MULTI-RELAY COOPERATIVE NB-LDPC CODING WITH NON-BINARY REPETITION CODES OVER BLOCK-FADING CHANNELS

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ABSTRACT

In this paper, we propose a cooperative coding scheme to communicate efficiently over multiple-relay fading channels. The particularity of our approach is to rely on non-binary LDPC codes at the source, coupled with non-binary repetition codes at the relays. A simple joint decoding strategy is used at the receiver end, so that the decoding complexity is not increased compared to a system without relays, while preserving the coding gain brought by re-encoding the signal at the relays. We show by simulations that the proposed scheme allows maintaining a constant gap to the outage probability of the cooperative system, irrespective of the number of relays.

1. INTRODUCTION

Cooperative diversity techniques over wireless relay channels [1, 2], allow exploiting the broadcast nature and the inherent spatial diversity of wireless communications. A relay channel is a multi-terminal network consisting of a source, a destination, and a collection of relays that might be of different nature. The source broadcasts a message to both relays and destination, while the relays forward the message or modified versions of it to the destination. Subsequently, different cooperation protocols have been proposed, which can be classified into three major categories, namely the amplify-and-forward (AF), the compress-and-forward (CF), and finally the decode-and-forward (DF) protocol [3, 4]. The DF protocol allows each relay to decode the received signal, re-encode it, and forward it to the destination. The forwarded message can either be identical to, or part of the initial transmission [5] (repetition coding), or it can be obtained by using a dedicated coding scheme at the relays. The destination uses the global knowledge of the cooperative coding scheme to jointly decode the received signals both from the source and the relays. Distributed coding using parallel turbo-codes [6] or binary LDPC codes [7–12], has already been proposed in the literature. The existing approaches are either based on serial or parallel code concatenation, meaning that the graph of the LDPC code broadcasted from the source is a subgraph of the destination decoding graph, or based on punctured rate-compatible LDPC codes. A different approach was proposed in [13], where the relay generates extra parity-bits by splitting parity-checks of the LDPC code broadcasted by the source. When the number of relays increases, these methods suffer from a large increase of decoding complexity, while the coding gain they present becomes increasingly less important.

In this paper, we propose a new approach to the problem of cooperative coding in the case of multiple relays. The approach is based on non-binary LDPC (NB-LDPC) codes and the recently introduced technique of multiplicative non-binary coding [14], which will be referred to as non-binary repetition coding. In our setting, the source broadcasts a NB-LDPC codeword to the destination and the relays. When the relays successfully decode the received word, extra parity symbols are computed at the relays through optimized non-binary repetition codes, which are then sent to the destination. The receiver collects the original received word from the source and the non-binary extra symbols from the relays and combines them before the iterative decoding. The iterative decoding complexity is the same in the presence or the absence of relays, while combining the codeword and the additional non-binary repetition symbols brings an effective coding gain.

The paper is organized as follows. In Section 2, we describe the proposed cooperative coding scheme and discuss its advantages. The joint optimisation of both NB-LDPC and non-binary repetition codes is presented in Section 3. The performance of the proposed coding scheme over block-fading multi-relay channels is evaluated in Section 4. Finally, Section 5 concludes the paper.

2. PROPOSED COOPERATIVE CODING SCHEME

2.1. Non-Binary LDPC Codes and Decoding

Throughout the paper, GF(q) denotes the Galois field with q elements. A NB-LDPC code is defined by a sparse parity-check matrix H, with M rows and N columns, whose entries are taken from GF(q). We shall assume that H is full rank, hence the coding rate is $R = K/N$, where $K = N - M$ is the number of source symbols. A NB-LDPC code can be advantageously represented by a bipartite (Tanner) graph containing N symbol-nodes and M constraint-nodes, associated respectively with the N columns and M rows of H. A symbol-node and a constraint-node are connected by an edge if and only if the corresponding entry of H is non-zero. Every edge of the graph is further assumed to be "labeled" by the corresponding non-zero entry.

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The Belief-Propagation (BP) decoding passes messages along the edges of the graph, in both directions, which are iteratively updated by Bayesian rules. In the non-binary case, each message is a probability distribution vector on GF(q), which gives the probabilities of the incident symbol-node being equal to each of its possible values. These probability distributions are iteratively updated until a codeword has been found or a maximum number of iterations has been reached. We shall not present the BP decoding in this paper, but merely refer to [15] for details. We also note that the BP decoding can be efficiently implemented by using binary Fourier transforms [16]. Moreover, at the cost of a small performance degradation, several low-complexity decoding algorithms, which operate in the Log Likelihood Ratio (LLR) domain, have been proposed in the literature [17, 18].

2.2. Cooperative System Description

Throughout the paper, we will assume that the source broadcasts a NB-LDPC codeword to the destination and a given number N_r of relays. Parameters of the different links in the system will be indicated by using subscripts SD, SR$_i$, and R$_i$D, with obvious meaning. We assume QAM signalling constellations of order M_{SD}, M_{SR_i}, and $M_{\text{R}_i\text{D}}$. Since relays and destination receive the same modulated signal broadcasted by the source, we have by construction $M_{\text{SD}} = M_{\text{SR}_i}, \forall i = 1, \ldots, N_r$. Our cooperative coding scheme, depicted in Figure 1 in the case of two relays, can be described as follows:

[S] The source encodes the packet of information bits, generating a NB-LDPC codeword $\mathbf{c}$. It modulates $\mathbf{c}$ with the $M_{\text{SD}}-$QAM constellation and broadcasts the modulated symbols $\mathbf{x}$ to both relays and destination.

[R$_i$] Each relay R$_i$ decodes the received signal, so that to correct the transmission errors on $\mathbf{c}$. If decoding fails, the relay does not transmit any information to the destination. Otherwise, it generates a new sequence $\mathbf{c}^{(i)}$ of non-binary symbols using the *non-binary repetition* coding. Vector $\mathbf{c}^{(i)}$ needs not be of the same size as the original codeword $\mathbf{c}$, since the coding rates for the links relay-destination are typically higher. The vector $\mathbf{c}^{(i)}$ is modulated using the $M_{\text{R}_i\text{D}}-$QAM constellation, and the modulated symbols $\mathbf{x}^{(i)}$ are then sent to the destination.

[D] The destination receives noisy versions of $\mathbf{x}$ and $\mathbf{x}^{(i)}$ (from both source and relays), and performs a joint iterative decoding using only the parity check matrix H of the NB-LDPC code broadcasted by the source.

The next section explains in details how non-binary repetition symbols are generated, and how the joint decoding is performed at the destination.

2.3. Non-Binary Repetition coding and Joint Decoding

We assume that the parameters of the transmission system, namely the constellation orders $\{M_{\text{SD}}, M_{\text{R}_i\text{D}}\}$ and the coding rates $\{R_{\text{SD}}, R_{\text{R}_i\text{D}}\}$ are fixed *a priori*. When the i^{th} relay successfully decodes the codeword $\mathbf{c}$ broadcasted by the source,

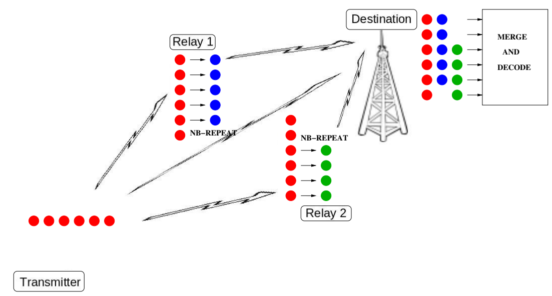

Fig. 1. Cooperative coding using non-binary repetition coding.

it generates $N_i = N\frac{R_{\text{SD}}}{R_{\text{R}_i\text{D}}}$ non-binary repetition symbols as follows:

- Select N_i symbols $\{c_{k_l^{(i)}}\}_{l=1,\ldots,N_i}$ among the N symbols of $\mathbf{c}$.
- Generate new symbols $c_l^{(i)} = h_l^{(i)} c_{k_l^{(i)}}$, where $h_l^{(i)} \in$ GF(q) are predetermined non-zero values, corresponding to the local NB-repetition code.

The vector $\mathbf{c}^{(i)}$ is then modulated and sent to the destination. As a particular case, choosing $h_l^{(i)} = 1, \forall l$, reduces to classical repetition coding. In our case, with a very limited extra complexity, we allow the use of non-binary repetitions with $h_l^{(i)} \neq 1$, which provides a non-negligible coding gain, as explained in Section 3.

We discuss now the joint decoding at the destination. To simplify the notation, we drop the index of the symbol within the codeword, and consider a particular code symbol $c \in$ GF(q). We denote by $\mathbf{x}$ the QAM symbols built from c and transmitted by the source. Similarly, we denote by $\mathbf{x}^{(i)}$ the QAM symbols corresponding to the non-binary repetitions of c transmitted by the relays. The destination receives channel values, denoted by $\left\{\mathbf{y}^{(0)}, \mathbf{y}_l^{(1)}, \ldots, \mathbf{y}_l^{(I)}\right\}$, from the source and I active relays, according to one row in Figure 1. These values are used to compute the joint-probability vector $\mathbf{P} = \{P(a)\}_{a \in \text{GF}(q)}$, where $P(a) = \Pr[c = a | \mathbf{y}^{(0)} \ldots \mathbf{y}^{(I)}]$, which merges the sufficient statistics of all active links. Using the fact that the SD and R$_i$D channels are conditionally independent, we can write:

$$P(a) = \prod_{i=0}^{I} \Pr[c = a | \mathbf{y}^{(i)}] = \prod_{i=0}^{I} \Pr[c^{(i)} = h^{(i)} a | \mathbf{y}^{(i)}],$$

where $h^{(i)}$ is the non-zero value used for the non-binary repetition encoding of symbol c at relay i, and $h^{(0)} = 1$. Consequently, the main transmission and the relay transmissions are combined into one joint-probability vector per code symbol, which is fed to the BP decoder.

The process is depicted in Figure 2, which shows the factor graph used for the joint decoding of the NB-LDPC code from the source and the repetition codes from the relays. We considered in this figure the case of 3 relays, each of them sending $N_i = 2N/3$ extra repetition symbols, such that the destination receives 3 probability measures for each coded symbol: one from the source, and two from the relays.

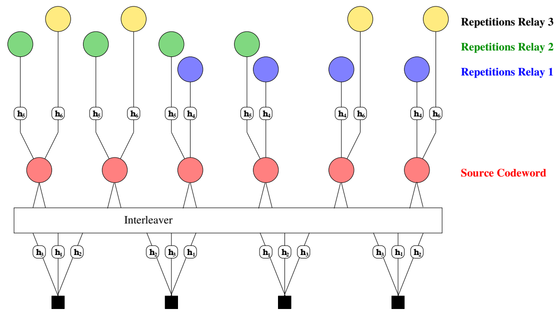

Fig. 2. Tanner graph of the joint-receiver at the destination

3. OPTIMIZATION OF THE PROPOSED COOPERATIVE CODING SCHEME

3.1. NB-LDPC code Optimization at the Transmitter

For codes defined over GF(q), it has been shown in [19] that selecting carefully the non binary entries of the parity-check matrix H can improve the overall performance of the code. The approach proposed in *loc. cit.* consists in choosing the non-zero entries of H such that the binary image of each non-binary check-node has the maximum *minimum Hamming distance* D_{min}, together with the minimum multiplicity of codewords with Hamming weight D_{min}. Although locally optimal, this strategy is not optimal when used in a message passing iterative decoder, where extrinsic vector messages are propagated along graph's edges.

In this paper, we propose a new selection criterion for the non-zero entries of H. In the sequel, we should also refer to a non-binary parity check as a *component code*; it is thus determined by the non-zero entries within a row of H. Our strategy is to optimize the balance between sub-codes of the component code, which is especially efficient when the code is a regular ultra-sparse code with $d_v = 2$. Since the message-passing decoder will propagate d_c extrinsic messages computed from the incoming message at each iteration, it is better to build extrinsic messages which statistically behave equally. In other words, the extrinsic messages should have their quantity of mutual information as close as possible to their average. Our optimization criterion for component code selection is described in Algorithm 1 below. This algorithm ensures that all the sub-codes of the component code have good and equally distributed error correction capabilities. This new optimization criterion is indeed interesting since we saw slight improvement in the waterfall region compared to codes that use existing sets of non-zero values. The best sets of $d_c = 4$ values for GF(64) and GF(256) that have been optimized with the new criterion are given in Table 1. The GF(q) elements are denoted by $\{0, \alpha^0, \alpha^1, \ldots, \alpha^{(q-2)}\}$, where α is a primitive element of the field. Four sets of values were found to have the exact same performance with respect to the criterion of the optimization algorithm.

Table 1. Best rows with $d_c = 4$, for GF(64) and GF(256)

GF(64)	$(\alpha^0, \alpha^9, \alpha^{26}, \alpha^{46})$	$(\alpha^0, \alpha^{17}, \alpha^{26}, \alpha^{43})$
	$(\alpha^0, \alpha^{17}, \alpha^{37}, \alpha^{54})$	$(\alpha^0, \alpha^{20}, \alpha^{37}, \alpha^{46})$
GF(256)	$(\alpha^0, \alpha^8, \alpha^{173}, \alpha^{183})$	$(\alpha^0, \alpha^{10}, \alpha^{82}, \alpha^{90})$
	$(\alpha^0, \alpha^{72}, \alpha^{80}, \alpha^{245})$	$(\alpha^0, \alpha^{165}, \alpha^{175}, \alpha^{247})$

Algorithm 1 Component Code Optimization

1. Consider a non-binary parity check of degree d_c with non-zero values $\{h_1 \ldots h_{d_c}\}$
2. Let $\mathcal{S}_{\text{cc}}(k)$ be the binary code determined by the binary images of $d_c - 1$ values in the set $\{h_1 \ldots h_{d_c}\}$ except h_k.
3. Choose for $\{h_1 \ldots h_{d_c}\}$ the field values in GF(q) such that:

$$\{h_1 \ldots h_{d_c}\} = \underset{\{h_1 \ldots h_{d_c}\}}{\text{argmax}} \left(\sum_{k=1}^{d_c} \mathrm{D}_{\text{min}}(\mathcal{S}_{\text{cc}}(k)) \right)$$

constrained to $|\mathrm{D}_{\text{min}}(\mathcal{S}_{\text{cc}}(k)) - \mathrm{D}_{\text{min}}(\mathcal{S}_{\text{cc}}(k'))| \leq 1$

3.2. Repetition code Optimization at the Relays

We now discuss the impact of the non-binary repetition symbols built by the relays and used in the joint-probability computation at the destination. Let us first concentrate on the case of a single repetition. Let c be the symbol to be repeated and $h^{(i)}c$ be the repeated Galois field value. The destination receives noisy values on both c and $h^{(i)}c$, corresponding to the same code symbol. It follows that the demodulation actually acts as a maximum-a-posteriori decoder of the code built from the concatenation of the two Galois field values $[1, h^{(i)}]$. Now the coding gain is increasing with the minimum distance of the binary image of $[1, h^{(i)}]$. In case of simple a copy, *i.e.* $h^{(i)} = 1$, the binary minimum distance is $\mathrm{D}_{\text{min}} = 2$ and no coding gain can be achieved, while for non-binary repetitions, this minimum distance is typically larger $\mathrm{D}_{\text{min}} \geq 3$ when the field size q is sufficiently large. Additionally, the non-zero repetition values $h_l^{(i)}$ need to be optimized with the knowledge of the non-zero values which have been used in the source NB-LDPC code. Indeed, during the iterative decoding algorithm, the extrinsic vector messages will be computed using the joint-probabilities, that is, with the modified parity-check nodes, including the repetition nodes as well. The modified parity-check nodes act then as the new *component codes* of the joint coding scheme.

The optimization criterion for non-binary repetition code selection is described in Algorithm 2. We advise in particular

Algorithm 2 Non-Binary Repetition Code Optimization

1. Let a parity-check equation have fixed non-zero values given by the set $\{h_1, h_2, \ldots, h_{d_c}\}$. Let H_0 be the binary image of the equivalent code. Set $i = 1$.
2. Consider the modified binary code H_i, build from H_{i-1} and the repetition codes with the same $h^{(i)}$ on all the d_c symbols,
3. Choose $h^{(i)} \in$ GF(q) such that the the minimum distance of H_i is maximum. If several values $h^{(i)}$ have the same minimum distance, choose the one with minimum multiplicity,
4. $i = i + 1$, goto step 2).

Table 2. Optimum values used at the relays for repetition coding

Relay #	1	2	3	4	5	6	7	8
GF(64)	α^{26}	α^{41}	α^{52}	α^{6}	α^{56}	α^{17}	α^{50}	α^{11}
D_{min}	8	14	20	25	31	37	43	49
GF(256)	α^{15}	α^{165}	α^{71}	α^{150}	α^{128}	α^{122}	α^{113}	α^{104}
D_{min}	10	17	24	32	39	46	54	62

to use the same non-zero value $h^{(i)}$ for all the repeated symbols at the relay i. The optimization algorithm stops when the maximum number of potential relays I has been reached. Table 2 shows the optimized repetition values for NB-LDPC codes over GF(64) and GF(256), with $d_c = 4$. The non-zero entries of the NB-LDPC parity check are defined according to any of the four sets of best rows presented in Table 1.

4. PERFORMANCE EVALUATION

4.1. Cooperation scenario

The performance of the proposed cooperative coding scheme has been assessed by Monte-Carlo simulation under the following scenario. The source broadcasts a NB-LDPC code with $N = 2K$ (rate $R = 1/2$). Each relay decodes the received signal, then computes an exact number of K non-binary repetition symbols, which are sent to the destination. Whether the relays operate in the half-duplex or in the full-duplex mode is out of the scope of this paper, however we mention that in our cooperation scenario both modes are possible. We consider that the source and the relays use multi-carrier modulation with orthogonal frequency-division multiplexing (OFDM), and that the different signals, from the source and the relays, can be separated at the destination. This can be done by using different multiple access methods, in which the source and the relays are separated either in time, in frequency, or by using spatial diversity. Finally, we shall further assume that the signals transmitted by the source and the relays use the same QAM modulation, hence $M_{\text{SD}} = M_{\text{R}_i\text{D}}$, and the transmission of the N coded symbols at the source and of the K non-binary repetition symbols at the relays takes the same period of time. Therefore, we assume that the source transmits on two frequency chunks, while each relay transmits on one frequency chunk. In case of spatial division multiple access (SDMA), relays can transmit in any of the frequency chunks allocated to the source, which ensures an efficient use of the frequency spectrum.

According to the a above scenario, the wireless channels can be modeled as block-fading channels, with flat Rayleigh fading, constant on each block and i.i.d. on different blocks [20]. The N coded symbols broadcasted by the source span two fading blocks, while the K repetition symbols transmitted by each relay span exactly one fading-block. Consequently, the number of fading blocks is given by $n_f = 2 + N_r$. For each fading block $j = 1, \ldots, n_f$, the baseband equivalent channel model is given by:

$$y_i^{(j)} = \sqrt{\rho^{(j)}} f^{(j)} x_i^{(j)} + z_i^{(j)},$$

where $x_i^{(j)}$ and $y_i^{(j)}$ denote the i^{th} QAM symbol transmitted in block j and the corresponding received symbol, $f^{(j)}$ is the fading coefficient of block j, and $z_i^{(j)} \sim \mathcal{CN}(0,1)$ is the i.i.d. circular complex Gaussian noise. We assume that the QAM-constellation has unit energy and that the fading is normalized on each block, *i.e.* $\mathbb{E}\left[|f^{(j)}|^2\right] = 1$. It follows that $\rho^{(j)}$ is the average received SNR on block j, thus, $\rho^{(1)} = \rho^{(2)} = \text{SNR}_{\text{SD}}$ and $\rho^{(2+i)} = \text{SNR}_{\text{R}_i\text{D}}$. We shall assume that $\rho^{(j)}$ and $f^{(j)}$ are perfectly known at the receiver.

For a given set of SNR values $\rho = (\rho^{(1)}, \ldots, \rho^{(n_f)})$, we denote by $I_\rho(\mathbf{f})$ the mutual information between channel input and output, assuming that the fading coefficients are given by $\mathbf{f} = (f^{(1)}, \ldots, f^{(n_f)})$, and that the channel input is uniformly distributed over the complex QAM constellation. Hence, $I_\rho(\mathbf{f})$ is a random variable taking values in $[0, m]$, where $m = \log_2(M_{\text{SD}}) = \log_2(M_{\text{R}_i\text{D}})$ is the number of bits per QAM symbol. Let R_{D} be the *overall coding rate* of the cooperative system, defined as the ratio between the number of information symbols K and the number of non-binary symbols received at the destination both from the source and the relays. According to the previous settings, $R_{\text{D}} = 1/n_f$. The *outage probability*, given by:

$$P_{\text{out}}(\rho, R_{\text{D}}) = \Pr\left(I_\rho(\mathbf{f}) < mR_{\text{D}}\right)$$

is the probability of the instantaneous mutual information $I_\rho(\mathbf{f})$ being less than the information rate value mR_{D}. Since this probability is non-zero, it follows that the Shannon capacity of the channel is 0. As a consequence, the performance of a code over the block fading channel is usually evaluated in terms of the SNR gap to the outage probability [20]. If this gap is maintained constant for arbitrarily large SNR values, the code is said to be *full diversity*.

4.2. Simulation results

A class of full-diversity LDPC codes over block-fading channels, referred to as *root-LDPC* codes, was proposed in [21]. Since the N coded symbols transmitted by the source span two fading blocks, for the NB-LDPC code at the source we chose a rate $1/2$ root-NB-LDPC code with $(d_v = 2, d_c = 4)$, defined over GF(64). Namely, the bipartite graph of the NB-LDPC code was designed according to the approach proposed in [21], while the non-zero entries on each row of the parity-check matrix were randomly selected among the four sets of "best rows" given in Table 1. The code has $K = 50$ information symbols, corresponding to 300 information bits.

We consider here the case of 4 relays, $\text{R}_1, \ldots, \text{R}_4$. The non-binary symbols transmitted by both the source and the relays are modulated using QPSK modulation. The Frame Error Rate (FER) performance of the root-NB-LDPC code broadcasted by the source is shown in Figure 3, and corresponds to the case when no relay is activated (the right-most curve). Consequently, we shall assume that $\text{SNR}_{\text{SR}_i} > 24\,\text{dB}$, for each relay R_i, such that the probability that the relay fails decoding the signal broadcasted by the source is less than 10^{-4}.

Moreover, we consider that $\mathrm{SNR}_{\mathrm{R}_1\mathrm{D}} = \mathrm{SNR}_{\mathrm{SD}} + 6\,\mathrm{dB}$, $\mathrm{SNR}_{\mathrm{R}_2\mathrm{D}} = \mathrm{SNR}_{\mathrm{SD}} + 4\,\mathrm{dB}$, and $\mathrm{SNR}_{\mathrm{R}_3\mathrm{D}} = \mathrm{SNR}_{\mathrm{R}_4\mathrm{D}} = \mathrm{SNR}_{\mathrm{SD}} + 2\,\mathrm{dB}$. The K symbols transmitted by R_1 and R_3 are non-binary repeated versions of the K information symbols, while the K symbols transmitted by R_2 and R_4 are non-binary repeated versions of the K parity symbols. The non-zero field values used at the relays for repetition coding are chosen according to Table 2. Specifically, R_1 and R_2 use $h^{(1)} = \alpha^{26}$ (together, they send to the destination a first repeated version of the N coded symbols), while R_3 and R_4 use $h^{(2)} = \alpha^{41}$ (they send to the destination a second repeated version of the N coded symbols).

Figure 3 shows the FER performance of the proposed cooperative coding scheme in case that 1, 2 or 4 relays are activated. For each case, we plotted the corresponding outage probability (dotted curves, asterisk markers) and the FER of the proposed cooperative coding scheme with optimized non-binary repetition coefficients (solid curves, full markers). For comparison purposes, we have also plotted the FER of the the proposed cooperative coding scheme with classical repetition coding (same *optimized* NB-LDPC code at the source, but $h^{(1)} = h^{(2)} = 1$; dotted curves, empty markers). It can be seen that the proposed coding scheme achieves full-diversity in every case, hence fully exploiting the spatial diversity brought by the existence of relays. When no relay is activated, the gap between the FER of the NB-LDPC code used at the source and the outage probability is about 1 dB. More important, the non-binary repetition coding proves to be strong enough to maintain the same gap to the outage probability, irrespective of the number of activated relays.

5. CONCLUSIONS

We have introduced and optimized a new cooperative coding scheme based on NB-LDPC coding at the source and NB-repetition coding at the relays. For cooperative systems with block-fading wireless channels, we showed that the proposed scheme archives full diversity, with constant gap to the outage probability of the system, irrespective of the number of relays. Additionally, our scheme is independent of the channel model or of the order of the modulation used for each link in the system, which allows preserving all the advantages shown in this paper with advanced link-adaptation and channel estimation techniques. This will be the purpose of a future work.

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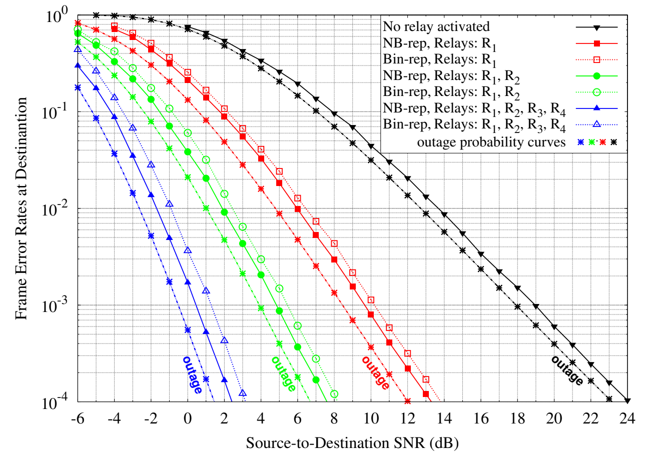

Fig. 3. FER performance of the proposed cooperative coding scheme

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