PARAMETRIC WAVEFORM DESIGN FOR IMPROVED TARGET DETECTION

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ABSTRACT

We address a parametric waveform design approach for improved detection of a Gaussian point target, which is embedded in signal-dependent clutter and noise. Unlike canonical waveform design schemes, the proposed transmit waveform is represented as a weighted sum of discrete prolate spheroidal sequences. In the optimization problem, the detection performance is maximized with respect to the weighting factors of the discrete prolate spheroidal sequence under the transmit energy constraint. The main benefit of the proposed approach is the direct acquisition of the optimal waveform rather than its energy spectral density. Simulation results demonstrate the merits and demerits of the parametric waveform design approach in contrast to a canonical approach. Furthermore, the superiority of the discrete prolate spheroidal sequences in parametric waveform modeling is exemplified.

1. INTRODUCTION

Optimal waveform design is a key technique in numerous engineering applications including radar, sonar, and communication systems. The ability to design the transmitted waveform in an optimal way strongly improves follow-on tasks such as target detection or classification and will play a key role in the future system design [1].

In modern radar systems, an optimal waveform is frequently used to meet the ever increasing demands of improved performance in target detection and identification. A manifold of work has been done in the past for this purpose. In [2], two optimal waveform design schemes for extended target detection in noise only environment are proposed. In the first approach, both target and noise are modeled as Gaussian random processes. Based on this model, the mutual information (MI) between the ensemble of target impulse response and the receiver filter output is maximized. Consequently, the waveform obtained is called MI-based waveform. In the second approach, the target is assumed to be deterministic. A signal-to-noise-ratio (SNR) criterion is then used for the optimal waveform design. The waveform obtained in this way is called SNR-based waveform. In [3], the authors generalized the SNR-based waveform design technique by taking signal-dependent interference into account, and an iterative solution was proposed.

There has been no analytical solution to the signal-dependent noise problem, until Kay proposed an optimal waveform design approach for the detection of a Gaussian point target in [4]. A Neyman-Pearson (NP) detector is constructed for optimal detection. The detection performance of this NP detector is subsequently optimized, which does not lead immediately to the desired waveform, but rather to its energy spectral density (ESD).

Many prevalent waveform design approaches, such as those proposed in [2, 4, 5], arrive at the ESD of the desired waveform rather than the waveform itself. In this paper, we refer to them as nonparametric waveform design approaches. In these approaches, the extra step for time series synthesis results in degradation of the detection performance. In order to conquer this drawback, we are motivated to parametrize the transmit waveform as a weighted sum of basis sequences. The detection performance is maximized with respect to the corresponding weighting factors, and the desired waveform is obtained immediately. This approach is referred to as parametric waveform design approach in the sequel.

The paper is organized as follows. In Section 2, we give a short review of the discrete prolate spheroidal sequence (DPSS), followed by the motivation for parametric modeling of transmit waveform in Section 3. Section 4 defines the system modeling and states the problem at hand. Kay's nonparametric waveform design approach is reviewed in Section 5, followed by our parametric waveform design approach in Section 6. Section 7 gives the simulation results. Finally, Section 8 concludes the paper.

2. REVIEW OF DPSS

Consider a discrete time sequence $v(n), n \in \mathbb{Z}$, of bandwidth W with finite energy and a sampling interval $\Delta t = 1$. Amongst all the sequences that are band-limited to the frequency interval $[-W,W]$ with $|W| < 1/2$, Slepian sought in [6] the one that maximizes the energy concentration

$$\alpha(N,W) = \frac{\sum_{n=0}^{N-1} |v(n)|^2}{\sum_{n=-\infty}^{\infty} |v(n)|^2}. \tag{1}$$

During the calculation of this maximum, in total N DPSS's are calculated from the corresponding discrete prolate spheroidal wave functions (DPSWF's). The zeroth order DPSS, which is calculated from the zeroth order DPSWF, maximizes $\alpha(N,W)$ in Eq. (1). DPSS's are actually doubly infinite sequences, but their subsequences of length N, are of particular interest. In [7], these N subsequences were also called DPSS's and one method for generating them was proposed. This method is shortly reviewed in the following.

Reconsider a sequence $v(n), n = 0,1,...,N-1$, with finite energy and a sampling interval $\Delta t = 1$. Amongst all the sequences that are index-limited to $[0,N-1]$, the authors sought in [7] the one that maximizes

$$\beta(N,W) = \frac{\int_{-W}^{W} |V(F)|^2 \mathrm{d}F}{\int_{-1/2}^{1/2} |V(F)|^2 \mathrm{d}F}, \tag{2}$$

where $V(F)$ denotes the discrete time Fourier transform (DTFT) of $v(n)$. In the sequel, $\beta(N,W)$ is first transformed into the time domain, and then maximized with respect to $v(n)$. It is proven [7] that the sequence $v(n)$, $n = 0,...,N-1$ that maximizes $\beta(N,W)$ must also satisfy

$$\sum_{n=0}^{N-1} \frac{\sin[2\pi W(n-m)]}{\pi(n-m)} v(n) = \lambda(N,W)v(m), \quad (3)$$

for $m = 0,1,...,N-1$. Eq. (3) can be further reformulated in matrix form as

$$\mathbf{A}\mathbf{v} = \lambda(N,W)\mathbf{v}. \quad (4)$$

The candidate real valued N-element column vector solutions $\mathbf{v}$ are the eigenvectors of the $N \times N$ matrix $\mathbf{A}$. There are in total N eigenvectors $\mathbf{v}_k$ and their associated eigenvalues $\lambda_k(N,W)$ for $k = 0,1,...,N-1$. The elements in the vector $\mathbf{v}_k$ construct the k-th order DPSS $v_k(n;N,W)$, and the eigenvalue $\lambda_k(N,W)$ indicates the energy concentration defined in Eq. (2).

In the following, some important properties of DPSS are summarized from [7], they are:

1) When the N eigenvectors are normalized, they satisfy the following orthonormal property

$$\mathbf{v}_i^T \mathbf{v}_j = \sum_{n=0}^{N-1} v_i(n;N,W) \cdot v_j(n;N,W) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad (5)$$

for $i, j = 0,1,...,N-1$.

2) These N eigenvectors form a basis for the N-dimensional Euclidean space.

3) The eigenvalue $\lambda_k(N,W)$ is close to unity when $k < 2NW$, and close to zero when $k > 2NW$. The time-bandwidth product $2NW$ is supposed to be an integer in our paper.

3. PARAMETRIC MODELING OF TRANSMIT WAVEFORM

In [6], DPSS's were applied to waveform representation. That is, for large N if $v(n)$ is band-limited to $[-W,W]$ and concentrates most of the energy in the index set $[0,N-1]$, then it can be well approximated by the weighted sum of the first $2NW$ DPSS's. This motivates us to model the transmit waveform according to

$$z(n;\mathbf{c}) = \sum_{k=0}^{2NW-1} c_k \cdot v_k(n;N,W), \quad (6)$$

for $n = 0,1,...,N-1$. In Eq. (6), $\mathbf{c} = [c_0, c_1, ..., c_{(2NW-1)}]$ is a vector of weighting factors c_k, which can take any complex value. DPSSs have shown great superiority in parametric waveform modeling, since the number of basis sequences $2NW$ is tunable with W. Especially when $W \ll 1/2$, i.e., $2NW \ll N$, the savings of orthonormal basis sequences is considerable for this modeling as compared to the most general one given by

$$z(n;\mathbf{c}) = \sum_{k=0}^{N-1} c_k \cdot \phi_k(n), \quad (7)$$

where $\phi_k(n)$s are orthonormal basis sequences that span the N dimensional Euclidean space. Since $z(n;\mathbf{c})$ is parameterized in $\mathbf{c}$, we call it parametric waveform in our paper. In the frequency domain, the DTFT of the parametric waveform $z(n;\mathbf{c})$ is defined as

$$Z(F;\mathbf{c}) = \sum_{k=0}^{2NW-1} c_k \cdot V_k(F;N,W), \quad (8)$$

where $V_k(F;N,W)$ denotes the DTFT of the k-th DPSS. Note that, in the following sections, we will use digital angular frequency $\omega = 2\pi F$, and denote $Z(F;\mathbf{c})$ newly as $Z(\omega;\mathbf{c})$.

4. SYSTEM MODELING AND PROBLEM STATEMENT

A simple receiver model is illustrated in Figure 1, where a discrete-time signal model is utilized in order to facilitate digital processing. As transmit signal we use the waveform $z(n)$, for $n = 0,1,...,N-1$, $n \in \mathbb{Z}$. The target response is composed of two parts. A stochastic part $A \sim N_1^C(0,\sigma_A^2)$ models the reflectivity of the target, where N_1^C denotes the univariate complex Gaussian distribution. The deterministic part $\delta(n)$ models the impulse response of the point target, where $\delta(n)$ stands for the kronecker delta function. The clutter impulse response $h(n)$ as well as the receiver noise $w(n)$ are modeled as discrete-time complex and stationary Gaussian random process with zero mean and known power spectral density (PSD) $C_{hh}(\omega)$, and $C_{ww}(\omega)$ respectively. Throughout this paper, we assume that the real and imaginary parts of a complex process are independent and identically Gaussian distributed real processes. Besides, the random variable A and random processes $h(n)$ and $w(n)$ are all independent.

In the absence of a target, the received signal $r(n)$, $n = 0,...,N-1$ is a superposition of receiver noise $w(n)$ and clutter return $x(n) = h(n) * z(n)$, where $*$ denotes the convolution. When the target is present, we have additionally the target return $A \cdot z(n)$. The detection problem is hence formulated as a binary hypothesis testing in the following.

$$\mathbf{H} \; : \; r(n) = x(n) + w(n) \quad (9)$$

$$\mathbf{K} \; : \; r(n) = A \cdot z(n) + x(n) + w(n), \quad (10)$$

for $n = 0,1,...,N-1$.

5. NONPARAMETRIC WAVEFORM DESIGN

In [4], a canonical nonparametric waveform design approach was proposed for the same detection problem but using a continuous time signal model. We will consider a similar design approach for our model.

The finite Fourier transform and its properties [8] are used for the derivation of an optimal NP-detector. The finite Fourier transform of $Y(n)$ is defined by

$$Y_N(\omega) = \sum_{n=0}^{N-1} Y(n) e^{-j\omega n}, \quad -\infty < \omega < \infty. \quad (11)$$

In the sequel, we denote the finite Fourier transform of $w(n)$, $x(n)$, $r(n)$ and $z(n)$ by $d_w(\omega)$, $d_x(\omega)$, $d_r(\omega)$ and $Z(\omega)$ respectively.

If $Y(n)$ is a complex and stationary Gaussian random process with zero mean and PSD $C_{YY}(\omega)$, it can be proven that asymptotically for large N, $Y_N(\omega) \sim N_1^C(0, N \cdot C_{YY}(\omega))$ for any ω. Furthermore, for pairwise distinct frequencies

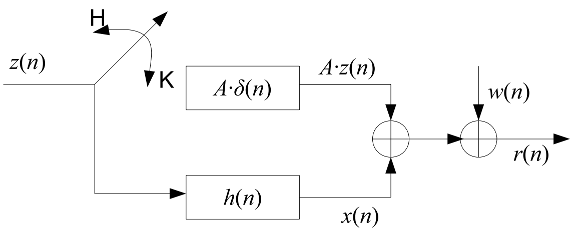

Figure 1: Modeling the received waveform.

$\omega_i = 2\pi i/N$, for $i = 0, 1, .., N-1$, $Y_N(\omega_i)$ are asymptotically independent variables. These can be easily proven by exploiting the properties of the finite Fourier transform introduced in [8, Chap.4]. Considering the vectors

$$\begin{aligned}\mathbf{d}_r &= [d_r(\omega_0), ..., d_r(\omega_{N-1})]^T \\ \mathbf{d}_x &= [d_x(\omega_0), ..., d_x(\omega_{N-1})]^T \\ \mathbf{d}_w &= [d_w(\omega_0), ..., d_w(\omega_{N-1})]^T \\ \mathbf{Z} &= [Z(\omega_0), ..., Z(\omega_{N-1})]^T ,\end{aligned}$$

the detection problem can be expressed in the frequency domain as follows:

$$\mathbf{H} \;:\; \mathbf{d}_r = \mathbf{d}_x + \mathbf{d}_w \tag{12}$$

$$\mathbf{K} \;:\; \mathbf{d}_r = A \cdot \mathbf{Z} + \mathbf{d}_x + \mathbf{d}_w. \tag{13}$$

Since vectors $A \cdot \mathbf{Z}$ and $\mathbf{d}_x$ and $\mathbf{d}_w$ are all multivariate complex Gaussian distributed and independent, we have

$$\mathbf{H} \;:\; \mathbf{d}_r \sim N_N^C(\mathbf{0}, \mathbf{T}_0) \tag{14}$$

$$\mathbf{K} \;:\; \mathbf{d}_r \sim N_N^C(\mathbf{0}, \mathbf{T}_1), \tag{15}$$

where N_N^C denotes the N-element multivariate complex Gaussian distribution. The covariance matrix $\mathbf{T}_0$ is given by

$$\mathbf{T}_0 = E\left[(\mathbf{d}_x + \mathbf{d}_w)(\mathbf{d}_x + \mathbf{d}_w)^H\right]. \tag{16}$$

$\mathbf{T}_0$ is a diagonal matrix with the i-th diagonal element equal to $[\mathbf{T}_0]_{ii} = N \cdot (C_{xx}(\omega_i) + C_{ww}(\omega_i))$, where $C_{xx}(\omega_i) = C_{hh}(\omega_i)|Z(\omega_i)|^2$. Analogue to $\mathbf{T}_0$, $\mathbf{T}_1$ can be calculated according to

$$\begin{aligned}\mathbf{T}_1 &= E\left[(A \cdot \mathbf{Z} + \mathbf{d}_x + \mathbf{d}_w)(A \cdot \mathbf{Z} + \mathbf{d}_x + \mathbf{d}_w)^H\right] \\ &= \mathbf{T}_0 + \sigma_A^2 \cdot \mathbf{Z}\mathbf{Z}^H. \end{aligned} \tag{17}$$

Having $\mathbf{T}_0$ and $\mathbf{T}_1$, we can easily express the log-likelihood ratio and further find the test statistic

$$T(\mathbf{d}_r) = \left|\mathbf{Z}^H \mathbf{T}_0^{-1} \mathbf{d}_r\right|^2. \tag{18}$$

In Eq. (18), the kernel $R = \mathbf{Z}^H \mathbf{T}_0^{-1} \mathbf{d}_r$ is a linear transformation of the complex Gaussian random vector $\mathbf{d}_r$, thus a complex Gaussian random variable. Besides, R differs only in variance under the two hypotheses. In [4], the probability of detection P_D is proven to relate to a given probability of false alarm P_{FA} according to

$$P_D = P_{FA}^{\frac{1}{1+d^2}}, \tag{19}$$

where d^2 is dependent on the variances of R under both hypotheses. Based on our signal model, d^2 in Eq. (19) is given by

$$d^2 = \sigma_A^2 \sum_{i=0}^{N-1} \frac{|Z(\omega_i)|^2}{N\left(C_{hh}(\omega_i)|Z(\omega_i)|^2 + C_{ww}(\omega_i)\right)}, \tag{20}$$

where $|Z(\omega)|^2$ is the ESD of the waveform $z(n)$, and it is denoted as $E_z(\omega)$. For the waveform design, we perform the following approximation. Assume N is large and $\Delta\omega = 2\pi/N$, Eq. (20) can be approximated by

$$\begin{aligned} d^2 &= \frac{\sigma_A^2}{\Delta\omega} \sum_{i=0}^{N-1} \frac{E_z(\omega_i)}{N\left(C_{hh}(\omega_i)E_z(\omega_i) + C_{ww}(\omega_i)\right)} \Delta\omega \\ &\approx \frac{\sigma_A^2}{2\pi} \int_0^{2\pi} \frac{E_z(\omega)}{C_{hh}(\omega)E_z(\omega) + C_{ww}(\omega)} \mathrm{d}\omega. \end{aligned} \tag{21}$$

For a given P_{FA}, P_D increases monotonically with d^2, thus we maximize it under the energy constraint

$$\frac{1}{2\pi} \int_0^{2\pi} E_z(\omega) \mathrm{d}\omega \leq E_z, \tag{22}$$

where E_z is the maximal available transmit energy. The optimization problem can be resolved by the Lagrangian multiplier method. As a result, the ESD of the optimal waveform that leads to the global maximal d^2 is given by

$$E_z^{\mathrm{opt}}(\omega) = \max\left(\frac{\sqrt{C_{ww}(\omega)/\lambda_L} - C_{ww}(\omega)}{C_{hh}(\omega)}, 0\right), \tag{23}$$

where the parameter λ_L can be found by meeting the energy constraint. Durbin's algorithm is then utilized to synthesize the nonparametric waveform $z^{\mathrm{np}}(n)$ from $E_z^{\mathrm{opt}}(\omega)$.

6. PARAMETRIC WAVEFORM DESIGN

The disadvantage of the nonparametric waveform design approach from Section 5 is illustrated in Figure 2. An extra step is needed to synthesize the actual optimal waveform from the obtained ESD. During this additional synthesizing step, a loss in performance generally occurs. The proposed parametric method on the other hand can be formulated as an optimization problem to directly yield a transmit waveform which is close to the theoretical optimal waveform. In this way, the performance loss can be successfully remedied.

In this section, we use the parametric waveform $z(n; \mathbf{c})$ derived in Eq. (6) as transmit signal. Following the steps described in Eq. (12) to Eq. (22), we can formulate the optimization problem with respect to $\mathbf{c}$ as follows:

$$\max_{\mathbf{c}} \quad \frac{\sigma_A^2}{2\pi} \int_0^{2\pi} \frac{E_z(\omega; \mathbf{c})}{C_{hh}(\omega)E_z(\omega; \mathbf{c}) + C_{ww}(\omega)} \mathrm{d}\omega \tag{24}$$

$$\text{s.t.} \quad \frac{1}{2\pi} \int_0^{2\pi} E_z(\omega; \mathbf{c}) \mathrm{d}\omega \leq E_z, \tag{25}$$

The optimization problem formulated in Eqs. (24)-(25) can not be solved by the Lagrangian multiplier method anymore. Thus, no closed-form solution for the weighting factors c_i,

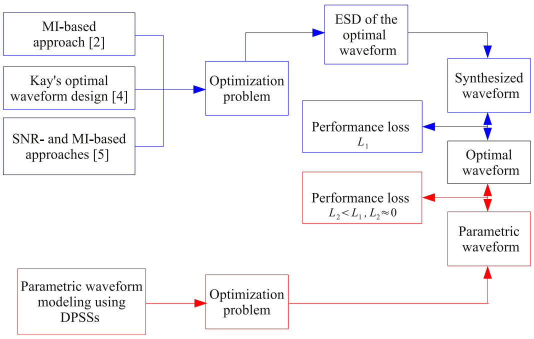

Figure 2: Comparison between parametric and non-parametric waveform design approach.

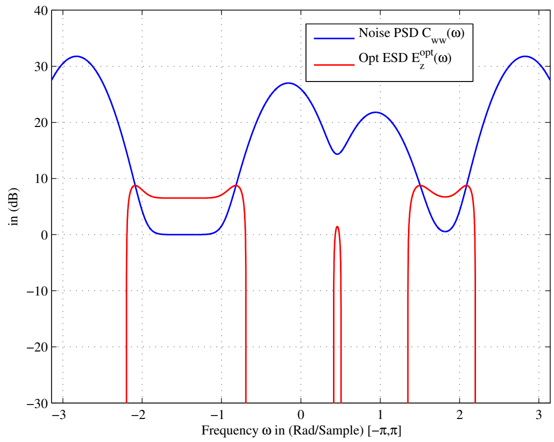

Figure 3: Noise PSD vs. the optimal ESD $E_z^{\text{opt}}(\omega)$, that leads to the global maximal probability of detection.

$i = 0, \ldots, 2NW - 1$ can be obtained and the optimization problem has to be solved numerically instead. For a computer based simulation, the integral expressions are replaced by

$$d^2 \approx \sigma_A^2 \sum_{i=0}^{N'-1} \frac{E_z(\omega_i; \mathbf{c})}{N' \left(C_{hh}(\omega_i) E_z(\omega_i; \mathbf{c}) + C_{ww}(\omega_i)\right)}, \tag{26}$$

and

$$\frac{1}{N'} \sum_{i=0}^{N'-1} E_z(\omega_i; \mathbf{c}) \leq E_z. \tag{27}$$

where $N' \gg N$. The reasons for optimizing Eq. (26) rather than Eq. (20) are as follows. Firstly, d^2 in both expressions are approximately equal. Secondly, the ESD of the waveform $E_z(\omega; \mathbf{c})$ should be optimal for every frequency grid $\omega_i = 2\pi i/N', N' \to \infty$, not only for the finite N grids $\omega_i = 2\pi i/N$.

The objective function derived in Eq. (26) is a summation of ratios, whose numerator and denominator are both quadratic functions of $\mathbf{c}$. Furthermore the constraint function in Eq. (27) is also a summation of quadratic functions of $\mathbf{c}$. Thus, the whole optimization problem shows high nonlinearity. Besides, the number of optimization variables $2NW$ is normally large. According to [9], such a large scale nonlinear problem can be solved efficiently by the interior point

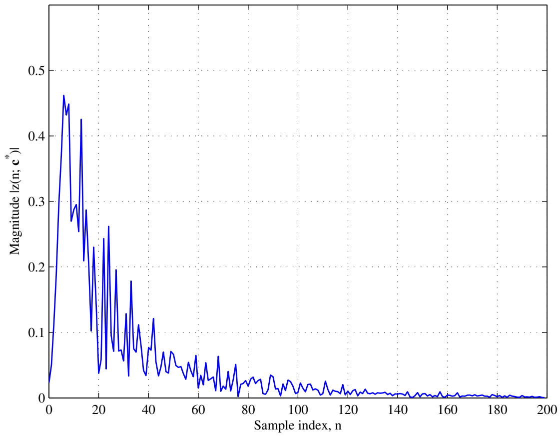

Figure 4: Magnitude of the parametric waveform $z(n; \mathbf{c}^*)$.

algorithm numerically. After the problem is solved, we obtain a vector solution $\mathbf{c}^*$ and the desired parametric waveform $z(n; \mathbf{c}^*)$ at the same time.

7. NUMERICAL RESULTS

We consider a simulation scenario with a system bandwidth $B = 5000$ Hz. The baseband transmit waveform is defined within $|f| \leq 2500$ Hz using a sampling frequency $f_s = 5000$ Hz. The PSD of the clutter impulse response $C_{hh}(\omega) = 1$ watts/(Rad/Sample) and the PSD of the noise process $C_{ww}(\omega)$ in one principle period $[-\pi, \pi]$ is given by

$$C_{ww}(\omega) = 1 + \sum_{k=1}^{4} P_k \cdot \exp[-(\omega - \omega_k)/2\omega_B], \tag{28}$$

where $P_1 = P_2 = 1500$, $P_3 = 500$ and $P_4 = 150$. The center frequencies are $\omega_1 = -0.9\pi$, $\omega_2 = 0.9\pi$, $\omega_3 = -0.05\pi$ and $\omega_4 = 0.3\pi$. ω_B is set to 0.016π. The reflect factor $A \sim N_1^C(0, 20)$ and the energy of the transmit waveform is set to $E_z = 2$ Joule. Under these assumptions, we can find the ESD of the optimal signal $E_z^{\text{opt}}(\omega)$ according to Eq. (23), which leads to the global maximal $d_{\text{opt}}^2 = 4.91807$. The Noise PSD $C_{ww}(\omega)$ and ESD of the optimal waveform $E^{\text{opt}}(\omega)$ are plotted in Figure 3. Based on this model, parametric and nonparametric waveforms will be designed in the following.

We start by demonstrating the performance of the proposed parametric waveform design technique. In our scenario, the noise is very large when $|\omega| > \omega_c = 0.8\pi$, the clutter is weak, and the available transmit energy is small. We are assuming the a priori knowledge that no energy should be distributed in the frequency band $|\omega| > \omega_c$. Having this prior knowledge, the parameter W for generating the DPSS's is chosen as $W = \omega_c/2\pi = 0.4$. Furthermore, the length of the sequences is selected as $N = 200$. As a result, 160 DPSS's are generated to represent $z(n; \mathbf{c})$ according to Eq. (6). In total 40 orthonormal basis sequences are saved for DPSS as compared with those from other orthonormal waveform families. The initial value of the to be optimized vector $\mathbf{c}$ is set to $\mathbf{1} + \mathbf{1}j$, where vetcor $\mathbf{1}$ is of dimension $2NW$ and contains all ones. N' is set to 1024. Subsequently, the optimization problem is resolved with respect to $\mathbf{c}$ via the interior point algorithm. As a result, a vector solution $\mathbf{c}^*$ is

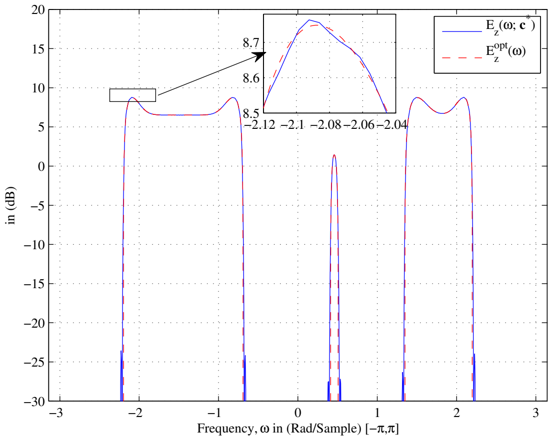

Figure 5: ESD of the parametric waveform $z(n;\mathbf{c}^*)$.

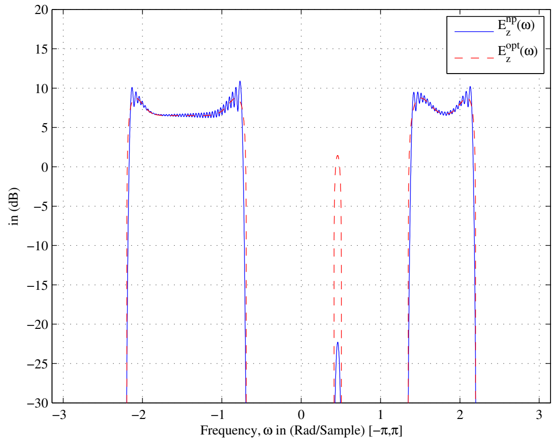

Figure 6: ESD of the nonparametric waveform $z^{\text{np}}(n)$.

obtained and the corresponding $d_{\text{p}}^2 = 4.91800$. Placing $\mathbf{c}^*$ into Eq. (6) we can immediately obtain the desired parametric waveform $z(n;\mathbf{c}^*)$. The optimal waveform $z(n;\mathbf{c}^*)$ and its ESD $E_z(\omega;\mathbf{c}^*)$ are shown in Figure 4 and 5, respectively.

Next, the nonparametric waveform design approach is considered. A nonparametric waveform $z^{\text{np}}(n)$ is synthesized from $E_z^{\text{opt}}(\omega)$ via Durbin's algorithm. Within this algorithm an exceedingly higher order autoregressive filter model is selected for better synthesis. The ESD of the waveform $E_z^{\text{np}}(\omega)$ is presented in Figure 6. In this case $d_{\text{np}}^2 = 4.90259$ is obtained. For a better comparison, the optimal ESD is also depicted in red in Figure 5 and 6. As can be seen, the ESD of the nonparametric waveform shows ripples and a poor performance in the narrow frequency band around 0.15π. While the ESD obtained via the parametric waveform design clearly shows a better fit.

We now compare the detection performance for our parametric waveform with the nonparametric waveform and traditional impulse waveform with the same time duration and energy. The impulse waveform is defined by $z^{\text{imp}}(n) = [\sqrt{E_z}, 0, ..., 0]$. For $z^{\text{imp}}(n)$, we have $d_{\text{imp}}^2 = 3.66468$. Suppose $P_{FA} = 0.1$, take the resultant d^2 for each designed waveform into Eq. (19), we can obtain the probability of detection

$$\begin{aligned} P_D^{\text{opt}} &= 0.677682 & P_D^{\text{p}} &= 0.677679 \\ P_D^{\text{np}} &= 0.676991 & P_D^{\text{imp}} &= 0.610412 \end{aligned}$$

As can be seen, the proposed parametric waveform design approach yields a probability of detection P_D^{p} that is slightly higher than the one obtained by the nonparametric waveform design approach and is closer to the global maximum.

8. CONCLUSION

We have proposed to parametrize a transmit waveform as a weighted sum of discrete prolate spheroidal sequences. An optimization problem is maximized with respect to the waveform parameters and as such yields the optimal waveform. The advantage of the proposed waveform design method as compared to nonparametric approaches is the fact that the optimization problem directly yields the transmit waveform rather than its ESD and thus saves the extra step for waveform synthesis. The obtained waveform ESD as well as the obtained detection performance outperform the existing nonparametric approach and are closer to the ones obtained by using the theoretically derived optimum waveform.

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