on the functionality of the relay terminal [3]. These two classes are also commonly referred to as decode forward and amplify forward respectively. In the regenerative case the relay terminal decodes the received signal streams and then retransmits the regenerated symbols to the destination [3]. For non-regenerative relaying, which is the least complex of the two approaches, the relay antennas simply amplify their received signal before forwarding to the next relay.

Most works in the area of MIMO relaying have focussed on the design of linear transceivers to enhance system performance in some manner. The design of such transceivers is highly dependant on the availability of CSI at each stage of the network. In [1] the authors derive the optimal relay precoder that maximises the mutual information between the source and destination, under the assumption that the source distributes the available power budget uniformly across the antennas. This requires the relay to have access to full CSI of both transmission channels. In [3] the authors also focus on the relay precoder design to maximise mutual information but introduce linear equalisation at the destination. The introduction of the linear equaliser improves the system information rate compared to [1] but requires that the receiver also has full CSI of both channels.

As well as the maximisation of channel capacity other design criteria such as MMSE have been well studied. In [4] a co-operative relay strategy was designed to minimise the MSE subject to an average power constraint at the relay. This work showed that co-operation between the relay nodes could enhance bit error ratio (BER) performance compared to schemes that do not allow signal mixing at the relay terminal [5]. However, co-operation among relays is only possible when the nodes are clustered together with short range local links [4]. When the relay antennas cannot exchange information with each other reliably then non-cooperative

relay protocols provide a more practical solution. In [6] a joint MMSE optimisation strategy is suggested where the Wiener filter is utilised at the receiver and the relay precoder is designed to minimise the system arithmetic MSE subject to an average power constraint.

A unified framework for the design of linear transceivers in conventional two-hop non-regenerative MIMO relay networks, where it is assumed that there is no direct link between the source and destination, has been presented in [7]. This work presents solutions to the joint optimisation of the source, relay, and destination processors for a broad range of objective functions and is an extension of the results derived in [8] which were obtained for the case of point to point MIMO. Assuming Wiener filtering at the destination the authors derive the optimal source and relay precoders for Schur convex and Schur concave objective functions which cover most reasonable design criteria. This work was later generalised to the case of multi-hop MIMO relay networks in [9] where it was shown that, as in the case of a two-hop network, the optimal source and relay precoder structures diagonalise the MIMO relay system into a set of parallel single input single output (SISO) sub-channels.

In this paper we consider the joint design of linear processors for a non-regenerative two-hop relay system with decision feedback detection at the destination. As in many works e.g. [1], [4], [5], and [7] we assume that, due to high attenuation, the destination does not receive any information directly from the source. It is also assumed that the source, relay, and receiver have access to full CSI of both transmission channels. We aim to jointly design the processors in the network to minimise the system arithmetic MSE whilst abiding by power constraints at the source and relay stages. The joint design of linear processors for point to point MIMO with decision feedback detection was studied extensively in [10]. Our work can be viewed as an extension of that in [10] such that the transceiver designs are applicable to two-hop non-regenerative MIMO relaying.

The remainder of this paper is organised as follows: In Section 2 we introduce the signal model for the communications system under consideration. We then formulate a constrained optimisation problem for the transceiver designs in Section 3. The problem is formulated by first deriving an expression for the MSE and then obtaining and minimising a lower bound on the MSE subject to average power constraints. From this constrained optimisation problem both the ZF and MMSE processors are derived. In Section 4 the performance of the proposed designs are evaluated through simulations and compared to benchmark linear designs. Finally conclusions are drawn in Section 5.

*Notation:* In our notation vectors and matrices are denoted by lower and upper case bold font respectively. The sets of real and complex numbers are  $\mathbb{R}$  and  $\mathbb{C}$ , which in the case of vector/matrix quantities indicate dimensions by means of a superscript. The operators  $\mathcal{E}\{\cdot\}$ ,  $\text{tr}\{\cdot\}$ ,  $(\cdot)^H$ , and  $|\cdot|$  denote the expectation, trace, hermitian transpose, and determinant respectively. The operator  $[\cdot]^+$  signifies taking the maximum value of the term inside the bracket and zero.

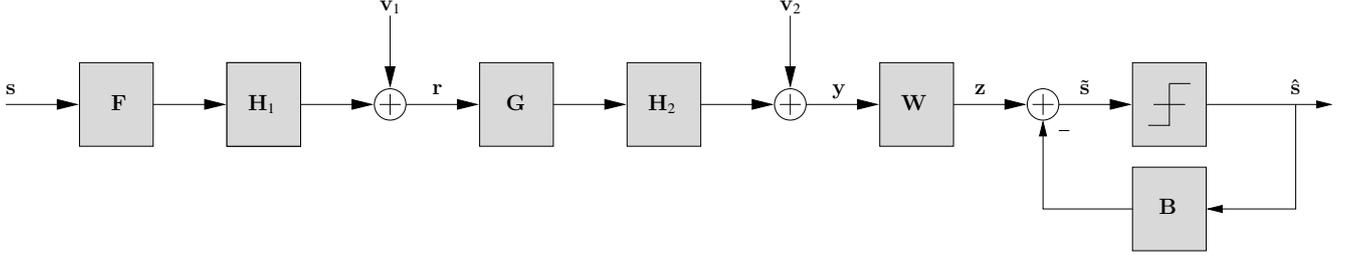


Figure 1: MIMO relay system model with decision feedback detection.

## 2. SIGNAL MODEL

In this section we develop the signal model for a conventional MIMO relay system with  $M$  antennas at both the source and destination, and  $N \geq M$  antennas in the relaying layer. For the purposes of interference cancellation we admit linear processors in each stage of the network and, furthermore, employ decision feedback detection at the destination. This configuration can be seen in Figure 1.

For the half duplex relaying system shown in Figure 1 the transmission of data between the source and destination is carried out in two separate time slots. In the first time slot the symbols from the source  $\mathbf{s} \in \mathbb{C}^M$ , which we assume are uncorrelated with covariance  $\mathbf{R}_{\mathbf{s}\mathbf{s}} = \mathcal{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}$ , are linearly precoded by the source matrix  $\mathbf{F} \in \mathbb{C}^{M \times M}$  and then transmitted over the first channel  $\mathbf{H}_1 \in \mathbb{C}^{N \times M}$  to the  $N$  relaying antennas. The data vector  $\mathbf{r} \in \mathbb{C}^N$  received by the relay layer can thus be written as

$$\mathbf{r} = \mathbf{H}_1 \mathbf{F} \mathbf{s} + \mathbf{v}_1, \quad (1)$$

where  $\mathbf{v}_1 \in \mathbb{C}^N$  is an Additive White Gaussian Noise (AWGN) vector with covariance  $\mathbf{R}_{\mathbf{v}_1\mathbf{v}_1} = \mathcal{E}\{\mathbf{v}_1\mathbf{v}_1^H\} = \sigma_{v_1}^2 \mathbf{I}$ . In the second time slot the relays precode the received data by the matrix  $\mathbf{G} \in \mathbb{C}^{N \times N}$  and transmit across the second stage channel  $\mathbf{H}_2 \in \mathbb{C}^{M \times N}$  giving the received signals at the destination

$$\mathbf{y} = \mathbf{H}_2 \mathbf{G} \mathbf{r} + \mathbf{v}_2, \quad (2)$$

where again the vector  $\mathbf{v}_2 \in \mathbb{C}^M$  contains AWGN samples and has covariance  $\mathbf{R}_{\mathbf{v}_2\mathbf{v}_2} = \mathcal{E}\{\mathbf{v}_2\mathbf{v}_2^H\} = \sigma_{v_2}^2 \mathbf{I}$ . The received data is processed by the linear equaliser  $\mathbf{W} \in \mathbb{C}^{M \times M}$ , resulting in  $\mathbf{z} = \mathbf{W} \mathbf{y}$ . Using (1) and (2) we can write the output of the equaliser as

$$\mathbf{z} = \mathbf{W} \mathbf{H} \mathbf{F} \mathbf{s} + \mathbf{W} \mathbf{v}, \quad (3)$$

where for convenience we define  $\mathbf{H} = \mathbf{H}_2 \mathbf{G} \mathbf{H}_1$  to be the compound MIMO channel between the source and destination antennas and  $\mathbf{v} = \mathbf{H}_2 \mathbf{G} \mathbf{v}_1 + \mathbf{v}_2$  to be the total noise at the input to the equaliser with covariance  $\mathbf{R}_{\mathbf{v}\mathbf{v}} = \mathcal{E}\{\mathbf{v}\mathbf{v}^H\} = \mathbf{H}_2 \mathbf{G} \mathbf{R}_{\mathbf{v}_1\mathbf{v}_1} \mathbf{G}^H \mathbf{H}_2^H + \mathbf{R}_{\mathbf{v}_2\mathbf{v}_2}$ . After processing by  $\mathbf{W}$ , successive interference cancellation (SIC) is performed. The feedback matrix  $\mathbf{B} \in \mathbb{C}^{M \times M}$  is strictly upper right triangular with co-efficients

$$\mathbf{B} = \begin{bmatrix} 0 & b_{12} & \cdots & b_{1M} \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & b_{(M-1)M} \\ 0 & \cdots & \cdots & 0 \end{bmatrix}. \quad (4)$$

The SIC performed at the receiver is then as follows [11]:

$$\hat{s}_m = C \left[ z_m - \sum_{n=m+1}^M b_{mn} \hat{s}_n \right], \quad m = M, M-1, \dots, 1, \quad (5)$$

where the operator  $C[\cdot]$  denotes quantisation to the nearest symbol in the transmitted symbol constellation. This operation is equivalent to successively making decisions [10] on

$$\tilde{\mathbf{s}} = \mathbf{W} \mathbf{H} \mathbf{F} \mathbf{s} + \mathbf{W} \mathbf{v} - \mathbf{B} \hat{\mathbf{s}}. \quad (6)$$

The error between the input to the decision device and the transmitted data vector provides a useful measure of quality for the decision feedback transceiver and is constructed as  $\mathbf{e} = \tilde{\mathbf{s}} - \mathbf{s}$ . Using (6) the error can be written as

$$\mathbf{e} = (\mathbf{W} \mathbf{H} \mathbf{F} - \mathbf{B} - \mathbf{I}) \mathbf{s} + \mathbf{W} \mathbf{v}, \quad (7)$$

where for mathematical tractability we have assumed correct decisions on the past data symbols in  $\tilde{\mathbf{s}}$  i.e.  $\hat{\mathbf{s}} = \mathbf{s}$ .

## 3. TRANSCEIVER DESIGN

The transceivers in this paper aim to minimise the system arithmetic MSE under average power constraints at the source and relay. From this optimisation problem the ZF and MMSE designs are obtained depending on whether or not the equaliser  $\mathbf{W}$  performs a regularised inversion. In the following sub-sections we begin by formulating the constrained optimisation problem before presenting the individual ZF and MMSE solutions.

### 3.1 MSE Problem Formulation

Using (7) the covariance of the error  $\mathbf{R}_{\mathbf{e}\mathbf{e}} = \mathcal{E}\{\mathbf{e}\mathbf{e}^H\}$  can be expanded as

$$\mathbf{R}_{\mathbf{e}\mathbf{e}} = (\mathbf{W} \mathbf{H} \mathbf{F} - \mathbf{B} - \mathbf{I})(\mathbf{W} \mathbf{H} \mathbf{F} - \mathbf{B} - \mathbf{I})^H + \mathbf{W} \mathbf{R}_{\mathbf{v}\mathbf{v}} \mathbf{W}^H. \quad (8)$$

The arithmetic MSE is simply given by the average of the diagonal elements in (8) and can be stated as

$$\bar{\sigma}^2 = \text{tr} \left\{ (\mathbf{W} \mathbf{H} \mathbf{F} - \mathbf{U})(\mathbf{W} \mathbf{H} \mathbf{F} - \mathbf{U})^H + \mathbf{W} \mathbf{R}_{\mathbf{v}\mathbf{v}} \mathbf{W}^H \right\} / M, \quad (9)$$

where for convenience we have used the substitution  $\mathbf{U} = \mathbf{B} + \mathbf{I}$ . Although the processors could be designed to minimise (9) we can obtain a lower bound on the MSE that will lead to transceivers with better performance.

#### 3.1.1 MSE Lower Bound

The geometric MSE provides a lower bound to (9) which is a simple consequence of the arithmetic geometric mean inequality [12] which, for an  $M \times M$  positive semi-definite matrix  $\mathbf{X}$ , states that

$$|\mathbf{X}|^{1/M} \leq \text{tr}\{\mathbf{X}\} / M, \quad (10)$$

which holds with equality when  $\mathbf{X} = \beta \mathbf{I}$  with  $\beta \geq 0$ . Using (8) and (10) we can obtain the following bounds on the MSE

$$\begin{aligned} & |(\mathbf{W} \mathbf{H} \mathbf{F} - \mathbf{U})(\mathbf{W} \mathbf{H} \mathbf{F} - \mathbf{U})^H + \mathbf{W} \mathbf{R}_{\mathbf{v}\mathbf{v}} \mathbf{W}^H|^{1/M} \\ & \leq \text{tr} \left\{ (\mathbf{W} \mathbf{H} \mathbf{F} - \mathbf{U})(\mathbf{W} \mathbf{H} \mathbf{F} - \mathbf{U})^H + \mathbf{W} \mathbf{R}_{\mathbf{v}\mathbf{v}} \mathbf{W}^H \right\} / M. \end{aligned} \quad (11)$$

In the following designs we shall use the lower bound in (11) as our objective function. However, the main goal of the transceivers is to minimise the arithmetic MSE which is given by the upper bound in (11). The transceivers may only achieve the lower bound on the arithmetic MSE if (11) holds with equality [10]. Thus,  $\mathbf{F}$ ,  $\mathbf{G}$ ,  $\mathbf{W}$ , and  $\mathbf{U}$  must be designed such that the error covariance matrix has the structure  $\mathbf{R}_{ee} = \beta \mathbf{I}$ .

### 3.1.2 Constrained Optimisation Problem

Using the lower bound in (11) as our objective function and with the source and relay terminals having limited average transmit power we can formulate the constrained optimisation problem as

$$\min_{\mathbf{F}, \mathbf{G}, \mathbf{W}, \mathbf{U}} |(\mathbf{W}\mathbf{H}\mathbf{F} - \mathbf{U})(\mathbf{W}\mathbf{H}\mathbf{F} - \mathbf{U})^H + \mathbf{W}\mathbf{R}_{vv}\mathbf{W}^H|^{1/M} \quad (12)$$

$$\text{subject to} \quad \text{tr}\{\mathbf{F}\mathbf{F}^H\} = P_s, \quad \text{and} \quad (13)$$

$$\text{tr}\{\mathbf{G}(\mathbf{H}_1\mathbf{F}\mathbf{F}^H\mathbf{H}_1^H + \mathbf{R}_{v_1v_1})\mathbf{G}^H\} = P_r, \quad (14)$$

where (13) and (14) are the source and relay power constraints respectively and (12) is the geometric MSE objective function.

### 3.1.3 Channel and Precoder Decomposition

As will be seen in the following sections the optimisation problem stated in (12), (13) and (14) can be vastly simplified if we consider the channel matrices  $\mathbf{H}_1$  and  $\mathbf{H}_2$ , in terms of their singular value decompositions

$$\mathbf{H}_1 = \mathbf{U}_1\mathbf{\Lambda}\mathbf{V}_1^H \quad \mathbf{H}_2 = \mathbf{U}_2\mathbf{\Delta}\mathbf{V}_2^H. \quad (15)$$

Although our designs can accommodate other cases, for simplicity we assume that  $\mathbf{H}_1$  and  $\mathbf{H}_2$  are full rank with rank  $M$ . We denote the first  $M$  columns of matrices  $\mathbf{U}_1$  and  $\mathbf{V}_2$  as  $\bar{\mathbf{U}}_1$  and  $\bar{\mathbf{V}}_2$  respectively. As well as this the upper left  $M \times M$  sub-matrices of  $\mathbf{\Lambda}$  and  $\mathbf{\Delta}$  are denoted by  $\bar{\mathbf{\Lambda}}$  and  $\bar{\mathbf{\Delta}}$  and contain the non-zero singular values,  $\lambda_{ii}$  and  $\delta_{ii}$ , of channels  $\mathbf{H}_1$  and  $\mathbf{H}_2$ .

It is also convenient to represent the source and relay precoders by the following decompositions

$$\mathbf{F} = \mathbf{\Theta}\mathbf{\Gamma}\mathbf{\Psi} \quad \mathbf{G} = \mathbf{\Xi}\mathbf{\Phi}\mathbf{\Upsilon}. \quad (16)$$

In the remainder of this paper we assume that the precoder  $\mathbf{F}$  distributes power uniformly across the source antennas and thus has rank  $M$  with  $\mathbf{\Gamma} = \gamma\mathbf{I}$  where  $\gamma = \sqrt{P_s/M}$  is such that the power constraint in (13) holds with equality. Although an  $N \times N$  matrix we will see in the following sub-sections that  $\mathbf{G}$  can be at most rank  $M$ . For later convenience we define  $\bar{\mathbf{\Xi}}$  and  $\bar{\mathbf{\Upsilon}}$  to contain the first  $M$  columns of  $\mathbf{\Xi}$  and  $\mathbf{\Upsilon}$  and the diagonal matrix  $\bar{\mathbf{\Phi}}$  to be the upper left  $M \times M$  submatrix of  $\mathbf{\Phi}$  containing the non zero singular values  $\phi_{ii}$ .

## 3.2 ZF Transceiver Design

The goal is now to design the processors  $\mathbf{F}$ ,  $\mathbf{G}$ ,  $\mathbf{W}$  and  $\mathbf{U}$  to minimise the geometric MSE in (12) under the power constraints (13) and (14) whilst also meeting the ZF criterion.

### 3.2.1 ZF Constrained Optimisation Problem

As previously mentioned, the functionality of our designs is dependant on the equaliser  $\mathbf{W}$ . In the case of the ZF transceiver the equaliser is required to eliminate all interference between transmitted data symbols and perfectly reconstruct the signals in the absence of noise. This requirement can be stated mathematically as

$$\mathbf{W}\mathbf{H}\mathbf{F} = \mathbf{U}. \quad (17)$$

For a given  $\mathbf{F}$ ,  $\mathbf{G}$  and  $\mathbf{U}$  the optimal ZF equaliser can be calculated directly from (17) as

$$\mathbf{W} = \mathbf{U}(\mathbf{F}^H\mathbf{H}^H\mathbf{H}\mathbf{F})^{-1}\mathbf{F}^H\mathbf{H}^H, \quad (18)$$

where we have used the minimum norm pseudo inverse of the product  $\mathbf{H}\mathbf{F}$ . Substituting (17) and (18) in (8) the error covariance matrix for the ZF transceiver can be written as

$$\mathbf{R}_{ee,ZF} = \mathbf{U}(\mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F})^{-1}\mathbf{U}^H. \quad (19)$$

From (11) we obtain the lower and upper bounds on the MSE as

$$|\mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F}|^{-1/M} \leq \text{tr}\left\{\mathbf{U}\left(\mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F}\right)^{-1}\mathbf{U}^H\right\}/M, \quad (20)$$

where we have used the fact that  $|\mathbf{U}| = 1$  since  $\mathbf{U}$  is a unit diagonal upper right triangular matrix. Using the lower bound in (20) as our objective function we can then state the constrained optimisation problem for the ZF transceiver to be

$$\max_{\mathbf{F}, \mathbf{G}} |\mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F}| \quad (21)$$

$$\text{subject to} \quad \text{tr}\left\{\mathbf{G}(\mathbf{H}_1\mathbf{F}\mathbf{F}^H\mathbf{H}_1^H + \mathbf{R}_{v_1v_1})\mathbf{G}^H\right\} = P_r, \quad (22)$$

where we have used the fact that minimising  $|\mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F}|^{-1/M}$  is equivalent to maximising  $|\mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F}|$ . We also note that the source power constraint in (13) has been eliminated as it is guaranteed to be met with  $\mathbf{F}$  chosen as in sub-section 3.1.3.

### 3.2.2 ZF Processors

We now go on to present the ZF processors  $\mathbf{F}$ ,  $\mathbf{G}$  and  $\mathbf{U}$  that maximise (21) under the constraint (22). Using the channel and precoder decompositions in sub-section 3.1.3, we can state from the Hadamard determinant inequality [12] that

$$|\mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F}| \leq \prod_{i=1}^M \frac{\gamma^2 \lambda_{ii}^2 |\phi_{ii}|^2 \delta_{ii}^2}{|\phi_{ii}|^2 \delta_{ii}^2 \sigma_{v_1}^2 + \sigma_{v_2}^2}, \quad (23)$$

where the bound holds with equality when  $\mathbf{\Theta} = \mathbf{V}_1$ ,  $\bar{\mathbf{\Xi}} = \bar{\mathbf{V}}_2$  and  $\bar{\mathbf{\Upsilon}} = \bar{\mathbf{U}}_1^H$ . We have thus established the following sets of source and relay precoders

$$\mathbf{F} = \gamma\mathbf{V}_1\mathbf{\Psi} \quad \mathbf{G} = \bar{\mathbf{V}}_2\bar{\mathbf{\Phi}}\bar{\mathbf{U}}_1^H, \quad (24)$$

where the unitary matrix  $\mathbf{\Psi}$  provides a degree of freedom that shall be exploited later in the design. Substituting such precoders in (21) and (22) the optimal  $\bar{\mathbf{\Phi}}$  can be calculated by solving the following optimisation problem

$$\max_{\phi_{ii}} \prod_{i=1}^M \frac{\gamma^2 \lambda_{ii}^2 |\phi_{ii}|^2 \delta_{ii}^2}{|\phi_{ii}|^2 \delta_{ii}^2 \sigma_{v_1}^2 + \sigma_{v_2}^2} \quad (25)$$

$$\text{subject to} \quad \sum_{i=1}^M |\phi_{ii}|^2 (\gamma^2 \lambda_{ii}^2 + \sigma_{v_1}^2) = P_r, \quad |\phi_{ii}|^2 \geq 0. \quad (26)$$

Since the objective function and inequality constraint are convex and the equality constraint is affine with respect to the design variable  $|\phi_{ii}|^2$  the solution to this problem can be obtained from the Karush Kuhn Tucker (KKT) conditions of optimality [13] and is given by

$$|\phi_{ii}|^2 = \left[ \sqrt{\frac{\mu \sigma_{v_2}^2}{\delta_{ii}^2 \sigma_{v_1}^2 (\gamma^2 \lambda_{ii}^2 + \sigma_{v_1}^2)} + \frac{\sigma_{v_2}^4}{4\delta_{ii}^4 \sigma_{v_1}^4}} - \frac{\sigma_{v_2}^2}{2\delta_{ii}^2 \sigma_{v_1}^2} \right]^+, \quad (27)$$

where  $\mu$  is a constant required to fulfill the constraint in (26) and is akin to the waterlevel in waterfilling procedures [14]. Having calculated the optimal  $\bar{\mathbf{\Phi}}$  we now focus on computing a unitary matrix  $\mathbf{\Psi}$  such that the bound in (20) holds with equality. We firstly note

that substituting (24) in (20) the lower bound on the MSE can be calculated to be

$$\bar{\sigma}_{ZF}^2 = \prod_{i=1}^M \left( \frac{\gamma^2 \lambda_{ii}^2 |\phi_{ii}|^2 \delta_{ii}^2}{|\phi_{ii}|^2 \delta_{ii}^2 \sigma_{v_1}^2 + \sigma_{v_2}^2} \right)^{-1/M}. \quad (28)$$

From the arithmetic geometric mean inequality we can state that (20) holds with equality if, and only if, the error covariance matrix in (19) is diagonal with equal diagonal elements. Thus, the ZF transceiver achieves the lower bound in (28) only if

$$\mathbf{U}(\mathbf{F}^H \mathbf{H}^H \mathbf{R}_{vv}^{-1} \mathbf{H} \mathbf{F})^{-1} \mathbf{U}^H = \bar{\sigma}_{ZF}^2 \mathbf{I}. \quad (29)$$

Using the channel decompositions in (15) and the precoders in (24) we can re-write (29) as

$$\mathbf{U} \Psi^H \left( \gamma^2 \bar{\Lambda}^2 \bar{\Phi}^2 \bar{\Delta}^2 (\bar{\Phi}^2 \bar{\Delta}^2 \sigma_{v_1}^2 + \sigma_{v_2}^2 \mathbf{I})^{-1} \right)^{-1} \Psi \mathbf{U}^H = \bar{\sigma}_{ZF}^2 \mathbf{I}. \quad (30)$$

Taking the Cholesky factor and re-arranging we arrive at

$$\left( \gamma^2 \bar{\Lambda}^2 \bar{\Phi}^2 \bar{\Delta}^2 (\bar{\Phi}^2 \bar{\Delta}^2 \sigma_{v_1}^2 + \sigma_{v_2}^2 \mathbf{I})^{-1} \right)^{1/2} = \mathbf{Q} \bar{\mathbf{U}} \Psi^H, \quad (31)$$

where  $\mathbf{Q} \in \mathbb{C}^{M \times M}$  is a unitary matrix and  $\bar{\mathbf{U}} = (1/\bar{\sigma}_{ZF})\mathbf{U}$  is an upper right triangular matrix with equal diagonal elements given by  $1/\bar{\sigma}_{ZF}$ . The matrix decomposition in (31) is the geometric mean decomposition [11] of  $(\gamma^2 \bar{\Lambda}^2 \bar{\Phi}^2 \bar{\Delta}^2 (\bar{\Phi}^2 \bar{\Delta}^2 \sigma_{v_1}^2 + \sigma_{v_2}^2 \mathbf{I})^{-1})^{1/2}$ . This matrix decomposition is also commonly referred to as the equal diagonal QR decomposition [15].

The ZF transceiver that achieves the lower MSE bound in (28) is thus constructed as follows. The channels  $\mathbf{H}_1$  and  $\mathbf{H}_2$  are decomposed using the singular value decompositions in (15). The power allocation matrix  $\Gamma$  for the source is constructed as  $\Gamma = \gamma \mathbf{I} = \sqrt{(P_s/M)} \mathbf{I}$  and the relay power allocation matrix  $\bar{\Phi}$  is calculated with diagonal elements satisfying (27). The geometric mean decomposition in (31) is then computed from which the feedback matrix  $\mathbf{B} = \bar{\sigma}_{ZF} \bar{\mathbf{U}} - \mathbf{I}$  can be calculated. Finally the source and relay precoders are given by (24) and the DFE feedforward matrix from (18).

### 3.3 MMSE Transceiver Design

Although the ZF solution completely eliminates interference between transmitted data symbols it can also amplify noise in the receiver which can result in poor performance. For this reason we now consider an MMSE solution where the equaliser takes into account the noise components in the system.

#### 3.3.1 MMSE Constrained Optimisation Problem

The optimal equaliser  $\mathbf{W}$  that minimises the MSE at the input to the decision device is provided by the well known Wiener solution and is given by

$$\mathbf{W} = \mathbf{U} \mathbf{F}^H \mathbf{H}^H (\mathbf{H} \mathbf{F} \mathbf{F}^H \mathbf{H}^H + \mathbf{R}_{vv})^{-1}. \quad (32)$$

Substituting (32) in (8) we can write  $\mathbf{R}_{ee}$  for the MMSE transceiver as

$$\mathbf{R}_{ee,MMSE} = \mathbf{U} (\mathbf{I} + \mathbf{F}^H \mathbf{H}^H \mathbf{R}_{vv}^{-1} \mathbf{H} \mathbf{F})^{-1} \mathbf{U}^H, \quad (33)$$

and from the arithmetic geometric mean inequality in (10) we can obtain the following lower and upper MSE bounds

$$\begin{aligned} & |\mathbf{I} + \mathbf{F}^H \mathbf{H}^H \mathbf{R}_{vv}^{-1} \mathbf{H} \mathbf{F}|^{-1/M} \\ & \leq \text{tr} \left\{ \mathbf{U} \left( \mathbf{I} + \mathbf{F}^H \mathbf{H}^H \mathbf{R}_{vv}^{-1} \mathbf{H} \mathbf{F} \right)^{-1} \mathbf{U}^H \right\} / M. \end{aligned} \quad (34)$$

As with the ZF design we use the lower MSE bound as our objective function and under the constraint of limited average power at the relay we can state the optimisation problem as

$$\max_{\mathbf{F}, \mathbf{G}} |\mathbf{I} + \mathbf{F}^H \mathbf{H}^H \mathbf{R}_{vv}^{-1} \mathbf{H} \mathbf{F}| \quad (35)$$

$$\text{subject to} \quad \text{tr} \left\{ \mathbf{G} (\mathbf{H}_1 \mathbf{F} \mathbf{F}^H \mathbf{H}_1^H + \mathbf{R}_{v_1 v_1}) \mathbf{G}^H \right\} = P_r. \quad (36)$$

We note that maximising the objective function in (35) is equivalent to maximising the mutual information.

#### 3.3.2 MMSE Processors

Applying the Hadamard determinant inequality to the determinant in (35) we can state that

$$|\mathbf{I} + \mathbf{F}^H \mathbf{H}^H \mathbf{R}_{vv}^{-1} \mathbf{H} \mathbf{F}| \leq \prod_{i=1}^M \left( 1 + \frac{\gamma^2 \lambda_{ii}^2 |\phi_{ii}|^2 \delta_{ii}^2}{|\phi_{ii}|^2 \delta_{ii}^2 \sigma_{v_1}^2 + \sigma_{v_2}^2} \right), \quad (37)$$

where the bound holds with equality when the source and relay precoders have the same structure as given in (24). Substituting (24) in (35) and (36) we arrive at the following scalar optimisation problem

$$\max_{\phi_{ii}} \prod_{i=1}^M \left( 1 + \frac{\gamma^2 \lambda_{ii}^2 |\phi_{ii}|^2 \delta_{ii}^2}{|\phi_{ii}|^2 \delta_{ii}^2 \sigma_{v_1}^2 + \sigma_{v_2}^2} \right) \quad (38)$$

$$\text{subject to} \quad \sum_{i=1}^M |\phi_{ii}|^2 (\gamma^2 \lambda_{ii}^2 + \sigma_{v_1}^2) = P_r, \quad |\phi_{ii}|^2 \geq 0. \quad (39)$$

The solution to this optimisation problem provides the optimal diagonal elements of  $\bar{\Phi}$  for the MMSE relay and can be calculated from the KKT conditions to be

$$|\phi_{ii}|^2 = \left[ \frac{-b_i + \sqrt{b_i^2 - 4a_i c_i}}{2a_i} \right]^+, \quad (40)$$

with the variables

$$\begin{aligned} a_i &= \delta_{ii}^4 \sigma_{v_1}^2 (\gamma^2 \lambda_{ii}^2 + \sigma_{v_1}^2) & b_i &= \delta_{ii}^2 \sigma_{v_2}^2 (\gamma^2 \lambda_{ii}^2 + 2\sigma_{v_1}^2) \\ c_i &= \left( \sigma_{v_2}^4 - \frac{\mu \gamma^2 \lambda_{ii}^2 \delta_{ii}^2 \sigma_{v_2}^2}{\gamma^2 \lambda_{ii}^2 + \sigma_{v_1}^2} \right), \end{aligned} \quad (41)$$

where again  $\mu$  must be calculated to satisfy the equality in (39). We now need to calculate the matrix  $\Psi$  such that  $\mathbf{R}_{ee,MMSE}$  is a diagonal matrix with equal diagonal elements and thus (34) holds with equality. With the precoders in (24) and the channel decompositions in (15) we can calculate the lower MSE bound in (34) to be

$$\bar{\sigma}_{MMSE}^2 = \prod_{i=1}^M \left( 1 + \frac{\gamma^2 \lambda_{ii}^2 |\phi_{ii}|^2 \delta_{ii}^2}{|\phi_{ii}|^2 \delta_{ii}^2 \sigma_{v_1}^2 + \sigma_{v_2}^2} \right)^{-1/M}. \quad (42)$$

Arguing as done for the ZF design we can state that the MMSE transceiver only achieves the lower bound in (42) if

$$\mathbf{U} (\mathbf{I} + \mathbf{F}^H \mathbf{H}^H \mathbf{R}_{vv}^{-1} \mathbf{H} \mathbf{F})^{-1} \mathbf{U}^H = \bar{\sigma}_{MMSE}^2 \mathbf{I}. \quad (43)$$

Substituting (15) and (24) in (43) we can write

$$\begin{aligned} & \mathbf{U} \Psi^H \left( \mathbf{I} + \gamma^2 \bar{\Lambda}^2 \bar{\Phi}^2 \bar{\Delta}^2 (\bar{\Phi}^2 \bar{\Delta}^2 \sigma_{v_1}^2 + \sigma_{v_2}^2 \mathbf{I})^{-1} \right)^{-1} \Psi \mathbf{U}^H \\ & = \bar{\sigma}_{MMSE}^2 \mathbf{I}. \end{aligned} \quad (44)$$

Similar to the ZF transceiver design the optimal matrix  $\Psi$  can be extracted from the following geometric mean decomposition

$$\left( \mathbf{I} + \gamma^2 \bar{\Lambda}^2 \bar{\Phi}^2 \bar{\Delta}^2 (\bar{\Phi}^2 \bar{\Delta}^2 \sigma_{v_1}^2 + \sigma_{v_2}^2 \mathbf{I})^{-1} \right)^{1/2} = \mathbf{Q} \bar{\mathbf{U}} \Psi^H, \quad (45)$$

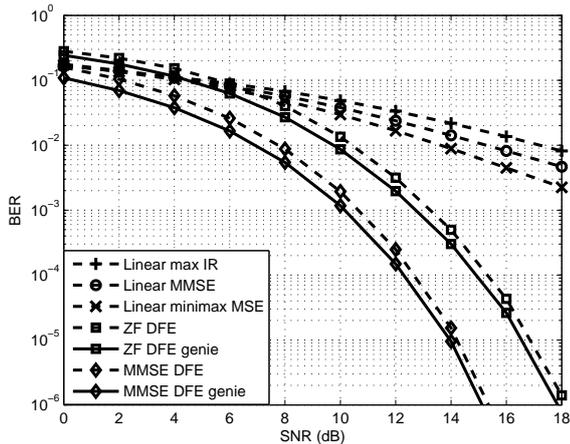


Figure 2: BER results for proposed and benchmark systems

where  $\bar{\mathbf{U}} = (1/\bar{\sigma}_{MMSE})\mathbf{U}$  is an upper right triangular matrix with equal diagonal elements given by  $1/\bar{\sigma}_{MMSE}$ .

This concludes the derivation of our MMSE transceiver design. The MMSE transceiver that achieves the lower bound in (42) is constructed as follows: Given the transmission channel decompositions in (15) the source precoder is constructed as in (24) where  $\gamma = \sqrt{P_s/M}$  and the unitary matrix  $\Psi$  is calculated from the geometric mean decomposition in (45). The diagonal matrix  $\bar{\Phi}$  for the relay precoder is calculated with diagonal elements satisfying (40). The feedback matrix is calculated from  $\mathbf{B} = \bar{\sigma}_{MMSE} \bar{\mathbf{U}} - \mathbf{I}$  where again  $\bar{\mathbf{U}}$  results from the decomposition in (45). Finally the feedforward filter is given by (32).

#### 4. SIMULATIONS AND RESULTS

In this section we evaluate the performance of the proposed ZF DFE and MMSE DFE transceivers in terms of BER and compare them to the linear MMSE, max IR, and minimax MSE designs presented in [6], [3], and [7] respectively.

##### 4.1 Simulation Parameters

In all simulations we assume  $N = M = 4$  antennas in each layer of the relay network with the resulting MIMO channels having complex Gaussian entries with zero mean and unit variance. The symbols from the source antennas are selected from QPSK constellations with unit variance. The Signal to Noise Ratio (SNR) for the two transmission stages are defined as  $SNR_1 = P_s/N\sigma_{v_1}^2$  and  $SNR_2 = P_r/M\sigma_{v_2}^2$ , with the source and relay power budgets being set as  $P_s = P_r = 4$ . The linear transceiver design in [3] was derived under the assumption that  $\sigma_{v_1}^2 = \sigma_{v_2}^2$  and so, for a fair comparison, when assessing BER against SNR in simulations we set  $SNR = SNR_1 = SNR_2$ .

##### 4.2 Results

Figure 2 shows the uncoded BER results for the proposed designs as well as the three linear benchmark systems. The solid curves shown in Figure 2 are the results obtained for the proposed designs in the absence of error propagation in the feedback loop. These results are purely theoretical but provide an insight into the effect that incorrect decisions causes on the BER performance of the proposed transceivers. Clearly both the ZF and MMSE DFE designs far outperform the linear benchmarks, particularly at high SNR values, with the MMSE solution providing the best performance over all values of SNR.

## 5. CONCLUSIONS

In this paper we have presented two transceiver designs for non-regenerative MIMO relay systems that utilise linear processors in the different stages of the network as well as decision feedback detection at the receiver. Under the assumption of full CSI at each stage of the network, the processors for both designs were jointly optimised to minimise the system arithmetic MSE under average power constraints at the source and relay terminals. From this constrained optimisation problem the ZF and MMSE solutions were derived depending on the functionality of the equaliser. Simulation results have shown that both the ZF and MMSE DFE's provide far superior performance in terms of BER over linear designs available in the literature.

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