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AN ADAPTIVE MODULATION PRECODING SCHEME FOR MEAN BER MINIMISATION OF ISI MIMO CHANNELS

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ABSTRACT

This paper considers a non-linear precoding scheme with adaptive bit loading in order to minimise the average BER for a given target data rate when transmitting over a frequency-selective (FS) multiple-input multiple output (MIMO) channel. The proposed design utilises a recently developed polynomial singular value decomposition (PSVD) to decouple the FS MIMO channel into a number of independent FS single input single output (SISO) subchannels, whereby spectral majorisation inherent on the PSVD leads to a natural ordering of channel gains. The latter is exploited to adaptively allocated the transmission bits to the resulting FS SISO subchannels according to their SNRs. Simulation results show that BER performance can be enhanced for different data rate targets compared to a previously proposed heuristic method as well as to uniform bit loading.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) technology has received considerable attention due to the promise of a tremendous increase in system capacity when utilising the spatial dimension [1, 2]. This has motivated the growth of high data-rate applications in commercial wireless communication systems. The communication channels of these systems cannot be considered as a frequency-flat channel and are usually referred to as wideband, broadband, or frequency selectice (FS), hence incurring inter-symbol-interference (ISI) in the temporal dimension along with co-channel-interference (CCI) in the spatial dimension. In order to realise the anticipated high capacity gain of MIMO broadband systems, sophisticated transceiver designs have been developed [3, 4, 5, 6, 7, 8] to combats these interferences while not admit redundancy in transmission, as for example found wideband solutions based on orthogonal frequency division multiplexing or other block-based approaches [9, 10].

Typical non-block based precoding/equalisation approaches include decision feedback equalisation (DFE) [6], V-BLAST approaches [3] adopted for the broadband case or a mixture thereof [7], as well as Tomlinson-Harashima precoding (THP) [4, 5]. In [8] a non-block based approach, which is based on a generalisation of the singular value decomposition (SVD) to the broadband case, namely polynomial SVD (PSVD) [11] technique is proposed. The PSVD decomposition is achieved by an iterative algorithm to approximately transfer the energy of the channel matrix onto the main diagonal and eliminate the off-diagonal elements of the ISI MIMO channel. This approach is applied to decouple the broadband MIMO system into a number of frequency selective (FS) single-input single-output (SISO) subchannels of ordered qualities. These broadband SISO subchannels are individually equalised using non-linear THP whereby the decision delay can be independently optimised for every subchannel [8].

The bit loading optimisation problem is widely considered in the literature for different scenarios [12, 13, 14, 15]. However, most of the reported work focusses only on flat-fading channels and/or linear precoding/equalisation approaches. Therefore in this paper we are interested in FS MIMO channels with non-linear approaches. Different from our previous work presented in [16], which assumes a heuristic bit loading, the work presented here adaptively allocates the transmission bits over the resulting FS SISO subchannels according to their SNR conditions and aims to minimise the mean BER under the constraints of a fixed modulation and specified target data rate.

The rest of this paper is organised as follows. In Sec. 2, the broadband MIMO system model is presented and the PSVD algorithm to mitigate CCI is first reviewed followed by the formulation of the mean BER optimisation problem highlighted by ISI mitigation using THP. Bit loading schemes to minimise the mean BER while achieving a target data rate are proposed in Sec. 3. Simulation results are conducted in Sec. 4 to investigate the mean BER performances of the proposed bit loading schemes while conclusion is drawn in Sec. 5.

2. SYSTEM MODEL, DECOUPLING, AND PRECODING

Below, we first review the description of an FS MIMO channel by means for a polynomial matrix, which can be decoupled for CCI cancellation, requiring a THP scheme to remove ISI from the remaining FS SISO subchannels.

2.1 Broadband MIMO System Description

A broadband MIMO system of N_t transmit and N_r receive antennas is considered in this paper. This type of system is often described by a MIMO FS channel with finite impulse response (FIR) which incurs both CCI in the spatial dimension and ISI in the time coordinate such that

$$\mathbf{H}(z) = \sum_{l=0}^{L} \mathbf{H}_l z^{-l} , \tag{1}$$

where L is the channel order and $\mathbf{H}_l$ is an $N_r \times N_t$ matrix representing the lth matrix valued channel coefficient, with $h_{ij}[l]$ being the complex baseband channel coefficient from the jth transmit antenna to the ith receive antenna at time index l.

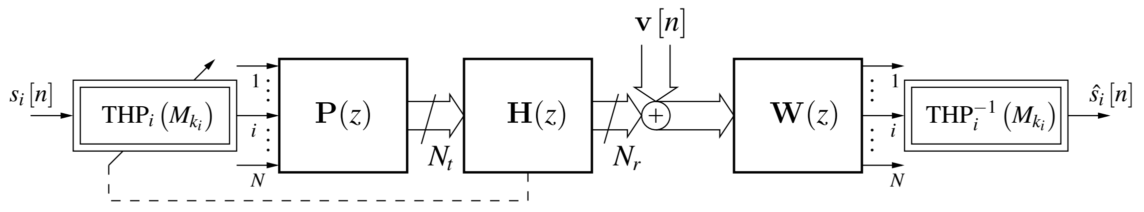

Figure 1: System model of a broadband MIMO channel $\mathbf{H}(z)$ precoding scheme with adaptive bit loading.

2.2 Polynomial Singular Value Decomposition

The ISI MIMO channel matrix $\mathbf{H}(z)$ in (1) can be decoupled into $N = \min(N_t, N_r)$ FS SISO subchannels by resorting to the PSVD algorithm detailed in [11, 17] which leads to the decomposition

$$\mathbf{H}(z) = \mathbf{U}(z)\mathbf{\Sigma}(z)\tilde{\mathbf{V}}(z), \tag{2}$$

where $\mathbf{U}(z) \in \mathbb{C}^{N_r \times N_r}(z)$ and $\mathbf{V}(z) \in \mathbb{C}^{N_t \times N_t}(z)$ are paraunitary [1] matrices and $\mathbf{\Sigma}(z) \in \mathbb{C}^{N_r \times N_t}(z)$ is a diagonalised and spectrally majorised matrix containing the FS SISO subchannels in the main diagonal such as

$$\mathbf{\Sigma}(z) = \mathrm{diag}\left\{\mathbf{\Sigma}_1(z), \mathbf{\Sigma}_2(z), \cdots \mathbf{\Sigma}_N(z)\right\}. \tag{3}$$

The spectral majorisation property of the PSVD algorithm [11] guarantees the SISO subchannels ordering

$$\mathbf{\Sigma}_1(e^{j\Omega}) \geq \mathbf{\Sigma}_2(e^{j\Omega}) \geq \cdots \geq \mathbf{\Sigma}_N(e^{j\Omega})\ \forall\Omega. \tag{4}$$

Since $\mathbf{U}(z)$ and $\mathbf{V}(z)$ are paraunitary matrices, the diagonalised system $\mathbf{\Sigma}(z)$ can easily be obtained between the end-to-end transmission system shown in Fig. 1 by applying a lossless precoder $\mathbf{P}(z)$ and equaliser $\mathbf{W}(z)$ defined respectively as [8]

$$\begin{aligned}
\mathbf{P}(z) &= \left[\ \underline{V}_1(z) \quad \underline{V}_2(z) \quad \cdots \quad \underline{V}_N(z)\ \right], && (5)\\
\mathbf{W}(z) &= \begin{bmatrix} \underline{\tilde{U}}_1(z) \\ \underline{\tilde{U}}_2(z) \\ \vdots \\ \underline{\tilde{U}}_N(z) \end{bmatrix}, && (6)
\end{aligned}$$

where the components $\underline{V}_n(z)$ and $\underline{\tilde{U}}_n(z)$ are the nth right and left singular vectors belonging to the matrices $\tilde{\mathbf{V}}(z)$ and $\mathbf{U}(z)$ in (2).

An illustrative example of the PSVD decomposition in (2) which leads to the diagonalised system in (3) with a spectrally majorised SISO subchannels in (4) is shown in Fig. 2. The original channel is a 4×4 MIMO system with 11-taps FS channel impulse responses derived from a Saleh-Valenzuela indoor channel model [18] with an exponentially decaying power delay profile.

The independent SISO subchannel gains $g_i, 1 \leq i \leq N$ can be deduced according to the Parseval's theorem as

$$g_i = \sum_{n=-\infty}^{\infty} |\sigma_i[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|\mathbf{\Sigma}_i(e^{j\Omega})\right|^2 d\Omega, \tag{7}$$

and the channel-to-noise ratio (CNR) of the ith subchannel can be defined as

$$\mathrm{CNR}_i = \frac{g_i}{\mathscr{N}_0}, \tag{8}$$

where $\mathscr{N}_0$ stands for the total noise power at the receiver. By allocating the transmit power budget P_{budget} equally among all subchannels, the subchannel SNRs γ_i are therefore

$$\gamma_i = P_i \times \mathrm{CNR}_i = \frac{P_{\text{budget}}}{N} \times \mathrm{CNR}_i, \tag{9}$$

where $P_i = {}^{P_{\text{budget}}}\!/_{N}$ is the amount of power allocated to the ith subchannel.

2.3 Mean BER Optimisation

The SISO subchannels resulting from the MIMO system decomposition via the PSVD are dispersive and independently incur ISI which we aim to mitigate using THP. Therefore, below we refer to this system as PSVD-THP. Fig. 1 shows the transceiver system model of the PSVD-THP, where $\mathbf{v}[n] \sim \mathscr{CN}(0, \mathscr{N}_0)$ is additive white Gaussian noise (AWGN) at the receiver. Assuming perfect diagonalisation in (2) and consequently complete suppression of CCI, the role of the THP block $\mathrm{THP}_i\left(M_{k_i}\right)$ is to remove ISI occurred by the ith SISO subchannel, while adaptively load it with $b_i = \log_2 M_{k_i}$ bits. An M-ary QAM transmission of order M_{k_i} is assumed , where $k_i, 0 \leq k_i \leq K$ is an integer denoting the index of the QAM modulation order M assigned to the ith subchannel. This, of course, requires the knowledge of CSI at the transmit side which has to be supplied by the receiver through a feedback channel (cf. Fig. 1) for an FDD channel or by assuming reciprocity for TDD channels [5]. Logically, strong subchannels will be loaded with higher QAM orders, while weak subchannels would be expected to support lower-order QAM transmission or even be switched off, leaving inactive subchannels. For more details of SISO-THP, please refer to [8, 16].

In this paper, we are interested in achieving a target data rate R while minimising the mean BER $\overline{\mathscr{P}}_b$ of the overall transmission. This problem can be formulated as a constrained optimisation problem

$$\begin{aligned}
&\text{minimise} && \overline{\mathscr{P}}_b = \frac{\sum_{i=1}^{N} b_i \mathscr{P}_{b,i}}{\sum_{i=1}^{N} b_i} && \text{(10a)}\\
&\text{subject to} && \begin{cases} \sum_{i=1}^{N} b_i = R, \\ 0 \leq b_i \leq b^{\max}, 1 \leq i \leq N, \end{cases} && \text{(10b)}
\end{aligned}$$

where b_i and $\mathscr{P}_{b,i}$ are, respectively, the number of bits and BER of the ith subchannel, while, for feasibility, $b^{\max} = \log_2 M_K$ is the maximum permissible number of bits per subchannel.

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[1] A paraunitary matrix $\mathbf{A}$ is defined as $\mathbf{A}(z)\tilde{\mathbf{A}}(z) = \tilde{\mathbf{A}}(z)\mathbf{A}(z) = \mathbf{I}$, where $\tilde{(\cdot)}$ denotes parahermitian operator defined as $\tilde{\mathbf{A}}(z) = \mathbf{A}^{\mathrm{H}}\left(z^{-1}\right)$.

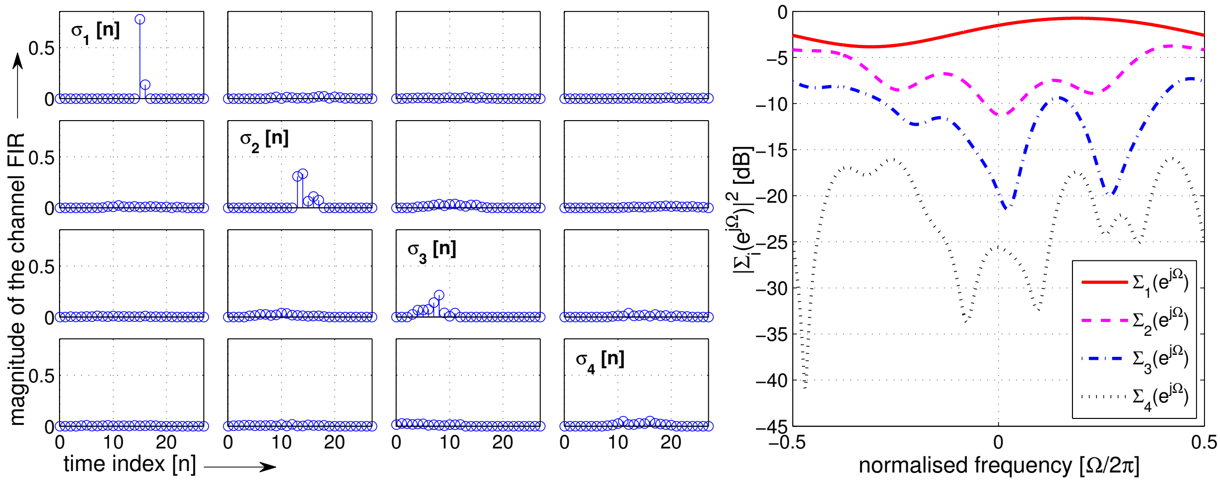

Figure 2: An illustrative example of the PSVD algorithm applied to a broadband 4 × 4 MIMO system where $\sigma_i[n], i = 1\cdots 4$ is shown to the left and its spectral majorisation property is shown to the right.

The theoretical BER $\mathscr{P}_{b,i}$ in (10a) of M-ary QAM modulation is given by [19]

$$\mathscr{P}_{b,i} = \mathscr{F}\left(\gamma_i, M_{k_i}\right) = \begin{cases} Q\left(\sqrt{2\gamma_i}\right) & \text{for BPSK}, \\ \dfrac{1-\left[1-2\left(1-\frac{1}{\sqrt{M_{k_i}}}\right)Q\left(\sqrt{\frac{3\gamma_i}{M_{k_i}-1}}\right)\right]^2}{\log_2 M_{k_i}} & \text{for } M_{k_i} \text{ QAM}, \end{cases} \tag{11}$$

where γ_i is the ith subchannel SNR in (9) and Q is the well-known complementary error function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du\,. \tag{12}$$

In the next section, different bit loading schemes are considered to solve the above optimisation problem and find the QAM order M_{k_i} and the allocated bit b_i for each subchannel i, $1 \leq i \leq N$.

3. BIT LOADING SCHEMES

3.1 Uniform Bit Loading (UBL)

To solve the optimisation problem in (10a) with the constraints in (10b), the easiest and most straightforward method is to uniformly distribute the total target bit rate R across all subchannels $i, 1 \leq i \leq N$, i.e.,

$$b_i = R/N\,. \tag{13}$$

The individual BERs are therefore computed for $M_{k_i} = 2^{R/N}\ \forall i$ leading to a mean BER of

$$\overline{\mathscr{P}}_b^{\mathrm{u}} = \frac{\sum_{i=1}^{N} \mathscr{P}_{b,i}}{N}\,. \tag{14}$$

Obviously, the performance of mean BER in (14) is dominated by the worst $\mathscr{P}_{b,i}$ of the poorest subchannel. This is expected due to the equal allocation of the number of bits b_i in (13) regardless of the subchannel gains γ_i. Moreover, to achieve the target bit rate R, b_i has to be an integer for practical modulation schemes, otherwise $b_i = \lfloor R/N \rfloor + c$, where c is a constant $\in \{0, 1\}$. This results in either violating the rate constraint in (10b) for $c = 0$ or even obtaining worse performance of $\overline{\mathscr{P}}_b^{\mathrm{u}}$ for $c = 1$.

Table 1: Heuristic bit loading for different data rates of the PSVD-THP system.

data rate R	8 bits	16 bits	24 bits
subchannel # 1	16-QAM	64-QAM	256-QAM
subchannel # 2	QPSK	64-QAM	256-QAM
subchannel # 3	QPSK	16-QAM	64-QAM
subchannel # 4	“off”	“off”	QPSK

3.2 Heuristic Bit Loading (HBL)

In order to obtain better mean BER performance than the UBL scheme, the SISO subchannel qualities should be taken into account. In [16] a bit loading scheme was proposed which matches the individual data rates of a prescribed target rate R across subchannels according to a heuristic fashion. This, in general, considers the allocation of transmission symbols of higher QAM orders to stronger subchannels while weak subchannels are loaded with lower QAM orders or even left inactive. Table 1 [16] shows a sample HBL scheme for a 4 SISO subchannel system resulting from a decoupled 4 × 4 MIMO system after applying the PSVD-THP algorithm [16].

Fixed allocated bits b_i are therefore assigned to the ith SISO subchannel according to Table 1 regardless of the CSI differences between various broadband MIMO channel realisations. The mean BER of this loading scheme is evaluated using

$$\overline{\mathscr{P}}_b^{\mathrm{h}} = \frac{\sum_{i=1}^{N} b_i \mathscr{P}_{b,i}}{\sum_{i=1}^{N} b_i}\,. \tag{15}$$

Table 2: Pseudo code of the ABL algorithm.

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Input: $M_k, 0 \leq k \leq K$, $b^{\max}$, and $\gamma_i, 1 \leq i \leq N$
Output: $b_i$ and $M_{k_i} = 2^{b_i}$

1.  Initiate all subchannels $i, 1 \leq i \leq N$ with
    $b_i = 0$, $k_i = 0$, and $\mathscr{P}_{b,i} = \mathscr{F}(\gamma_i, M_1)$
    using (11)
2.  while $\sum_{i=1}^{N} b_i < R$
3.      $j = \underset{1 \leq i \leq N}{\text{argmin}} \left(\mathscr{P}_{b,i}\right)$
4.      $k_j = k_j + 1$
5.      if $k_j = 1$
6.          $b_j = \log_2 M_1$, $\mathscr{P}_{b,j} = \mathscr{F}(\gamma_j, M_2)$
7.      elseif $k_j < K$
8.          $b_j = b_j + \log_2 \frac{M_{k_j}}{M_{k_j - 1}}$, $\mathscr{P}_{b,j} = \mathscr{F}\left(\gamma_j, M_{k_j + 1}\right)$
9.      else
10.         $b_j = b_j + \log_2 \frac{M_{k_j}}{M_{k_j - 1}}$, $\mathscr{P}_{b,j} = +\infty$
11.     end
12. end
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3.3 Adaptive Bit Loading (ABL)

In this section an iterative algorithm is proposed to adaptively allocate data bits across all subchannels to achieve the target rate constraint in (10b) while minimising the mean BER. Different from the HBL scheme, the ABL scheme adaptively allocates data bits according to the SISO subchannel SNRs γ_i. The ABL algorithm starts with computing $\mathscr{P}_{b,i}$ for all subchannels i if loaded with 1st QAM level of order M_1. Obviously, the strongest subchannel will demonstrate the best BER performance. Similar to the concept of the remaining SNR proposed in [20], the ABL algorithm proceeds with upgrading the strongest subchannel, which is associated with the minimum BER, to the next higher QAM level, M_2 in this case. At each iteration, a search for the subchannel that results in the minimum BER $\mathscr{P}_{b,i}$ if upgraded to the next higher QAM level is done (step 3 in Table 2). The algorithm continued until the sum of the allocated bits across all subchannels is equal to the target data rate R according to step 2 in Tab. 2.

The minimisation of the mean BER in the ABL algorithm is guaranteed due to the exhaustive search in step 3 of Tab. 2, which ensures that only subchannels capable of achieving the minimum BER are selected. Note that to keep the last constraint in (10b) of the permissible QAM levels, i.e. not to exceed the maximum order of $M_K = 2^{b^{\max}}$, $\mathscr{P}_{b,j}$ is set to infinity in step 10 of Tab. 2. The weighted mean BER is finally computed analogous to the HBL scheme in (15).

4. SIMULATION RESULTS

A 4×4 MIMO system with a normalised broadband channel of order $L = 5$ and exponentially decaying coefficients drawn from a complex Gaussian distribution is used to conducted the simulation below. Perfect CSI is assumed at both the transmit and receive sides of the communication link of Fig. 1. The PSVD-THP method [16] is applied to mitigate both CCI and ISI and obtaining an equivalent SISO subsystems with different subchannel SNRs. Bit loading schemes presented in Sec. 3 are applied to the resulting SISO subchannels aiming to minimise the mean BER and achieve a given target rate. Three different data rates of 8, 16, and 24 bits are considered to examine the proposed bit loading schemes. Square QAM modulation scheme is considered for transmitted symbols $s[n]$ with fixed modulation orders of $M = 2^b, b \in \{2, 4, 6, 8\}$ with $b^{\max} = 8$ bits. The results presented here are averaged over a considerably large number of channel realisations.

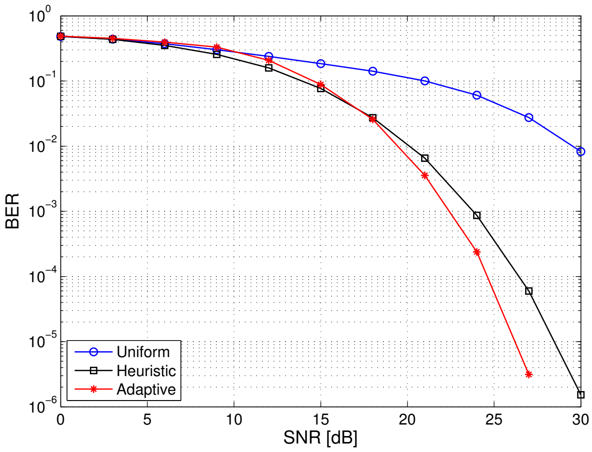

Figure 3: Mean BER performance of different bit loading schemes with a target data rate of $R = 8$ bits for a 4×4 broadband MIMO system after applying the PSVD-THP method.

Fig. 3 shows the mean BER performances of the all considered bit loading schemes for a target data rate of 8 bits. Obviously, UBL scheme shows a very inefficient mean BER performance due to the domination of the poorest subchannel (subchannel # 4) since all subchannels are loaded with 2 bits, i.e. QPSK transmission is considered for the UBL in this case. With the ABL scheme, an advantage of approximately 1.5 dB in SNR can be gained at BER $= 10^{-3}$ over the HBL proposed in [16]. This advantage reaches about 10% reduction in the SNR required for a mean BER performance of 10^{-5}. Higher data rate performances of 16 and 24 bits are shown in Fig. 4.

Both HBL and ABL achieve better mean BER compared to the UBL scheme, in particular in the high SNR region. Both schemes yield close performance, which can be attributed to the last constraint (10b) of allocating a maximum permissible number of bits to a subchannel with ABL. At higher data rate targets fewer degrees of freedom (DoF) are available for the ABL scheme to allocate bits across subchannels since all subchannels approach $b^{\max}$, particularly for $R = 24$ bits.

5. CONCLUSIONS

A broadband MIMO precoding scheme with adaptive bit loading has been proposed to minimise the mean BER performance under the constraints of a target data rate and a bounded QAM modulation scheme. The proposed scheme considers the polynomial singular value decomposition (PSVD) technique followed by a non-linear SISO-THP scheme to remove both CCI and ISI and obtain an equivalent

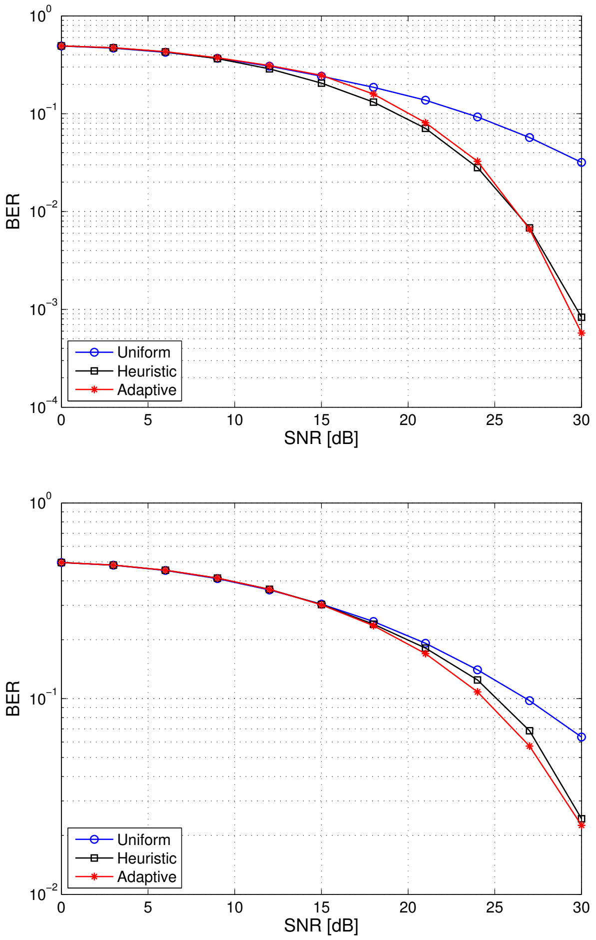

Figure 4: Mean BER performance of different bit loading schemes with a target data rate of (top) $R = 16$ bits and (bottom) $R = 24$ bits for a 4×4 broadband MIMO system after applying the PSVD-THP method.

ISI-free SISO subchannels system. The subsequent adaptive bit loading stage is based on the SISO subchannel SNRs, and aims to allocate bits across subchannels to satisfy the target data rate with a minimised mean BER. Simulation results show that better mean BER performance is achieved compared to uniform and heuristic bit loading approaches.

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