DISTRIBUTED SPECTRUM ALLOCATION WITH THE COURNOT COMPETITION

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ABSTRACT

The paper considers a new method of the spectrum scheduling process and QoS support in distributed wireless networks using a game theoretic-framework. The game uses incomplete information model, so the concept of the Bayesian Nash equilibrium is used. Moreover, we consider the new algorithm of distributed spectrum sharing based on the Cournot oligopoly competition. Multiple traffic classes of high and low priority have been used with special parameters which represent priority classes and the volume of users' individual demands. The proposed resource-sharing algorithm may be used in distributed opportunistic or cognitive wireless networks.

1. INTRODUCTION

In contemporary wireless communication, radio spectrum is the scarce resource, particularly in scenarios, where there exist multiple players (network users and nodes) with high Quality of Service (QoS) demands. This spectrum has to be efficiently and fairly shared by the players. In a centralized architecture, it is easier to solve the problem of resource scheduling, because at the central element of that network (the resource manager or the spectrum broker) usually the channel state information (CSI) for all players is available. In the networks without that element a fair spectrum sharing algorithm is difficult to design. In this paper, we discuss the Cournot spectrum-sharing competition, which can be used for the distributed resource assignment, and which incorporates some fairness and QoS-support mechanisms. Our approach proposed below can be used in the cognitive wireless networks with a decentralized architecture, in which some information is made available to all network devices, e.g. in the case, when one of the cognitive devices becomes a central (master) node, has some management (or broadcasting) rights, but is not interested in sharing the computing resources with other players. The model of the considered network is shown on Figure 1.

The idea of the Cournot competition has been first shown and applied in economy. Studies of that game have been presented in [1] and [2]. The same idea has been adopted in radio communication for resource-sharing in [3] and [4]. Here below, we contribute to the advance of the work presented in [3] and [4]. We adopt the same utility function, extend the Cournot model to the distributed spectrum management game with incomplete information, and analyze the Bayesian Nash Equilibrium (BNE) in this game.

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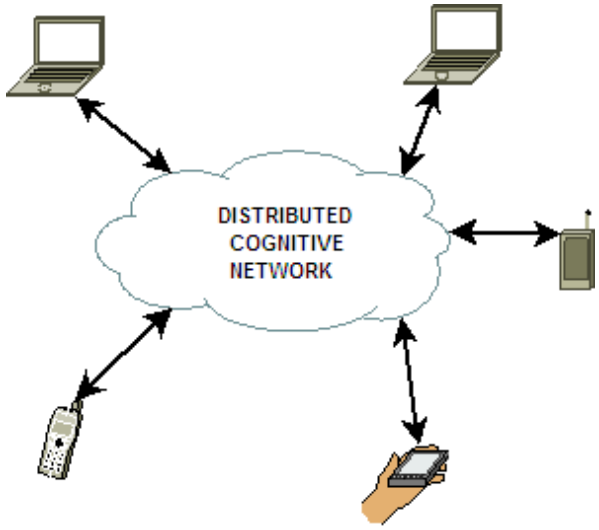

Figure 1 – Cognitive devices in a distributed network

2. GAME MODEL

A. System description

Let us consider a decentralized network with N users. We assume that there is some spectrum available, and its size is given by B. The demand of an i-th user for some portion of that spectrum is denoted by b_i. We assume that the users monitor the behaviour of the rest of the players, and that their *effective SNR*'s are made known to all of them (e.g. by the master node). This concept is used to hide the CSI in one parameter called *effective SNR* denoted by γ_i for the i-th player [6]. The values of γ_i may be easily sent in a signalization channel (e.g. in the Cognitive Pilot Channel – CPC [5]) without wasting much of the spectrum for that purpose. Each player maps her frequency-selective channel characteristic and the associated SNRs in the considered band to one effective γ_i value. This value is used to make an efficient decision on the adaptive transmission parameters, e.g. coding scheme and rate, adopted power, modulation constellation. The effective-SNR concept is presented in [6].

In the effective scheduling process, the players use the continuous function of their spectral efficiency. For an i-th player it is defined as ([7]):

$$\eta_i = \log_2\left(1+\alpha_i \cdot \gamma_i\right), \tag{1}$$

where for the QAM modulation schemes:

$$\alpha_i = \frac{1.5}{\ln\left(\frac{0.2}{\mathrm{P}_{\mathrm{e}_i}}\right)}, \tag{2}$$

and $\mathrm{P}_{\mathrm{e}_i}$ is the target Bit Error Probability (BEP) of an i-th player. The α_i parameter allows to estimate the data rate and the spectral efficiency for a target BEP [7]. The players may transmit data of various QoS demands and priorities which are reflected in the pair of parameters ($\mathrm{P}_{\mathrm{e}_i}$, r_i) (r_i is the revenue per unit of the allocated spectrum, the parameter characterizing the traffic class of an i-th user). These pairs translate to traffic classes. We assume that the probability that other user wants to transmit the data of the k-th priority is given by p_k. We can also assume that users, which compete for the spectrum, always have enough data to send. In other case we can assume that they are out of the game.

B. The principles of the Cournot competition

Let us consider the static Cournot competition to model the spectrum sharing market. In that market, the players compete for an available bandwidth. If there are two players the market is called *duopoly*, if there are more than two players we have the oligopoly [3]. The strategy of each player is related to the amount of spectrum which they want to demand and use (b_i). The cost function (reflecting the cost of the acquired spectrum) for all competing users given by [4]:

$$c(\mathbf{b}) = x + y \cdot \left(\sum_{j=1}^{N} b_j\right)^{\tau}, \tag{3}$$

where $\mathbf{b}$ is the vector of (b_1, …, b_N), x is a fixed cost of spectrum sharing, y is the cost of the spectrum unit (assumed to be the same for all users), which affects (increases) the players' cost, and thus also reduces their demands, and τ is the factor additionally impacting the cost, as well as the fairness of spectrum sharing. We consider cost function with $\tau = 1$ for the oligopoly case. The revenue of the i-th player is related to the obtained spectral efficiency, and is given by [4, 7]

$$R_i = r_i \cdot \eta_i \cdot b_i, \tag{4}$$

where r_i (let us note again) is the revenue per unit of the allocated spectrum of an i-th user. The profit of the i-th player is given by the revenue minus the cost:

$$\pi_i(\mathbf{b}) = r_i \cdot \eta_i \cdot b_i - b_i \cdot c(\mathbf{b}). \tag{5}$$

The above profit function is a concave function of b_i [4], and thus, we can find its maximum by solving the equation:

$$\frac{\partial \pi_i}{\partial b_i} = 0. \tag{6}$$

For $\tau = 1$, and in the oligopoly case, we obtain the best-response function of an i-th player (for other players the formula is the same) which is given by

$$b_i = \frac{1}{2} \cdot \left(\frac{r_i \cdot \eta_i - x}{y} - \sum_{i \neq j}^{N} b_j\right). \tag{7}$$

To obtain the Nash Equilibrium (NE) of this game, the set of equations must be solved, as defined above for i = 1, …, N. For the duopoly case, the solution (NE) is given by the following equation:

$$b_i^* = \frac{2 \cdot r_i \cdot \eta_i - r_j \cdot \eta_j - x}{3 \cdot y}. \tag{8}$$

C. Enhancements of the Cournot competition model

Let us assume that the nature of the transmitted traffic in the network is unknown. Any station can transmit data with various priorities (the priority is the feature of a traffic class). Without the loss of generality, we will consider two players (duopoly case) and a finite number K of traffic classes, defined by priorities $r_j \in \{r_{j_k}\}$ (where k = 1,…, K). The players know only the effective SNRs of the rest of the players and the probabilities (p_k) of the traffic class of the opponent (which can be collected during the previous observations as this may be the characteristic cognitive feature of the cognitive radio). Equation (8) presents the Nash equilibrium for the case of the perfect and complete knowledge of the opponent's parameters: r_j and η_j. Now we can add the element of uncertainty to the game of two players. We assume that the target BEP of that both traffic classes is the same, and we use $x = 0$, $y = 1$, $\tau = 1$ in the cost function (3). The mobile device knows her own traffic class ($\mathrm{P}_{\mathrm{e}_i}$, r_i), and she also knows with probability p_k that the other player wants to transmit the data with the priority class k. She does not know the exact nature of the traffic that is in the buffers of the other player. According to the theory of games with incomplete information [2] we have to find the Bayesian Nash equilibrium, and we have to consider not only one opponent but actually K opponents with different traffic behaviour, denoted by j_k (for k = 1,…, K).

Let us assume that the particular player i wants to transmit data of the priority class r_i. The profit of the other (j-th) player in case of having priority r_{j_k} data in buffers is given by:

$$\pi_{j_k}(\mathbf{b}) = r_{j_k} \cdot \eta_j \cdot b_{j_k} - b_{j_k} \cdot (b_{j_k} + b_i). \tag{9}$$

and the profit of the considered (i-th) player is given by

$$\pi_i(\mathbf{b}) = r_i \cdot \eta_i \cdot b_i - b_i \cdot \left(b_i + \sum_{k=1}^{K} p_k b_{j_k}\right). \tag{10}$$

The sum of probabilities p_k must be equal to 1. Now, the profit maximization proceeds according to the same steps as in the classic Cournot competition (6):

$$\begin{cases} \dfrac{\partial \pi_i}{\partial b_i} = 0 \\ \dfrac{\partial \pi_{j_1}}{\partial b_{j_1}} = 0 \\ \vdots \\ \dfrac{\partial \pi_{j_K}}{\partial b_{j_K}} = 0 \end{cases}, \tag{11}$$

The result of (11) is the following set of equations:

$$\begin{cases} b_i\left(b_{j_1},...,b_{j_K}\right)=\frac{1}{2}\cdot\left(r_i\cdot\eta_i-\sum_{k=1}^{K}p_k b_{j_k}\right) \\ b_{j_1}\left(b_i\right)=\frac{1}{2}\cdot\left(r_{j_1}\cdot\eta_j-b_i\right) \\ \vdots \\ b_{j_K}\left(b_i\right)=\frac{1}{2}\cdot\left(r_{j_K}\cdot\eta_j-b_i\right) \end{cases}, \tag{12}$$

where $b_i(...)$ is the best-response spectrum demand of the i-th user. It can be written in the matrix form as

$$\mathbf{A}\cdot\mathbf{b}^{\mathrm{T}}=\mathbf{D}, \tag{13}$$

where

$$\mathbf{A}=\begin{bmatrix} 2 & p_1 & p_2 & \cdots & p_K \\ 1 & 2 & 0 & \cdots & 0 \\ 1 & 0 & 2 & \ddots & 0 \\ 1 & 0 & 0 & \cdots & 2 \end{bmatrix}, \tag{14}$$

$$\mathbf{D}=\begin{bmatrix} r_i\cdot\eta_i \\ r_{j_1}\cdot\eta_j \\ \vdots \\ r_{j_K}\cdot\eta_j \end{bmatrix}, \tag{15}$$

and

$$\mathbf{b}^{\mathrm{T}}=\begin{bmatrix} b_i \\ b_{j_1} \\ \vdots \\ b_{j_K} \end{bmatrix}. \tag{16}$$

The result (the BNE) is given by

$$\mathbf{b}^{*\mathrm{T}}=\mathbf{A}^{-1}\cdot\mathbf{D}. \tag{17}$$

For two players in the game (denoted by i and j) it is easy to write the full formula for the Bayesian Nash equilibrium as the result, and it is given by:

$$b_i^*=\frac{1}{3}\cdot\left(2\cdot r_i\cdot\eta_i-\sum_{k=1}^{K}p_k\cdot r_{j_k}\cdot\eta_j\right). \tag{18}$$

Formula (18) is the most important for us because in the duopoly case each player uses that formula for the decision on her strategy.

We have obtained the result with the element of uncertainty defined by the probability p_k which may be modelled by observing the character of the traffic in the network. For more than two players, we can start gathering the statistics of the other players as one and use these statistics for each player. Using one parameter as generalization of all users may be better and simpler than gathering statistics for each user separately, but in terms of game-solution effectiveness it may produce worst results. We suggest that the collection of data should be done separately for each user only when number of users is small and compressed to one number when there is more than an assumed threshold. One of the disadvantages of the Bayesian games is that when we have N players then we have to consider (NK-K+1) equations.

We can write presented game for N players. We assume x = 0, y = 1, τ = 1 . The profit of considered (i-th) player is given by

$$\pi_i(\mathbf{b})=r_i\cdot\eta_i\cdot b_i-b_i\cdot\left(b_i+\sum_{\substack{j=1\\ j\neq i}}^{N}\sum_{k=1}^{K}p_{j_k}b_{j_k}\right). \tag{19}$$

The profit of anyone of the rest of the players, e.g. of the j-th player having the k-th traffic class is given by:

$$\pi_{j_k}(\mathbf{b})=r_{j_k}\cdot\eta_j\cdot b_{j_k}-b_{j_k}\cdot\left(b_{j_k}+b_i+\sum_{\substack{n=1\\ n\neq j\\ n\neq i}}^{N}\sum_{l=1}^{K}p_{nl}b_{n_l}\right) \tag{20}$$

Now, the profit maximization proceeds according to the analogous steps as in the classic Cournot competition (6) and in the duopoly game (11). Then we can achieve the matrices **A**, **D**, and $\mathbf{b}^{*\mathbf{T}}$ in the same way as in the previous game.

Below we present the matrices **A**, **D**, and $\mathbf{b}^{\mathbf{T}}$ for the game with three players and K = 2.

$$\mathbf{A}=\begin{bmatrix} 2 & p_{1_1} & p_{1_2} & p_{2_1} & p_{2_2} \\ 1 & 2 & 0 & p_{2_1} & p_{2_2} \\ 1 & 0 & 2 & p_{2_1} & p_{2_2} \\ 1 & p_{1_1} & p_{1_2} & 2 & 0 \\ 1 & p_{1_1} & p_{1_2} & 0 & 2 \end{bmatrix}, \tag{21}$$

$$\mathbf{D}=\begin{bmatrix} r_1\cdot\eta_1 \\ r_{2_1}\cdot\eta_2 \\ r_{2_2}\cdot\eta_2 \\ r_{3_1}\cdot\eta_3 \\ r_{3_2}\cdot\eta_3 \end{bmatrix}, \tag{22}$$

$$\mathbf{b}^{\mathrm{T}}=\begin{bmatrix} b_1 \\ b_{2_1} \\ b_{2_2} \\ b_{3_1} \\ b_{3_2} \end{bmatrix}. \tag{23}$$

In that game each player solves equation (17) and is looking only for her solution (b_1). That value is processed in next stages of the new spectrum sharing algorithm. Observing (21), (22) and (23) it is easy to generalize those matrices for N players and K traffic classes. It is also worth to mention that r_i parameters have to be chosen correctly and are strictly dependent on the available bandwidth size.

3. DISTRIBUTED SPECTRUM SHARING ALGORITH

In the previous section we have formulated the enhancement of the Cournot competition and we added the elements of uncertainty into the players' strategies. Now, let us propose the new algorithm of the spectrum sharing in the distributed cognitive networks, where each player may demand a portion

of the spectrum from 0 to all available bandwidth. This algorithm is defined as follows:

1. One of the mobile devices starts to play a leading (master) role the distributed network and informs the players about the beginning of a new game and about the effective SNRs in the network.
2. The players solve the Cournot competition according to (17) or (18) for the game of two players or according to generalized (17) for n players.
3. The players' demands are sent to the network master node. Each player sends only her demand.
4. The master-node of the network makes an adjustment to the Available Bandwidth Size (ABS) according to the following formulas, as suggested by us in [8]:

$$D = \sum_i b_i^* \tag{24}$$

and

$$s_i = \left\lfloor \frac{S \cdot b_i^*}{D} \right\rfloor, \tag{25}$$

where D is the sum of players' demands, S is the number of available spectrum units (the element of the game) and s_i is number of assigned spectrum units. This is the only operation which is processed by the master node. This step may be omitted but the advantages of this step are that the entire available spectrum is utilized, and that the sum of demands does not exceed the ABS.

5. The master node informs the players about the amount of spectrum units they may use.
6. The players transmit their data and build their statistics which will be used in the next games.

4. NUMERICAL RESULTS

In this section we show the effect of uncertainty in the Cournot game presented in Section 2, which is the basis of the new spectrum sharing algorithm presented in Section 3.

We consider the game of two players and an algorithm with an adjustment to the ABS. As mentioned above this adjustment action is needed because for some values, the sum of demands may be higher than available spectrum. Demands for both players have been calculated as if they did not know the opponent's revenue parameter. We consider only two traffic classes of two priorities resulting in the revenue parameter values: $\rho_1 = 12$ for the high priority data (first traffic class) and $\rho_2 = 8$ for the low priority data (second traffic class), i.e. $r_1, r_2 \in \{\rho_1, \rho_2\}$. Moreover, we consider the system with the following parameters: $B = 15$ MHz of the ABS, $x = 0$, $y = 1$, $\tau = 1$, $P_{e_1} = P_{e_2} = 10^{-4}$ and $p_{11} = p_{21} = p$ being in the range [0, 1] (it implies $p_{12} = p_{22} = 1 - p$). An example of $\gamma_1 = \gamma_2 = 10$ has been considered for comparison of the game with complete and incomplete information.

First, in Table 1 the solutions are presented for the game with full information and for the game with incomplete information (concerning the traffic classes). These solutions are obtained after step 3 of the algorithm discussed in the previous section. It is easy to observe that all demands from Table 1 are in the area of the Bayesian game in Figure 2, where the Cournot-game solution options are presented. We can observe the effect of uncertainty in the second player demands, which are calculated may be not optimal. In case when one player has high priority data, her demands may be in the range of [6.29, 8.38] and are dependent on the traffic priority of the other player. The resulting demands of the player with the low-priority traffic are in the range of [2.10, 4.19]. The area of the Bayesian game shown on Figure 2 is dependent on the p parameter.

TABLE 1. Results of using BNE and NE in the Cournot competition for spectrum sharing.

Parameters used	**Results (MHz)**	**Remarks on the types of the game and solutions**
$r_1 = \rho_1, r_2 = \rho_1$	$b_1 = 6.29$ $b_2 = 6.29$	complete information, NE
$r_1 = \rho_1, r_2 = \rho_2$	$b_1 = 8.38$ $b_2 = 2.10$	complete information, NE
$r_1 = \rho_2, r_2 = \rho_1$	$b_1 = 2.10$ $b_2 = 8.38$	complete information, NE
$r_1 = \rho_2\ r_2 = \rho_2$	$b_1 = 4.19$ $b_2 = 4.19$	complete information, NE
$r_1 = \rho_1,\ r_{21} = \rho_1, r_{22} = \rho_2,$ $p = 1$	$b_1 = 6.29$ $b_{21} = 6.29$ $b_{22} = 3.14$	incomplete information, BNE
$r_1 = \rho_2,\ r_{21} = \rho_1, r_{22} = \rho_2,$ $p = 1$	$b_1 = 2.10$ $b_{21} = 8.38$ $b_{22} = 5.24$	incomplete information, BNE
$r_1 = \rho_1,\ r_{21} = \rho_1, r_{22} = \rho_2,$ $p = 0.5$	$b_1 = 7.34$ $b_{21} = 5.76$ $b_{22} = 2.62$	incomplete information, BNE
$r_1 = \rho_2,\ r_{21} = \rho_1, r_{22} = \rho_2,$ $p = 0.5$	$b_1 = 3.14$ $b_{21} = 7.86$ $b_{22} = 4.72$	incomplete information, BNE
$r_1 = \rho_1,\ r_{21} = \rho_1, r_{22} = \rho_2,$ $p = 0$	$b_1 = 8.38$ $b_{21} = 5.24$ $b_{22} = 2.10$	incomplete information, BNE
$r_1 = \rho_2,\ r_{21} = \rho_1, r_{22} = \rho_2,$ $p = 0$	$b_1 = 4.19$ $b_{21} = 7.34$ $b_{22} = 4.19$	incomplete information, BNE

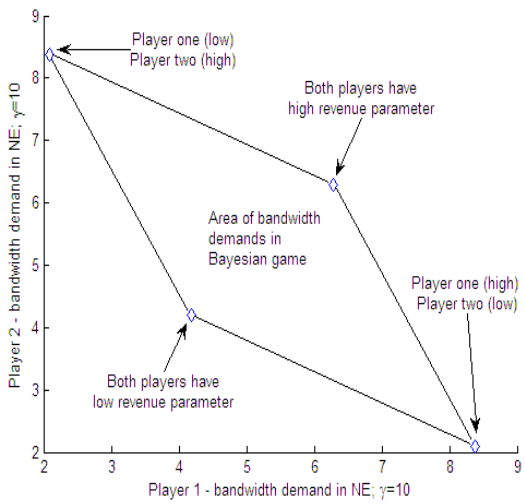

Figure 2. Demands of the players resulting from the Bayesian game

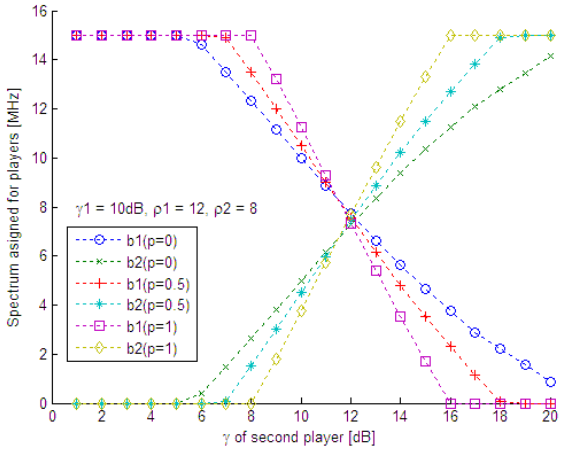

Figure 3 – Spectrum assigned to players (after the adjustment to ABS) versus the channel condition of player 2 in case of player 1 and 2 having the high and low priority traffic respectively

In Figures 3 and 4, the final results (in MHz) of the spectrum sharing algorithm are presented, that uses the Cournot competition and the final adjustments to the ABS. These presented results have been calculated based on the values obtained in spectrum units and on the total available bandwidth *B*. In both figures, the impact of the second player variable channel-quality on the game solution is presented, however for different distribution of traffic priorities. We can observe that when the quality of the second player's channel increases, her demands for the spectrum increase too. In the same time, the first player has to verify and decrease her demands, even though her channel quality is not decreasing.

5. CONCLUSIONS

The paper has presented the idea of sharing the resources in the distributed cognitive wireless network based on the Cournot competition with the elements of uncertainty about the players' strategies. The Bayesian Nash equilibrium, as stable solution of that game, has been examined. The possibility of the transmission of the traffic with various priorities has been also considered, as well as the uncertainty concerning the players' revenue parameters (that translate to traffic classes and their priorities). Moreover, the revenue parameters are strictly dependent on the available spectrum size and should be chosen correctly for efficient results.

Our presented idea may be used especially in the wireless distributed networks, where multiple stations take independent decisions, and one station (master) plays a leading role in that network and decides about the final resource allocation, based on the players' demands. Each station must solve a set of *KN-K*+1 equations (that set of equation is different for each of *n* stations) to obtain her own demand for the spectrum. Due to the fact that the network is distributed, each station must observe the network, collect data and build her own statistics on the traffic classes, because they are needed to make the best decisions. Moreover, the efficient SNR values of the other stations have to be obtained (e.g. from the CPC) by each player. Thus, the idea of the cognitive radio is perfectly fitted in our framework. The considered model can be directly applied in flat-fading environment. In the selective-fading scenario, our proposed model requires an amendment on the distribution of spectrum units acquired by the users according to their sub-band channel qualities. This is a subject of our ongoing work.

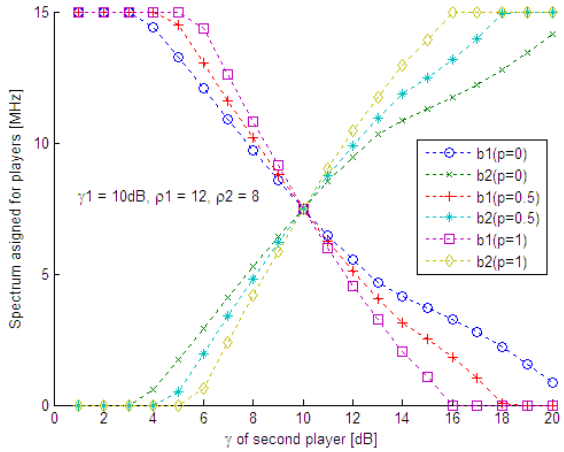

Figure 4 – Spectrum assigned to players (after the adjustment to ABS) versus the channel condition of player 2 in case of player 1 and 2 both having the high priority traffic.

REFERENCES

[1] E. Rasmusen, "Games and Information: An Introduction to Game Theory" Fourth Edition, 2006, Blackwell Publishers, Chapter 3 and 14.

[2] J. Watson, "Strategy Introduction to Game Theory", 2002, W.W. Norton & Company

[3] D. Niyato and E. Hossain, "A game-theoretic approach to competitive spectrum sharing in cognitive radio networks," in Proceedings of IEEE WCNC, Hong Kong, 11-15 March, 2007.

[4] D. Niyato and E. Hossain, "Microeconomic models for dynamic spectrum management in cognitive radio networks," chapter in Cognitive Wireless Communication Networks, (Eds. V.K. Bhargava and E. Hossain), Springer-Verlag, November 2007.

[5] J. Perez-Romero, O. Salient, R. Agusti, L. Giupponi, "A Novel On-Demand Cognitive Pilot Channel Enabling Dynamic Spectrum Allocation", 2nd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, DySPAN 2007, 17-20 April 2007, Dublin, Ireland, pp. 46-54.

[6] A. Kliks, A. Zalonis, I. Dagres, A. Polydoros, H. Bogucka "PHY Abstraction Methods for OFDM and NOFDM Systems", Journal of Telecommunications and Information Technology (JTIT) 3/2009

[7] A.J. Goldsmith and S.-G Chua, "Variable rate variable power MQAM for fading channels" IEEE Trans. Commun., vol.45, no.10, pp. 1218-1230, Oct. 1997

[8] M. Parzy, H. Bogucka, "QoS Support in Radio Resource Sharing with the Cournot Competition", in Proc. Int. Workshop on Cognitive Information Processing CIP'10, Elba, Italy, 14-16 June, 2010