CYCLOSTATIONARY AUTOCORRELATION BASED CFO ESTIMATORS

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ABSTRACT

We derive three novel data-aided CFO estimators based on a new autocorrelation function which is defined using cyclostationary properties of the repetitive preamble. The first estimator is a generalization of a classic estimator, and the other two take advantage of the structure of the new autocorrelation function to improve the estimation for low CFO. In addition, the performance of the proposed schemes is studied using simulations and compared with classical implementations. Comparing to the state-of-the-art CFO estimators, the proposed algorithms improve the performance, in terms of mean-squared error (MSE), by 6 % at an SNR of 2 dB and by 100 % at an SNR of -5dB.

1. INTRODUCTION

Initial time and carrier frequency offset (CFO) estimation are important tasks in every communication system. In particular, OFDM is sensitive to CFO as shown in [1].

CFO estimators can be generally divided into data-aided (DA), and non-data-aided (NDA) or blind estimators. DA estimators make use of a frame structure that typically includes a preamble or pilots to perform time and/or frequency synchronization [2, 3, 4]. The NDA estimators, on the other hand, exploit the available statistical information of the transmitted and received signals to perform the estimation.

Although NDA estimators are bandwidth efficient, they require large amount of data in general. Therefore, they are not appropriate for acquisition, because processing a large number of samples would require longer time than DA techniques [5, 6]. Advanced wireless communication systems for frequency selective channels generally employ DA synchronization [6, 5, 7]. Therefore, development of DA estimators for initial synchronization remains an important topic. Avoiding the high complexity of maximum likelihood estimators (MLE), suboptimal DA estimators typically employ a training sequence (TS) divided into J identical parts. These solutions are based on the autocorrelation function which provides $J-1$ lags. In [2] an estimator is derived that employs two TSs. Later in [8], Morelli and Mengali proposed an improved estimator, the Morelli and Mengali (MM) estimator, that uses only one TS with a repetitive pattern. To avoid a range loss, the algorithm combines constant phase differences using the best linear unbiased estimator (BLUE) to obtain the CFO. The MM estimator is not able to exploit the information of autocorrelation (AC) lags greater than $J/2$. This fact degrades the performance of the algorithm. In [9], Minn (MTB) proposes a two-step procedure to remedy that. First, a large-range coarse CFO estimation is subtracted from the received data to obtain a low-range residual unknown CFO. Then, a BLUE combines the information of the $J-1$ AC lags in order to obtain the residual CFO. As shown in the following sections, despite that this strategy allows the use of the information associated to all autocorrelation lags, the performance for low SNRs can be improved considerably.

This paper presents three novel CFO estimators based on the cyclostationary properties of the periodic TS. This new approach introduces a generalization of the classical AC, named averaged cyclic autocorrelation (ACA), which is statistically different from AC and leads to novel approximations. The first estimator is a reformulation of MM algorithm which is able to employ the $J-1$ correlation lags. The other two, take advantage of the ACA formulation to perform an accurate CFO estimation of low range, using the BLUE.

This work is organized as follows. Section 2 introduces the signal model and the notation used in the paper. MM and MTB algorithms are described in Section 2.1. Section 3 presents the derivation of the novel CFO estimators, formulated in terms of the new ACA. Finally simulation results and performance comparison with state-of-the-art estimators are presented in Section 4.

2. SIGNAL MODEL AND PREVIOUS APPROACHES

We consider an OFDM system, where $q(n) = x(n)*h(n)$ (i.e., the convolution of the M-periodic training data $x(n)$ of length N, and the channel $h(n)$ of length L) is an M-periodic signal. The number of periods of $q(n)$ is $J = N/M$. It is useful to define $s(p) = q(p)$, for $0 \leq p \leq M-1$, i.e., one period of $q(n)$. A cyclic prefix (CP) of length N_{CP} is inserted before the training sequence and later removed at the receiver. The received signal becomes

$$y(n) = e^{j\frac{2\pi\epsilon}{N}n} q(n) + w(n) \tag{1}$$

where $w(n)$ is Gaussian noise and ϵ is the CFO normalized by the subcarrier spacing.

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A *quasi-synchronous scenario* [10] is assumed to decouple the time offset (TO) and the CFO estimation problems. This implies that the CP is assumed long enough to assure that the received signal is periodic in spite of the TO and the channel length.

2.1 Previous Approaches

CFO estimation is a nonlinear parameter estimation problem. As a consequence, low complexity algorithms for CFO estimation follow three basic steps: an indirect measure of the CFO (usually the phase of the autocorrelation); a differential phase computation to reduce ambiguity; and finally, a linear combination of these phase differences (usually performed using the best linear unbiased estimator, BLUE).

Following these steps, MM and MTB CFO estimation algorithms are based on the *sample autocorrelation* (AC) defined as

$$\hat{r}(k) = \frac{1}{N-kM}\sum_{n=kM}^{N-1} y(n)y^*(n-kM),\ 1 \le k \le J-1 \tag{2}$$

Substituting (1) in (2), results in

$$\hat{r}(k) = e^{\frac{j2\pi\epsilon k}{J}}\chi(k),\ 1 \le k \le J-1 \tag{3}$$

where $\chi(k)$ depends on $q(n)$ and $w(n)$, see [9] for further details. We note from (3) that the CFO information is divided in the $J-1$ components of $\hat{r}(k)$. Defining

$$\theta(k) \quad = \quad \arg\{\hat{r}(k)\} = \frac{2\pi\epsilon k}{J} + \arg\{\chi(k)\} \tag{4}$$

where $1 \le k \le J-1$, and considering $|\epsilon| < J/(2k)$, the function

$$\zeta(k) \quad = \quad \frac{J}{2\pi k}\theta(k) = \epsilon + \frac{J}{2\pi k}\arg\{\chi(p)\} \tag{5}$$

for $1 \le k \le J-1$, gives an estimate of ϵ. We see from (5) that the estimation range varies with k, and it is not possible to directly combine the available information without reducing the estimation range.

To avoid the range reduction the MM algorithm form the CFO estimate by properly combining the phase differences of contiguous samples of $\hat{r}(k)$. That is, we first form a set phase differences as

$$\phi(k) \quad = \quad [\theta(k) - \theta(k-1)]_{2\pi} = \frac{2\pi\epsilon}{J} + \gamma(k) \tag{6}$$

for $1 \le k \le J/2$, where $\gamma(k)$ is a function which depends on $q(n)$ and $w(n)$ (for further details see [8]) and $\phi(0) = 0$. We see from (6) that the estimation range is restricted by $|\epsilon| < J/2$. Finally, assuming a high SNR regime, the BLUE is applied to $\phi(k)$, to obtain the CFO estimate. Since the covariance matrix of $\phi(k)$ derived in [8], is singular for $k > J/2$, it is not possible to employ all available information.

The MTB algorithm estimates the CFO using the function $\zeta(k)$, defined in (5). In order to extend the range, $\zeta(1)$ is first used as a coarse CFO estimate. Thereafter, the received signal is compensated using the coarse estimation as $\tilde{y}(n) = \exp(-j2\pi\zeta(1)n/N)y(n)$. Replacing $y(n)$ by $\tilde{y}(n)$ in (2) and (5) the angles $\tilde{\zeta}(k)$ are obtained corresponding to the residual CFO, i.e. $\zeta(k) = \zeta(1) + \tilde{\zeta}(k)$ for $2 \le k \le J-1$. Finally, considering also a high SNR regime, the BLUE combines $\zeta(k)$ to estimate CFO. In [9], the authors present three covariance matrices for $\zeta(k)$, corresponding to different approaches. Two of them employ all available information ($J-1$ phases), the other one being singular for $k > J/2$, as in the MM algorithm.

In the following sections we elaborate on low complexity schemes for CFO estimation. The proposed estimators also base the estimation in the aforementioned three basic steps, i.e. autocorrelation, differential phase and linear combination, but they employ the ACA instead of the classic AC. The differences between them lie in the technique used to combine the information. The CFO estimators proposed in the following offer different tradeoffs between estimation accuracy and computational load.

3. NEW CFO ESTIMATION ALGORITHMS

The proposed estimators are based on the ACA, defined as

$$\hat{r}_c(p,k) = \frac{1}{J-k}\sum_{n=0}^{J-k-1} y(nM+p)y^*([n+k]M+p) \tag{7}$$

From definitions of the AC in (2) and the ACA in (7) we may conclude that

$$\hat{r}(k) = \frac{1}{M}\sum_{p=0}^{M-1}\hat{r}_c(p,k) \tag{8}$$

Substituting (1) into (7) and applying the *central limit theorem*, the ACA for a large J may be approximated by

$$\hat{r}_c(p,k) \approx e^{-j\frac{2\pi\epsilon k}{J}}|s(p)|^2 + w_c(p,k) \tag{9}$$

where $w_c(p,k)$ is a zero-mean complex joint Gaussian process for $0 \le p \le M-1$ and fixed k; or for $1 \le k \le J-1$ and fixed p. As can be noted from (9), the CFO can be estimated using the ACA as long as $|\epsilon| < J/(2k)$.

We note that the combination over p is inherent to the ACA and can be considered as an additional degree of freedom with respect to the classical AC function (2). On the other hand, the combination over k can be used with any of the classical techniques to obtain full range.

In the following we present three alternative low complexity CFO estimators based on the ACA.

3.1 Sum-based CFO estimator (SBE)

The first low complexity estimator consists of the following steps

- **Step 1**: Considering (9), the combination over p is given by

$$\xi(k) = \arg\left\{\sum_{p=0}^{M-1}\hat{r}_c(p,k)\right\} \tag{10}$$

where $\xi(k)$ is the partial CFO estimate for k.

- **Step 2**: From the relationship between the AC and the ACA of (8) it follows that (10) is equivalent to (4) for the same received signal $y(n)$. To avoid a range reduction, we take the phase differences of adjacent samples of sequence $\xi(k)$. For this purpose we define

$$\boldsymbol{\xi}_d = [\xi_d(1), \cdots, \xi_d(J-1)]^{\mathrm{T}}, \text{ where} \quad (11)$$
$$\xi_d(k) = [\xi(k) - \xi(k-1)]_{2\pi} \quad (12)$$

for $1 \le k \le J-1$, where $[\cdot]_{2\pi}$ is the modulo-2π operation and we also define $\xi(0) = 0$.
- **Step 3**: Finally, considering the approximate weights

$$a(k) = \frac{-J}{2\pi} \frac{(J-k)^2}{\sum_{k=1}^{J-1}(J-k)^2} = \frac{-1}{2\pi} \frac{6(J-k)^2}{(J-1)(2J-1)} \quad (13)$$

it is possible to find the CFO SBE as

$$\hat{\epsilon}_s = \mathbf{a}^{\mathrm{T}} \boldsymbol{\xi}_d \quad (14)$$

where $\mathbf{a} = [a(1) \cdots a(J-1)]^{\mathrm{T}}$.

Remarks:

- The third step is the main difference with the MM algorithm. The weighting factors in (14) employ $J-1$ autocorrelation lags, while the MM algorithm only employs $J/2$ lags. This leads to an improvement in the performance.
- The range of the algorithm is $J/2$, as noted from (6).
- In the derivation of the algorithm is assumed a high SNR regime.
- Approximate weighting factors are based on the assumption that the phase differences in (12) are independent. From [11], it is known that if samples of a linear statistical model are independent, the weights to obtain the parameter estimation are proportional to the normalized energy of each sample. Since $J-k$ in (7) is the normalization coefficient for the ACA, the selection of $a(k)$ is natural. Extensive simulations confirm that these weights lead to good results.

3.2 Direct combining CFO estimators (DCE)

For the remaining estimators to be introduced we consider $\alpha(p,k) = \arg\{\hat{r}_c(p,k)\}$. After some straightforward manipulation and assuming a high SNR, we obtain

$$\alpha(p,k) \approx -\frac{2\pi\epsilon k}{J} + \frac{\mathrm{Im}\{w_c(p,k)e^{\frac{j2\pi\epsilon k}{J}}\}}{|s(p)|^2} \approx -\frac{2\pi\epsilon k}{J} + w_\alpha(p,k) \quad (15)$$

where $w_\alpha(p,k)$ is a zero-mean real joint Gaussian process for $0 \le p \le M-1$ and fixed k; or for $1 \le k \le J-1$ and fixed p.

From (15) we can infer that the CFO estimate $\hat{\epsilon}$ can be obtained as long as $|\epsilon| < J/(2k)$. The linear dependence of ϵ in (15) suggests the application of the BLUE to combine the information contained in p and k to find the CFO estimation. Furthermore, since for high SNR $\alpha(p,k)$ is approximately Gaussian, the BLUE approximates well the MLE.

Depending on which order the indices of $\alpha(p,k)$ are processed (first combination over p, or first combination over k), we obtain two different algorithms, as discussed in the following.

3.2.1 *Direct combining estimator A (DCE-A)*

Step 1: For a fixed k, $\{\alpha(p,k)\}_{p=0}^{M-1}$ are statistically independent. Therefore, the BLUE weights that combine the information over p result in

$$f(p,k) = \frac{|s(p)|^2}{(J-k)(1+\sigma^2/(2|s(p)|^2)) - (J-2k)},$$
$$g(p,k) = \frac{|s(p)|^2}{(J-k)(1+\sigma^2/(2|s(p)|^2))},$$
$$K(k) = \begin{cases} \sum_{p=0}^{M-1} f(p,k) & \text{if } k < J/2 \\ \sum_{p=0}^{M-1} g(p,k) & \text{if } k \ge J/2 \end{cases},$$
$$b(p,k) = \frac{1}{K(k)} \begin{cases} f(p,k) & \text{if } k < J/2 \\ g(p,k) & \text{if } k \ge J/2 \end{cases},$$
$$\mathbf{b}(k) = [b(0,k), \cdots, b(M-1,k)]^{\mathrm{T}}, \quad (16)$$

Then, arranging the phases $\alpha(p,k)$ into the vector

$$\boldsymbol{\alpha}(k) = [\alpha(0,k), \ldots, \alpha(M-1,k)]^{\mathrm{T}}, \quad (17)$$

we can combine p components as

$$\lambda(k) = \mathbf{b}^{\mathrm{T}}(k)\boldsymbol{\alpha}(k) \quad (18)$$

Step 2: To avoid the range reduction, following MM approach, we define the vector

$$\boldsymbol{\lambda}_d = [\lambda_d(1), \cdots, \lambda_d(J-1)]^{\mathrm{T}}, \text{ where} \quad (19)$$
$$\lambda_d(k) = [\lambda(k) - \lambda(k-1)]_{2\pi} \quad (20)$$

and $\lambda(0) = 0$.

Step 3: In order to obtain the DCE-A, the CFO estimates $\lambda_d(k)$ are combined using the approximate weighting factors defined in (13) as

$$\hat{\epsilon}_a = \mathbf{a}^{\mathrm{T}} \boldsymbol{\lambda}_d \quad (21)$$

Remarks:

- Equation (18) employs a different combination for p than SBE in (10) and MM in (4). Like SBE, DCE-A also employs $J-1$ lags, which is possible due to the introduction of the ACA providing an additional degree of freedom (the index p).
- DCE-A depends on $|s(p)|^2$ but because of the unknown channel, this variable is not available. However, as noted from (9), $|s(p)|^2$ for a given k can be estimated as $|\hat{r}_c(p,k)|$.
- The angles resulting from $\arg\{\cdot\}$ lie between $\pm\pi$. As a consequence, there is a phase discontinuity in π. It is seen from (15) that for some ϵ and k, the elements of $\alpha(p,k)$ for $0 \le p \le M-1$ are close to the phase discontinuity. Due to the noise, these values may fall into different sides of the discontinuity. In these cases, the estimate becomes incorrect, because the derivation of the weighting factors assumes that the phase is unwrapped and not circular. For example, if $M = 2$ and phase estimates for some k are $-\pi + \delta$ and $\pi - \delta$ (with $0 < \delta < \pi/2$), with both weights $1/2$, the direct result of the weighting is 0, whereas it should be π. If the CFO is not large, errors due to discontinuities are unlikely and DCE-A works properly.

To avoid the phase-discontinuity problem that arises from the application of the weighting factors in Step 1, the next section introduces another direct combining estimator that invert the order in the combination of p and k.

3.2.2 Direct combining estimator B (DCE-B)

Step 1: Let us define the following phase differences for a fixed p

$$\psi(p,k) = \alpha(p,k) - \alpha(p,k-1) \tag{22}$$

where $\alpha(p,0) = 0$ for $0 \leq p \leq M-1$.
Step 2: Again using the approximate weighting factors defined in (13), the combination over k results in

$$\psi_p(p) = \mathbf{a}^{\mathrm{T}}\boldsymbol{\psi}(p) \tag{23}$$

where $\boldsymbol{\psi}(p) = [\psi(p,1), \cdots, \psi(p,J-1)]^{\mathrm{T}}$.
Step 3: Considering (3) and noting that the components of $\psi_p(p)$ are independent for different p (because $\alpha(p,k)$ are independent), it is possible to define the following approximate weighting factors

$$c(p) = \frac{|s(p)|^2}{\sum_{p=0}^{M-1}|s(p)|^2} \tag{24}$$

By grouping $\psi_p(p)$ in vector $\boldsymbol{\psi}_p = [\psi_p(0), \cdots, \psi_p(M-1)]^{\mathrm{T}}$ and $c(p)$ in $\mathbf{c} = [c(0), \cdots, c(M-1)]^{\mathrm{T}}$, the final CFO estimate becomes

$$\hat{\epsilon}_b = \mathbf{c}^{\mathrm{T}}\boldsymbol{\psi}_p \tag{25}$$

Remarks:

- Combination over k is not as robust as in DCE-A or SBE, because it depends on a single component p. This can be seen by comparing (23) with (14) and (21). This is particularly notorious for large CFO. As a consequence DCE-B works properly only for small CFO.
- DCE-B also depends on $|s(p)|^2$, but in this case is more appropriate to estimate it as $1/(J-1)\sum_{k=1}^{J-1}|\hat{r}_c(p,k)|$.

4. SIMULATIONS

In this section the performances of the proposed CFO estimators are evaluated and compared with those of the MM and the MTB algorithms. We employ the same sequence to evaluate each method. The MTB uses Method B described in [9]. As a benchmark we consider the Cramer-Rao lower bound (CRLB) for CFO estimation employing a periodic TS [3].

Although (16) reveals that DCE-A algorithms depends on σ^2, extensive simulations show that the dependence on this value has a small effect in the weighting factors. Therefore, the simulations assume that σ^2 is perfectly known.

The training sequence has $N = 64$ samples and period $M = 8$. The length of the cyclic prefix is 16, and the channel taps are given by $\{h_k(l)\}_{l=0}^{L}$, $L = 10$, with exponential decay profile $E\{|h_k(l)|^2\} = Ge^{-l/\gamma}$, where G is chosen such that $\sum_{l=0}^{L} E\{|h_k(l)|^2\} = 1$ and $\gamma = 5$. The CFO estimates are averaged over $N_c = 100$ different channel realizations, and each channel realization is averaged over $N_n = 100$ noise realizations for each SNR.

Figure 1 shows the performance of the proposed algorithms for low CFO. It can be noted that DCE-A and DCE-B have the best performance for low SNR. SBE works better than MM for any SNR but especially for low values. At high SNR (> 0 dB), DCE, SBE and MTB have the same performance and all are better than MM. This is because MM does not employ all available information. All algorithms converge to the CRLB for high SNR.

The performance of SBE and DCE algorithms demonstrates that it is possible to employ the information available for correlation lags greater than $J/2$ for the differential phase method proposed in [8]. This refutes the hypothesis of [3] that states there is no extra information in these terms.

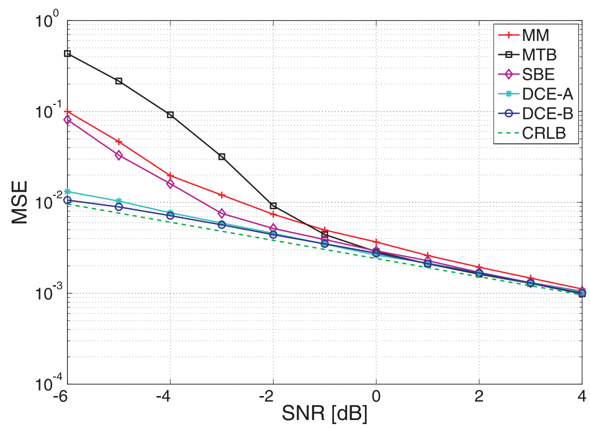

Figure 1: MSE versus SNR for $\epsilon = 0.1$.

Figure 2 illustrates the behavior for a large CFO. Although the DCE based estimators are tailored for low CFO, they still outperform the MM algorithm for any SNR, and the MTB algorithm for low SNR (< -1dB). Also in this case, SBE outperforms MM. On the other hand, CFO levels remain moderate in practical wireless communication systems. and the reduced range of DCE-based estimators should not pose a problem. For example, LTE [12] requires 1 ppm accuracy from the oscillators. Together with typical mobile speeds this gives rise to CFO levels on the order of hundreds of Hz while subcarrier spacing is 15 kHz.

In order to give a qualitative measure of the performance improvement obtained with the proposed estimators, Table 1 shows the MSE of each method relative to the CRLB, for SNRs of -5dB and 2dB; and a $\epsilon = 0.1$. The relative MSE is defined as $MSE_{\mathrm{r}} = Q/Q_{CR}$, where Q and Q_{CR} are respectively the MSE of the evaluated method and the MSE of CRLB at specified SNR.

4.1 Computational load

To illustrate the complexity of the algorithms, Table 2 shows the approximate number of real multiplications needed to obtain an estimate for each algorithm. The

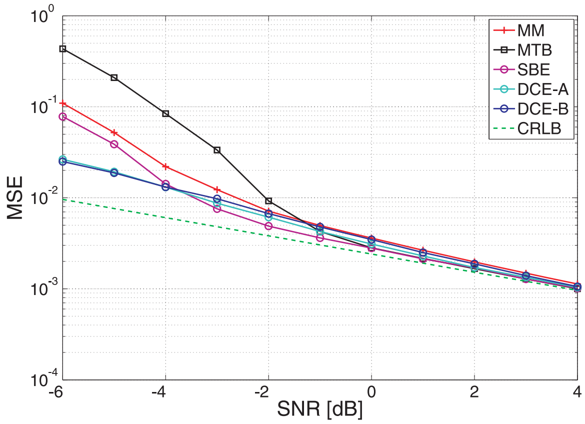

Figure 2: MSE versus SNR for $\epsilon = 0.3$.

Table 1: MSE relative to the CRLB

Estimator	MSE_r at -5dB	MSE_r at 2dB
MM	609 %	127 %
MTB	2830%	109 %
SBE	434 %	109 %
DCE-A	135 %	106 %
DCE-B	117 %	106 %

complexity calculation assumes that a complex multiplication is equivalent to four real multiplications and a real division is considered as one real multiplication. $N_b = J - 1$ is the number of correlation lags.

Table 2: Complexity comparison

Estimator	No. of real multiplications	$M = J = 8$
MM	$2J(3/4N - M/2)$	700
MTB	$N_b^3/3 + 2MN_b^2 + 4JN_b + 4N$	1400
SBE	$2N_b(N+1)$	900
DCE-A	$N_b(2N + 6M)$	1200
DCE-B	$N_b(2N + M) + 3M$	1000

Although the MM algorithm needs approximately 20% less operations than SBE, it only employs half of the available AC coefficients ($J/2$). The complexity of MTB is high because it needs matrix inversion [9].

5. CONCLUSIONS

We derived a family of novel carrier frequency offset (CFO) estimators, the sum-based CFO estimator (SBE) and two BLUE-based CFO estimators (DCE). The algorithms are based on a new interpretation of the cyclic autocorrelation function of the received training sequence.

SBE is a generalization of the classic estimator proposed by Morelli and Mengali and performs better for low and high SNRs. DCE algorithms are derived from the structure of the new cyclic autocorrelation. Although they provide an important improvement in the performance, they are not robust for large CFO. The proposed algorithms attain a better performance since they employ all autocorrelation lags, contrary to Morelli and Mengali algorithm. Even when more information is used, the computational complexity remains comparable.

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