A HYBRID ICA-HERMITE TRANSFORM FOR REMOVAL OF BALLISTOCARDIOGRAM FROM EEG

Saideh Ferdowsi[1], *Saeid Sanei*[1], *Judith Nottage*[2], *Owen O'Daly*[2] *and Vahid Abolghasemi*[3]

[1]Faculty of Engineering and Physical Sciences, University of Surrey, Guildford, UK
[2]Centre for Neuroimaging Sciences, King's College London, London, UK
[3]School of Engineering and Design, Brunel University, Uxbridge, UK

ABSTRACT

In this paper the problem of removing Ballistocardiogram (BCG) artifact from EEG signal is addressed. BCG removal is an important task in analysis of simultaneous EEG-fMRI data. We propose a new method by combining independent component analysis (ICA) and discrete Hermite transform (DHT) for this purpose. Discrete Hermite transform is a powerful technique which is able to model a signal with no assumption about its shape. This feature makes DHT an appropriate tool to be combined with ICA for removing the BCG artifact. We show that the proposed hybrid ICA-Hermite transform can compensate for the existing drawbacks of the two methods, when applied separately. A significant improvement over conventional methods is demonstrated with synthetic data, and supported by preliminary work with real EEG.

Index Terms— Artifact removal, Ballistocardiogram, Independent component analysis, Discrete Hermite transform.

1. INTRODUCTION

Simultaneous recording of electroencephalography (EEG) and functional magnetic resonance imaging (fMRI) is a powerful technique to study the human brain function. The first modality, EEG, represents the scalp potentials. The recorded voltage by EEG reflects the synaptic activities inside the brain with high temporal resolution in the order of millisecond. The second modality, fMRI, shows local hemodynamic changes in the brain which is known as blood oxygenation level dependent or BOLD signal. In contrast to EEG, fMRI provides a signal with high spatial resolution on the order of millimeter and low temporal resolution on the order of second. Fusion of these two modalities would allow researchers to combine high resolution spatial and temporal mapping of mental activities.

A major limitation of simultaneous EEG-fMRI is the effect of artifacts introduced in the EEG. EEG signals acquired in the magnetic field suffer from two major artifacts, gradient artifact and ballistocardiogram. The gradient artifact results from switching of magnetic field gradient used for image recording. This artifact is distinguished by large amplitude and high frequency content; Moreover, it does not show significant variability over time. These characteristics allow one to subtract the gradient artifact using an average template approach [1]. The EEG signal after removing the gradient artifact shows another severe artifact known as ballistocardiogram or BCG. The ballistocardiogram is caused by movements of the EEG electrodes in the magnetic field. There is a small movement in each electrode during the cardiac pulsation and as a result, a voltage will be induced into each electrode. The ballistocardiogram artifact obscures EEG at alpha frequencies (8–13 Hz) and below, with amplitudes around 150 μV inside a magnetic field with the strength of 1.5 T [2].

The task of removing BCG artifact plays a major role for a successful EEG-fMRI integration. Different data recording techniques and signal processing algorithms have been developed to eliminate this artifact. For example, Allen et al. [3] suggested an efficient technique to reduce the ballistocardiogram by firmly bandaging the electrodes and wires to the subject. Average artifact subtraction (AAS) is one of the first algorithms which has been proposed for cancelation of the BCG artifact [3]. In this method a template is built up for the artifact by averaging over the artifact trials. The ballistocardiogram can be removed from EEG signal by subsequent template subtraction from each trial. One of the major drawbacks of this widely used method is incompatibility with artifact changes over the time. Since the shape of BCG artifact is affected by both variation of heart beat and movements of subject, the assumption of occurring similar artifact in all trials is not always valid. Moreover, in all the methods based on averaging, the reference ECG channel is needed. Niazy et al. [4] developed an algorithm using principal component analysis (PCA) known as optimal basis set (OBS). In their method, first a mixture of artifact trials is formed for each channel, then, the principal components of this mixture are calculated. In the third step, few of the first principal components are selected as the basis set. In the last step a template is created using selected basis set and subtracted from each BCG trial. This method does not lead to good results when there is additional artifacts in EEG data due to subject movement. In [5], an algorithm using discrete Hermite transform (DHT) has been proposed for BCG removal. The main objective in this method is modeling the BCG artifact using discrete Hermite transform. The shape of BCG is modeled using Gaussian functions which are initial Hermite functions. These Gaussian functions are eigenvectors of a centered or shifted Fourier matrix. In [5], the Hermite transform of EEG signal is obtained by computing the inner product between the signal and the Gaussian functions. This provides a set of transformed values corresponding to a particular shape within the EEG signal. Then, the artifact template is built using some of the transformed values and subtracted from EEG signal. As already implied, AAS and OBS work only for stationary signals, while DHT is robust against changes in the shape of signal. The main superiority of DHT lies in the fact that it analyzes a single trial, containing one BCG artifact at each stage. This enable us to remove the artifact without interferences from other trials.

Another class of artifact removal methods is based on blind source separation (BSS). Independent component analysis (ICA) is a well known BSS technique and powerful statistical algorithm to remove the ballistocardiogram in EEG [6]. The blind source separation approaches are useful when no ECG signal is available for template matching. Moreover, they do not consider that the BCG artifact is predictable. Methods using the ICA assume that the recorded

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EEG signals can be represented by a linear mixture of neural activity inside the brain, and artifacts caused by muscles and noise. On the other hand, ICA decomposes the EEG signals into a set of independent components (ICs). Removing ICs containing BCG artifact and backprojecting the remaining ICs achieves the clean EEG signals. In several studies ICA has been used for removing the BCG artifact [7] [8]. Nakamura et al. [8] evaluated the performance of different ICA algorithms for removing BCG from EEG data. In a recent method, Ghaderi et al. [9] proposed a blind source extraction technique with cyclostationary constraint to extract the BCG sources.

In this paper we propose a new BCG removal method by combining ICA and DHT to take most advantages of both methods and overcome their existing drawbacks. The main advantage of the proposed method is robustness against changes in the shape of artifact over time. The proposed method alleviates the uncertainty in choosing the right number of sources to be deflated in ICA-based methods. Moreover, an adaptive parameter selection strategy is proposed which decreases the sensitivity of DHT-based methods to variations of the model parameters.

The next section briefly describes ICA algorithm. In section 3, we present mathematical details of discrete Hermite transform. The proposed method is given in section 4. The simulation results are presented in section 5. Finally, the paper is concluded in section 6.

2. INDEPENDENT COMPONENT ANALYSIS

ICA models a set of observed data as a linear mixture of statistically independent variables [10] [11]. The basic ICA can be formulated as follows:

$$\mathbf{y}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{s}(t) = [s_1(t), ..., s_m(t)]^T$ is the vector of m source signals at time t, $\mathbf{y}(t) = [y_1(t), ..., y_l(t)]^T$ is the $l \times 1$ observation vector, $\mathbf{A}$ is an $l \times m$ matrix known as mixing matrix and $\mathbf{n}(t)$ is $l \times 1$ noise vector. ICA problem can lead to a correct solution if two main conditions are satisfied. First, the observed data is assumed to be a linear mixture of independent source signals. Second, the factorized source signals should be statistically independent. In addition to these conditions, the dimension of observation data l should be larger than the dimension of source signal m. With these assumptions, the independent components (ICs) can be retrieved by determining an $m \times l$ matrix $\mathbf{W}$, namely unmixing matrix. In general, $\mathbf{W}$ can be represented by product of $\mathbf{A}^{-1}$ and scaling and permutation matrices. After obtaining the unmixing matrix the source signals are computed by:

$$\hat{\mathbf{s}}(t) = \mathbf{W}\mathbf{y}(t). \tag{2}$$

In this work we use Infomax to remove the BCG artifact. Infomax was proposed by Bell et al. [12] and is based on maximizing the output entropy. Consequently, it maximizes the mutual information between the observations and the separated signals.

3. DISCRETE HERMITE TRANSFORM

Continuous Hermite transform is a well-known signal processing method and has found many applications in biomedical signal processing [13] [14]. DHT is a version of continuous Hermite transform with capability of applying to digital signals. DHT of a digital signal is obtained using discrete basis functions, $\mathbf{h}_k$, which are generated as a set of eigenvectors of a centered or shifted Fourier transform of that signal [15]. The eigenvectors of shifted Fourier transform, $\mathbf{F}_c$, should satisfy:

$$\mathbf{F}_c\mathbf{h}_k = j^k\mathbf{h}_k \tag{3}$$

where $j = \sqrt{-1}$ and $k = 1, \ldots, n$. As it is mentioned before $\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_n$ are basis functions for a matrix with dimension n in discrete Hermite transform. In order to generate the basis functions, a tridiagonal matrix, $\mathbf{T}$, which has the same eigenvectors as $\mathbf{F}_c$ is used [13]. The eigenvectors of matrix $\mathbf{T}$ are orthogonal to each other, since $\mathbf{T}$ is a symmetric $n \times n$ matrix. The elements of $\mathbf{T}$ can be computed using the following equations [13].

- The k-th element on the main diagonal is computed by:

$$\mathbf{T}(k,k) = -2\cos(\frac{\pi}{\sigma^2})\sin(\frac{\pi k}{n\sigma^2})\sin(\frac{\pi}{n\sigma^2}((n-1)-k)) \tag{4}$$

  where $1 \leq k \leq n$

- And k-th off-diagonal element is computed by:

$$\mathbf{T}(k,k-1) = \mathbf{T}(k-1,k) = \sin(\frac{\pi k}{n\sigma^2})\sin(\frac{\pi}{n\sigma^2}(n-k)) \tag{5}$$

  where $2 \leq k \leq n-1$

- The remaining elements of $\mathbf{T}$ are set to zero.

Parameter $\sigma \geq 1$ in the above equations, known as dilation parameter, controls the width of the digital basis functions [5]. Choosing the value of the dilation parameter is an important issue. A proper value for this parameter allows the Hermite transform to model the signal with a minimum number of terms. The Hermite transform of a digital signal, $\mathbf{x}$ of size $1 \times n$, can be computed by the inner product between the input signal and the basis functions. The result of this product is a set of transform values which is shown as:

$$\mathbf{C}_k = <\mathbf{x}, \mathbf{h}_k> \qquad k = 1, \ldots, n \tag{6}$$

In fact, the coefficients of $\mathbf{C}_k$ represent the shape contents of the signal. Therefore, the result of applying this transform to $\mathbf{x}$ (the segments of ICs containing BCG artifact) implies that the BCG can be modeled by a number (e.g. p) of elements of $\mathbf{C}_k$. This can be achieved by inverse discrete Hermite transform:

$$\hat{\mathbf{x}} = \sum_{k=1}^{p} \mathbf{C}_k\mathbf{h}_k. \tag{7}$$

4. PROPOSED METHOD

In this section, we propose an ICA-based method, followed by an adaptive DHT, to remove BCG artifact from EEG signals. We call this method ICA-DHT. It has been shown that both ICA and DHT can be used for removing BCG [5] [8] but their performances are affected by some weaknesses. The most important issue in BCG removal by ICA, is choosing the number of sources that should be deflated. The results of applying ICA to EEG signals show that BCG artifact comprises of three to six independent components, added linearly to EEG data [7] [8]. Some methods use the correlation between the estimated independent components (ICs) and the ECG channel as a criterion to select the BCG components [7]. In another method [16] only the strongest component, in terms of power, is labeled as BCG. Selecting and removing only a small number of sources as BCG may leave some artifacts in the signals. In contrast, selecting and removing a large number of sources may eliminate useful information from the EEG signals. We have found DHT as a suitable complement which can soften this problem. This can be achieved by applying DHT to those ICA sources labeled as BCG artifact. In this work, we first detect the existing peaks in BCG sources using a simple correlation based method. Then, each BCG source is divided into segments centered at the detected peaks. The segments are chosen in a way such that we have one peak in each

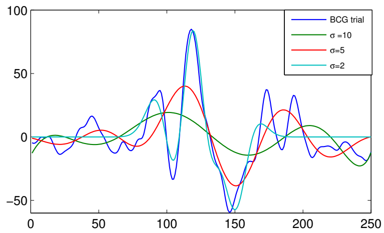


Fig. 1: BCG modeling with different values of the dilation parameter.

segment. Then, an adaptive DHT (which will be shortly introduced) will be applied to each segment in order to model the artifact. Finally, the obtained model is subtracted from each segment which gives a clean component with no artifact. All the obtained components based on this procedure will be projected back to the electrode space giving clean EEG signal.

However there are some drawbacks for DHT-based algorithms. One of the important weaknesses of DHT in BCG removal is its disability to model the whole artifact with minimum number of basis functions. In the BCG trial example, shown in Figure 1, the artifact template is made by 15 basis functions and different dilation parameters. As can be seen, the amplitude and length of reconstructed template are inversely related. Assuming a fixed number of terms, increasing the value of σ decreases the amplitude of basis functions, $\mathbf{h}_k$, and consequently decreases the amplitude of reconstructed model for BCG. Although reconstructed model by a large value for dilation parameter does not cancel the BCG artifact in terms of amplitude, it covers the whole BCG trial. In contrast, decreasing the value of σ increases the amplitude of template but decrease the length of reconstructed template. As a result it is not able to remove the whole BCG artifact. Based on these weaknesses, we need to use large number of coefficients of $\mathbf{C}_k$ to reconstruct the BCG template such that it covers the BCG artifact in terms of amplitude and length. Removing BCG with a large number of basis functions leads to removing EEG data as well.

We observed that the artifact is more localized in BCG sources than in the raw EEG signals. Moreover, when the artifacts are grouped into few components, their relative amplitudes will be larger than artifacts in raw EEG data. We found that these characteristics of BCG source (as a result of applying ICA on the raw EEG data) make them appropriate inputs for the DHT algorithm and can compensate for the drawbacks occurred by directly applying DHT to raw EEG data.

Another drawback of DHT based algorithms is sensitivity of these methods to selection of a proper value for the dilation parameter (σ in (4) and (5)). One way to decrease the sensitivity of the algorithm is by using an adaptive strategy which is proposed as follows:

$$\begin{cases} \sigma_{i+1} \leftarrow \sigma_i & cr \leq thr \\ \sigma_{i+1} \leftarrow \sigma_i - \beta \frac{cr}{max(cr)} & cr > thr \end{cases} \quad (8)$$

where cr is the normalized correlation between the ECG signal and BCG source after applying the proposed method. i is the current iteration of the algorithm. β is step size which is manually selected by user. The initial value of σ is manually selected between 10-15. The value of this parameter is updated until reaching to a predefined threshold, thr. Since BCG artifact is appeared as spike in the results of ICA, an initial large value for dilation parameter is selected. Selecting the value of σ in this range leads to removing a small portion of artifact in the first iterations due to obtaining a smooth template with low amplitude for the artifact. After several iterations the value of σ is decreased by the proposed iterative algorithm to obtain an optimal template for appeared spikes.

In what follows we summarize the steps of the proposed method for BCG removal:

- **Step 1:** Apply ICA to the EEG data contaminated by BCG artifact to separate BCG sources;
- **Step 2:** Select six ICA components which are more correlated with ECG channel;
- **Step 3:** Apply DHT with the proposed adaptive technique on each selected BCG source to find a template for the artifact. Repeat this step until $cr \leq thr$;
- **Step 4:** Subtract the template from each BCG source and backproject the residuals together with the remaining sources.

5. RESULTS

In this section, performance of the proposed algorithm for BCG removal is evaluated. Two data sets comprising synthetic and real EEG are used. The comparison between the obtained results using the proposed algorithm and other conventional methods confirms the effectiveness of the proposed methods to remove BCG artifact. We have observed that the combination of ICA and DHT can be more effective than applying any of these methods separately.

5.1. Synthetic Data

A set of five sources including artifacts and EEG rhythms are generated (Figure 2). In this figure, S1 and S3 are 9Hz and 12Hz sine waves respectively, S2 represents BCG artifact, S4 is random noise and S5 is a periodic signal. The sources are mixed by a random 5×5 matrix. Three methods, Infomax, DHT, and the proposed method are applied to synthetic EEG data set in order to remove the BCG artifact. In order to evaluate the results, the normalized correlation between the extracted sources and the actual EEG sources is computed. This value is calculated for two sources S1 and S3 as these sources are considered as EEG rhythms. Table 1 shows the obtained results when different noise levels are added to the mixtures. The number of coefficients of $\mathbf{C}_k$ used to model the artifact are 15 and 8 for DHT and ICA-DHT respectively. The dilation parameter when DHT is used to remove the BCG artifact, is selected 3. The obtained value of this parameter for ICA-DHT, using the proposed adaptive strategy, is 2.5. Comparing the obtained results in Table 1 shows that the performance of Infomax and DHT decreases at higher level of noise, while the proposed method is still able to remove the BCG artifact in these noise levels. Figure 3 shows a segment of reconstructed S3 using different methods at SNR=5dB.

5.2. Real Data

The real EEG data set used in this work comprises of 64 EEG channels with sampling rate of 10KHz from two subjects. In order to decrease the computational time, we selected 32 of these channels.

Table 1: Normalized correlation between the extracted and actual simulated EEG sources.

		No noise	SNR=20dB	SNR=10dB	SNR=5dB
Infomax	S1	0.926	0.882	0.622	0.574
	S3	0.998	0.928	0.723	0.649
DHT	S1	0.934	0.899	0.828	0.634
	S3	0.970	0.9387	0.818	0.369
ICA-DHT	S1	0.980	0.935	0.862	0.802
	S3	0.998	0.962	0.897	0.872

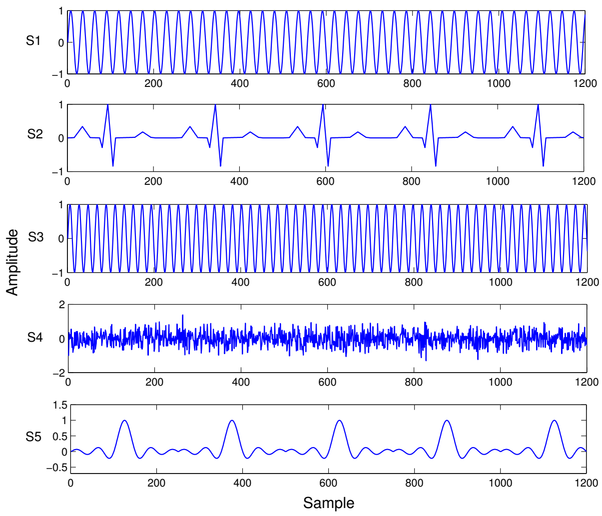


Fig. 2: Five simulated EEG sources.

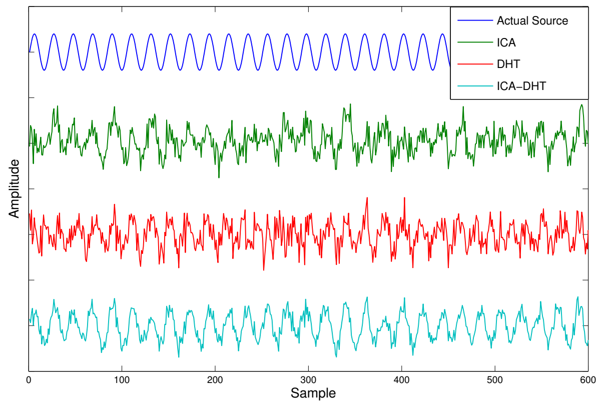


Fig. 3: Extracted sources for S3 after BCG removal when the level of noise is 5dB.

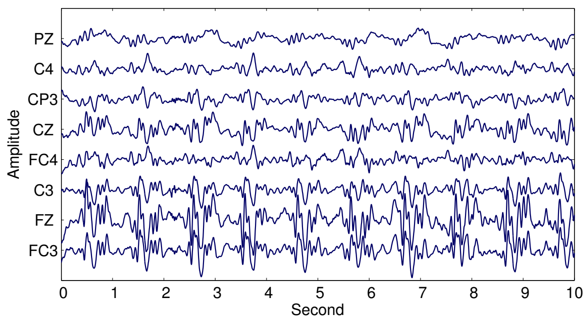


Fig. 4: 10 EEG channels contaminated with BCG artifact.

All the preprocessing steps used for this data set are: 1) removing gradient or imaging artifact using the method proposed by Niazy et al. [4]. 2) Down-sampling data to 250Hz. 3) Bandpass filtering using a Butterworth filter. The cutoff frequencies for low-cut and high-cut frequencies are selected as 0.5Hz and 45Hz respectively. A 5s segment of data from 8 channels after preprocessing is shown in Figure 4. After preprocessing, Infomax ICA is applied to EEG

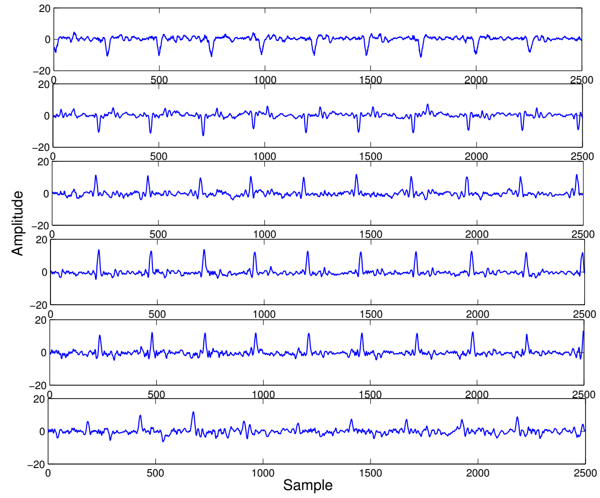


Fig. 5: Extracted BCG sources.

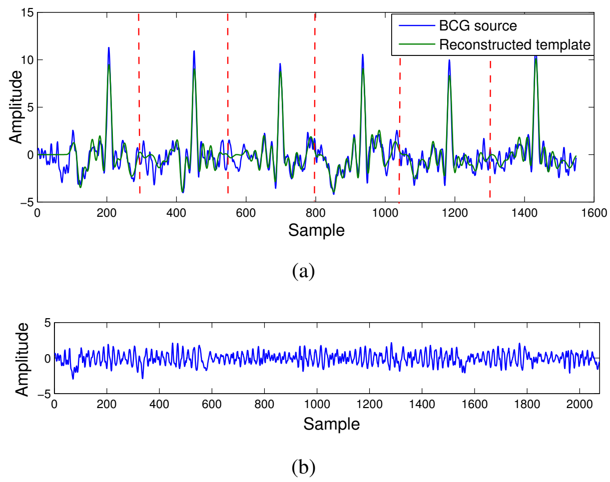
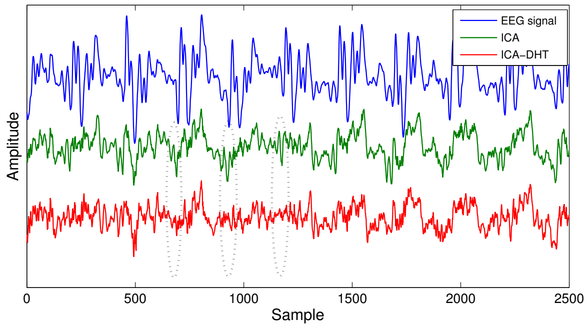
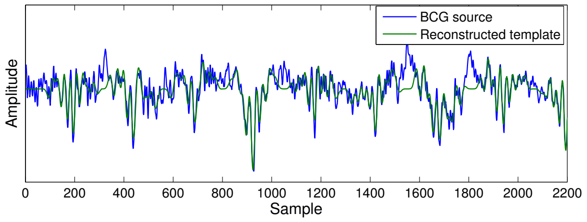


Fig. 6: (a) The reconstructed template and (b) the residual.

data to separate all the sources. The number of sources are selected equal to the number of sensors. We used 10-second segments of EEG for this purpose. After applying ICA, the extracted sources are clustered based on their correlations with ECG channel. The clustered BCG artifact sources for a sample segment are shown in Figure 5. As can be seen there are some spikes in these sources which are due to BCG artifact. Hence, DHT is used here for modeling and removing these spikes from the selected sources. For this purpose, these sources are segmented such that the spikes fall in the center of each segment. In order to have one artifact per each segment, with no overlap, the length of each segment is selected to be 256. This is due to the fact that the period of BCG artifact is approximately one second, and the sampling rate of data is 250Hz. The reconstructed template for BCG artifact for one of the sources is shown in Figure 6 (a). The residual as a result of subtracting the BCG source and the template is presented in Figure 6 (b). As can be seen, the residual contains brain rhythms retrieved by the proposed method. In this work, 15 coefficients (5% of total coefficients) from $\mathbf{C}_k$ are selected to model the artifact. The reason for selecting such a small fraction is to avoid losing useful EEG information while removing BCG. The main advantage of the proposed algorithm is its ability to remove the artifact from the BCG sources containing brain



(a)



(b)

Fig. 7: (a) Comparison between ICA and ICA-DHT for BCG removal from EEG. (b) The BCG source, not deflated in ICA, together with corresponding model obtained in ICA-DHT.

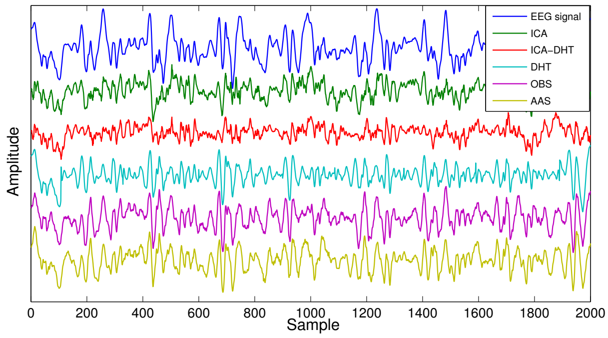


Fig. 8: Results of artifact removal from CZ channel

rhythms such as the source shown at the bottom of Figure 5. Figure 7 (a) shows the results of artifact removal from one "EEG channel" using ICA and ICA-DHT which clearly shows the advantage of the proposed method in removing more portions of artifact without losing any EEG information. The highlighted areas show some peaks of BCG artifact which have not been removed by ICA. The number of deflated sources in ICA is five, while we selected six sources to deflate in ICA-DHT method. Figure 7 (b) shows the BCG source which is not deflated in ICA.

Figure 8 shows the results of applying different methods for a segment of EEG signal in CZ channel. The results confirm that the proposed method removes the BCG artifact more efficiently than the other methods. Our experiments on BCG removal for real EEG data set show that the dilation parameter converges to a value around four. The averaged dilation parameter obtained as a result of applying the proposed method to 40 segments of EEG signals (each segment comprises of 32 channel and 10 second signal) was 4.25. The threshold used for normalized correlation between the ECG channel and the BCG sources after applying the ICA-DHT was set to 0.001.

6. CONCLUSION

In this paper the problem of BCG removal from EEG data has been addressed. A new method, called ICA-DHT, has been proposed for this purpose. The proposed method takes the advantages of both ICA and DHT for removal of BCG artifact. We also proposed an adaptive dilation parameter which decreases the sensitivity of DHT to the selection of this parameter. The experimental results have been promising and showed the effectiveness of the proposed method in removing the BCG artifacts.

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