INFORMATION RATE OF HUMAN MANUAL CONTROL OF UNSTABLE SYSTEMS

Mircea F. Lupu[1], *Mingui Sun*[2], *Zhi-Hong Mao*[1]

University of Pittsburgh
[1]Department of Electrical and Computer Engineering
[2]Department of Neurological Surgery
Pittsburgh, PA 15261 USA

ABSTRACT

Human operators are active components of feedback control systems. They are faced with challenges especially when the system is unstable or when time delay affects the feedback loop. In this study, the human operator is assumed to act as a dynamical system, and its complexity and performance is analyzed through the entropy rate of its generated output command.

We investigated the characteristics of the human operator when controlling an unstable system under different time delays and higher degrees of task instability. The entropy rate for dynamical systems, also called the Kolmogorov-Sinai entropy, was estimated based on the numerical method proposed by Grassberger and Procaccia.

Index Terms— Human-in-the-loop, Kolmogorov-Sinai entropy, entropy estimation, correlation sum, time delay

1. INTRODUCTION

The performance of human operators as components of feedback control systems is an important factor in human-in-the-loop (HIL) systems, especially when the controlled system is unstable or when time delay is present in the feedback loop. The human controller shown in Fig. 1 is expected to generate the sequence of commands $\mathbf{u}$ relative to the perceived sensory input $\mathbf{e}$ in order to achieve the desired outcome $\mathbf{y}$ of the closed-loop system. The human controller thus resembles characteristics of a dynamical system.

The goal of this paper is to evaluate the performance of the human controller by means of information exchange between human and machine. The sequence of control movements is assumed to be a time series generated by a dynamical system, and its entropy rate is to be estimated as a measure of the ability of the human controller to convey information. This analysis can aid in the design of efficient HIL systems by identifying the limitations of the human controller and matching its capabilities to the task requirement.

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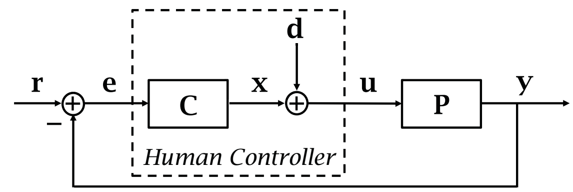

Fig. 1. HIL system: $\mathbf{r}$ - reference signal, $\mathbf{u}$ - human command, $\mathbf{d}$ - disturbance related to manual control, $\mathbf{y}$ - output signal.

Introduced by Shannon [1] for communication channels in what became information theory, the entropy rate was later extended by Kolmogorov [2] and Sinai [3] to quantify the complexity of dynamical systems. In this paper, the entropy rate is interchangeable with information rate. If the signal generated by the dynamical system follows some probability distribution, and the transition between the observed numbers is well-defined, then we can infer knowledge about the internal states of the system and their evolution over time. The internal states of the human sensory-motor system are very difficult, if not impossible, to determine. However, the entropy rate seems to provide insight about both the average information needed to encode a state with a certain accuracy, and the quantity of information needed to predict future observations given past measurements. The analysis is also linked to prediction abilities, because the entropy rate scales inversely with the time scale of prediction.

Fitts [4] was the first to suggest quantifying the amount of information of a movement (in bits) by its index of difficulty, and measuring the time it takes to perform that movement (in seconds). Then the human motor system was said to generate information at a rate measured in ”bits per second” when carrying out that movement. The observed human control signal, similar to the observation of any dynamical system, can convey information, and the entropy rate is the quantity that measures the average amount of information generated per unit time. Throughout our investigation the human controller is regarded as an information generation source, and its performance is assessed by its capacity to generate information while performing a manual control task.

2. METHODS

We characterize the human controller by estimating the entropy rate of its control command $\mathbf{u}$. This signal is in fact a time series $\{u(k\Delta t)\}_{k=1}^{N}$ because it is being recorded at a fixed sampling period of $\Delta t = 10$ ms (i.e. 100 Hz). The method used to estimate the entropy rate was developed by Gaspard and Wang [5] based on the idea of Grassberger and Procaccia [5, 6]. We refer to [7] as the main reference for the entropy estimation from a time series.

The recorded time series of the human control signal was obtained from an experiment where the human controller balanced an inverted pendulum simulation. Various scenarios were created by varying the pendulum length and including different time delays in the HIL system.

2.1. Entropy rate estimation

Consider a dynamical system with an D-dimensional phase space, and assume that the state of the system $\mathbf{x}(t)$ is measured at finite time intervals τ. If the phase space is covered by a partition $\mathcal{P}_\epsilon$, then the joint probability $p(\gamma_1, \gamma_2, ..., \gamma_m)$ is defined as the probability that the state of the system visits successively the partition elements $\gamma_1, \gamma_2, ..., \gamma_m$ at times $t, t+\tau, ..., t+(m-1)\tau$. The the order-$q$ Renyi block entropy of block size m can be formulated as

$$H_q(m, \mathcal{P}_\epsilon) = \frac{1}{1-q} \log \sum_{\mathcal{P}_\epsilon} p^q(\gamma_1, \gamma_2, ..., \gamma_m) \quad (1)$$

where the log is taken in base 2 to measure the entropy in *bits*. Then the order-q generalized entropies are

$$h_q(m, \mathcal{P}_\epsilon) = H_q(m+1, \mathcal{P}_\epsilon) - H_q(m, \mathcal{P}_\epsilon) = \frac{H_q(m, \mathcal{P}_\epsilon)}{m}$$

$$h_q = \sup_{\mathcal{P}_\epsilon} \lim_{m\to\infty} h_q(m, \mathcal{P}_\epsilon) \quad (2)$$

The Kolmogorov-Sinai entropy h_{KS} is obtained by taking the limit $q \to 1$ in (2). As a measure of the order of the system [5, 8], the entropy rate is able to asses the type of the dynamical system:

- $h_{KS} = 0 \Rightarrow$ system is deterministic;
- $h_{KS} = ct \neq 0 \Rightarrow$ system is deterministically chaotic;
- $h_{KS} \to \infty \Rightarrow$ system is stochastic.

Computing h_{KS} directly by taking the supremum over all ϵ-size partitions (usually implies $\epsilon \to 0$) and the limit $m \to \infty$ is impractical for a finite-sample time series. Instead, Gaspard and Wang [9] suggested analyzing the entropy rate $h_q(m, \epsilon)$ based on its scaling with the resolution ϵ and the block size m. They referred to it as *coarse grained dynamical entropy* or *ϵ-entropy per unit time*. This method is said to be a generalization of h_{KS} to stochastic systems because it considers the length scale ϵ-dependence in (2) to reveal additional information about the system.

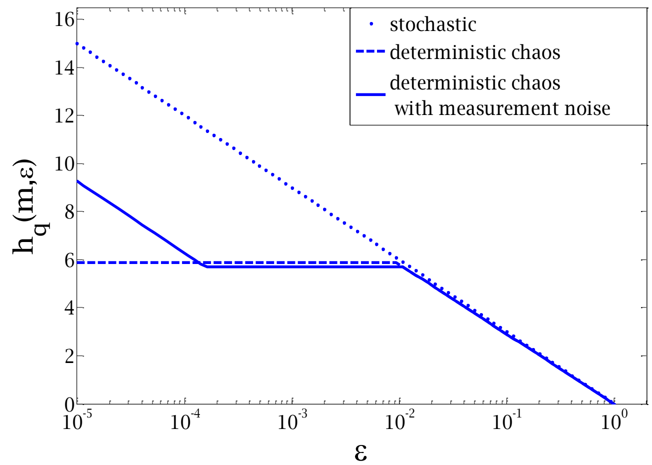

Fig. 2. Coarse grained entropy $h_q(m, \epsilon)$ for a stochastic signal (dotted line), deterministically chaotic signal (dashed line), and deterministically chaotic with measurement noise (solid line). The plateau for intermediate values of ϵ is estimated to be h_{KS}. Figure adapted from [7].

The general shape of $h_q(m, \epsilon)$ is shown in Fig. 2 for different length scales ϵ. If ϵ is large then we have that $h_q(m, \epsilon) = 0$, because the neighborhood size covers the whole range of observables. For deterministically chaotic systems (dashed curve), $h_q(m, \epsilon)$ scales to a non-zero constant for small values of ϵ and m sufficiently large. It was shown [5, 6, 9] that this value is a good numerical approximation to the Kolmogorov-Sinai entropy: $h_q(m, \epsilon) \approx h_{KS}$. For stochastic systems (or deterministic systems with m not sufficiently large) we have that $h_q(m, \epsilon) = -\log \epsilon + h_c(m)$ (dotted curve), where h_c is a constant which represents the continuous entropy rate [7]. Therefore, when $\epsilon \to 0$ the entropy rate diverges: $h_q(m, \epsilon) \to \infty$. For a deterministic system with measurement noise (solid curve) $h_q(m, \epsilon)$ exhibits a plateau in an intermediate range of ϵ (when $h_q(m, \epsilon) \approx h_{KS}$) and then scales like $-\log \epsilon$ for $\epsilon \to 0$.

Following the protocol of Grassberger and Procaccia [5], we consider the case $q = 2$ in (2) due to its more convenient and robust numerical computation. The relationship between h_2 and h_{KS} is that $h_2 \leq h_{KS}$, and they are both non-negative. Moreover, $h_2 \to \infty$ for stochastic systems and $h_2 = ct \neq 0$ for chaotic systems, which implies that $h_2 > 0$ is a sufficient condition for chaos.

The entropy rate $h_2(m, \epsilon)$ can be defined in terms of the correlation sum [5, 6]

$$h_2(m, \epsilon) = \frac{1}{\tau} \log \frac{C(m, \epsilon)}{C(m+1, \epsilon)} \quad (3)$$

The correlation sum computes the probability that two states of the system at different times are closer than a threshold ϵ

$$C(\epsilon) = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j \neq i}^{N} \Theta(\epsilon - ||\mathbf{u}(i) - \mathbf{u}(j)||) \quad (4)$$

where $\Theta(x)$ is the Heaviside step function: $\Theta(x) = 0$ if $x \leq 0$, and $\Theta(x) = 1$ if $x > 0$, and $||\cdot||$ is a norm. Notice that the states of the system are not known to us, but instead the scalar measurements $\{u(k\Delta t\}_{k=1}^{N}$ are available. Because our observables are only a projection of the internal states of the system onto an interval on the real axis, a reconstruction of the phase space is required. Takens's delay embedding theorem [10] suggests that the evolution of the delay vector

$$\mathbf{u}(k) = [u(k), u(k-\tau), ..., u(k-(m-1)\tau]^T \quad (5)$$

for $k = (m-1)\tau+1, ..., N$, reflects the same dynamics of the unknown state space (in the sense of mapping onto each other by a uniquely invertible smooth map) for the embedding dimension m sufficiently large. The lag τ can be larger than the sampling period Δt in order to avoid temporal correlation between samples. Consequently, the state of the original system in (4) was substituted by its vector approximation $\mathbf{u}$.

Thus, the computation of the entropy rate $h_2(m, \epsilon)$ by using the correlation sum is very attractive because it requires only the computation of the arithmetic average over the number of neighbors. Moreover, it was shown not to introduce any bias in the estimation of the correlation sum due to finite statistics [7].

The dependency of $h_2(m, \epsilon)$ on the embedding dimension m reveals important information about the dynamical system. For the delay vector to resemble the same behavior as the actual state of the system, an important requirement is that $m \geq 2D$, where D is the dimension of the attractor in the phase space [10]. This is the necessary condition for what we referred to earlier as m being "sufficiently large". Furthermore, the rate of convergence of $m \to \infty$ is proven to relate to the strength of correlations in the system [6], thus providing an insight into the memory of the system. However, careful interpretation of the results for very large values of m is required due to the finite-sample time series.

The estimation of the entropy rate $h_2(m, \epsilon)$ was performed by adapting the TISEAN software package from [7]. The time series was normalized to the interval $[0, 1]$, and a maximum embedding dimension of $m = 10$ was considered sufficient using the false nearest neighbors algorithm. The time delay τ from the delay vector (5) was estimated by determining the first minimum of the delayed mutual information according to [7].

2.2. Human Manual Control of an Inverted Pendulum

The HIL system considered in our study involves a human controller balancing a planar inverted pendulum simulation using a joystick (Fig. 3(a)). An inherently unstable system is

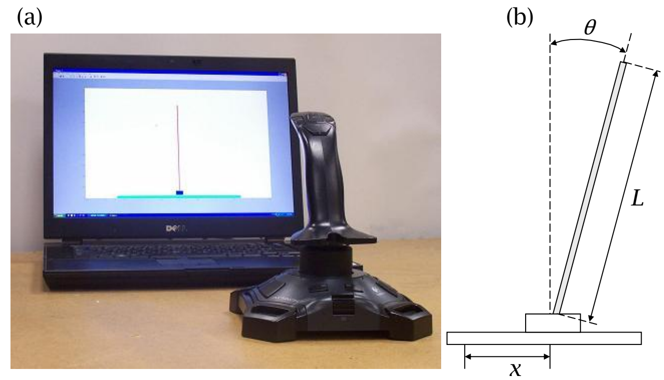

Fig. 3. Inverted pendulum restricted in a plane. (a) Computer simulation adapted from [11]. (b) Definition of variables: θ is the angle deviation from the upright position; L is the pendulum length; x is the applied displacement to the bottom tip of the pendulum.

very relevant in human-machine interaction applications such as rocket or missile guidance, piloting an unstable airplane, and teleoperation where the system exhibits unstable behavior.

The inverted pendulum system (Fig. 3(b)) considers the control variable to be the displacement x applied to the bottom tip of the pendulum. The output of the inverted pendulum system is the angle θ which is to be held as small as possible. The linearized dynamics of the inverted pendulum system yields two poles of which one is real and positive. This unstable pole varies inversely with the length of the pendulum: $p = \sqrt{3g/2L}$, where L is the length of the pendulum, and g is the gravitational acceleration. Its magnitude reflects the degree of instability of the system.

In order to apply the method for entropy rate estimation, the time series must be stationary. Therefore, the time series used for the computation is the velocity of the generated joystick movements (the time derivative of the displacement x), which is assumed to be stationary. It is common practice to use increments or returns to render a stationary signal from a non-stationary signal [7].

The performance of the human controller was analyzed during two scenarios: (1) when introducing various amounts of time delay in the feedback system, and (2) when changing the degree of instability of the feedback system by varying the pendulum length. For the former scenario a pendulum length of 20 m was used for the following amounts of time delay: 0, 200, 400, 600, 800, and 1000 ms. For the latter scenario the pendulum length was changed to 6 m and 3 m with no time delay. Ten trials were recorded for each time delay and each pendulum length. The duration of a trial was 60 seconds, unless the human operator dropped the pendulum which ended the trial.

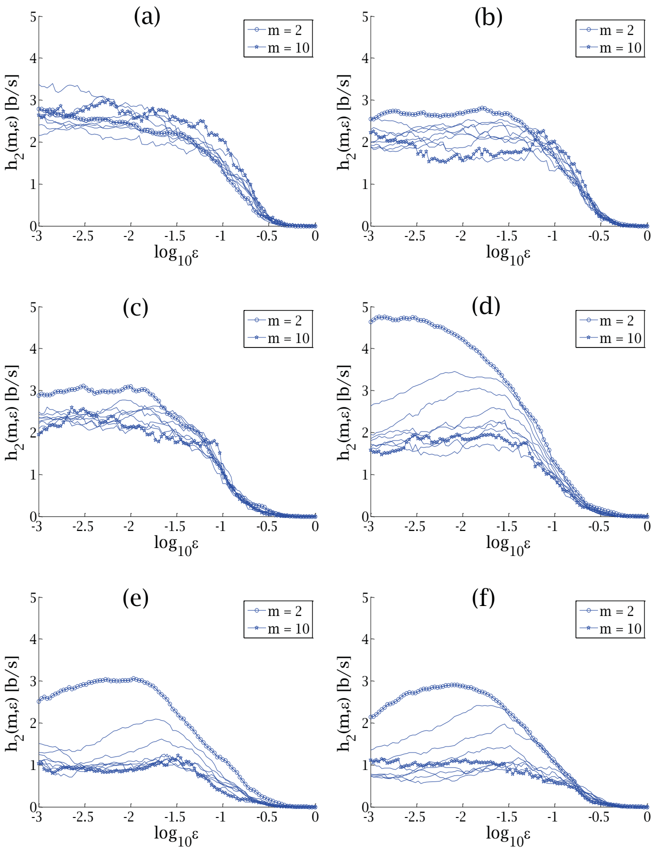

Fig. 4. Entropy rate estimation for (a) no time delay, (b) 200ms, (c) 400 ms, (d) 600 ms, (e) 800 ms, (f) 1000 ms time delay. The curves correspond to the embedding dimensions 2-10 while $m = 2$ (circles) and $m = 10$ (stars) are emphasized.

3. RESULTS

The entropy rate $h_2(m, \epsilon)$ as a function of the embedding dimension m and the neighborhood size ϵ is illustrated in Fig. 4 for different time delays, and in Fig. 5 for smaller pendulum lengths. With increasing m, the entropy rate settles to a plateau for smaller values of ϵ in all considered scenarios. As mentioned earlier, this is an indication that the human controller resembles characteristics of a deterministically chaotic system.

The information rate of the control movements was observed to decrease with the amount of time delay in the feedback loop and to increase for the shorter-length pendulum (Fig. 6).

These results can be correlated with our previous work [12] which showed that human operators have to adapt their control actions to low-frequency bandwidth movements in order to maintain stability of the system as more time delay affects the task. Therefore, the entropy rate as a measure of information content per unit time carried by the control movements implies that the human controller is limited to generating higher information-content movements when adapting to the bandwidth constraint.

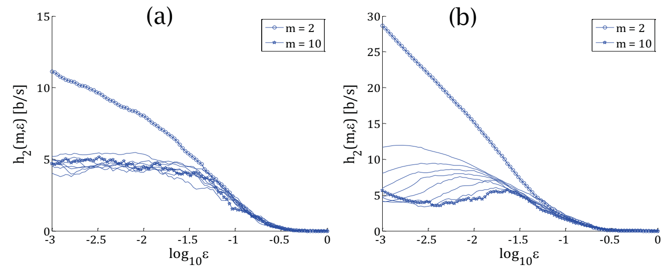

Fig. 5. Entropy rate estimation for shorter pendulum length: (a) 6 m, (b) 3 m. The curves correspond to embedding dimensions 2-10 while $m = 2$ (circles) and $m = 10$ (stars) are emphasized.

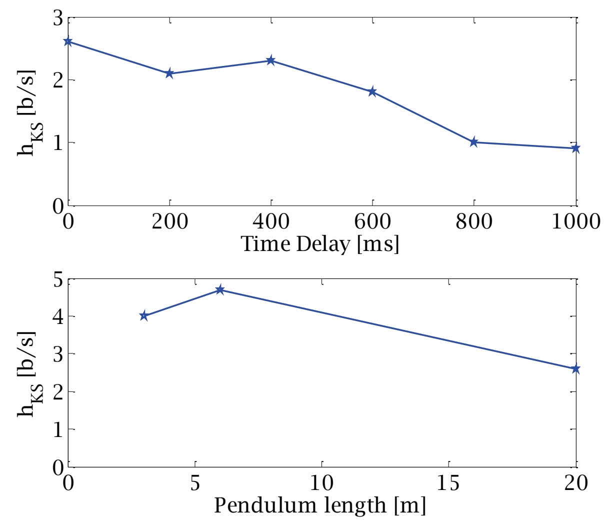

Fig. 6. The variation of the estimated entropy rates with time delay (above), and with pendulum length (below).

When the control task is made more difficult by decreasing the pendulum length, the human operator is constrained to generate higher frequency movements [12]. The estimated entropy rate increased accordingly from 2.6 b/s for a pendulum of 20 m to 4.7 b/s for a pendulum of 6 m. Thus, the human operator was able to compensate for the degree of instability of the task by delivering more information.

However, when the pendulum length was 3 m, the information rate was observed to slightly decrease. This result seems to imply that the human controller is not able to ramp up the frequency of generating movements without sacrificing the accuracy and the effectiveness of its control signal. This aspect of manual control can be explained by signal-dependent noise in motor control [13].

4. DISCUSSION

The estimated entropy rate showed that the capability of the human operator to generate information while performing a control task in a HIL system is reduced with increasing time delay in the feedback loop. Moreover, the human operator was observed to adapt to the tasks with a higher degree of instability by generating movements at a higher information rate. However, results show that this capability reaches a limit and even decreases after a certain degree of instability.

The idea that the information rate scales inversely with the amount of time delay implies that the human movements become more regular. The human controller apparently selected its actions more carefully, most probably using its prediction capabilities, under the challenge of time delay. Recent work [14] has proven that there should be a minimum amount of information flow in the feedback system to guarantee the stability. This minimum information rate increases with the degree of instability of the task according to $p \log_2(e)$ b/s. The limitation of the human controller can thus be predicted when the estimated entropy rate approaches this lower bound.

The entropy rate was more difficult to estimate in the situations when the time delay was large (i.e. 600-1000 ms) and when the length of the pendulum was very short (i.e. 3 m). The spurious spread of the curves in the entropy rate plot may be caused by the fact that less data points were recorded for these trials. Due to the increased difficulty to perform the task, the human controller dropped the pendulum before the 60 seconds elapsed. It is important to acknowledge that this method has its limitations when analyzing high dimensional attractors at small length scales ϵ due to sparse data. It has been suggested [7] that the minimum length of the time series should satisfy $N_{min} > \epsilon^D$ for a consistent result.

The information-rate analysis applies to any situation when the human output is a time series. It is able to encode the human controller's accuracy in sensing the feedback signal, its ability to mentally process the command, and the limitations of the muscles used in performing the task. Therefore, information-rate analysis may be used as a general statistic to measure human performance and predict its limitations.

5. CONCLUSIONS

This investigation analyzes human operator performance in a HIL system from the perspective of a dynamical system by estimating the entropy rate of its control signal.

The scaling of the entropy rate to a constant value suggests that the human controller system resembles characteristics of a deterministically chaotic system. The performance of the human operator as an information source diminishes with increasing time delay in the feedback system and with increasing degree of task instability. These preliminary conclusions are to be investigated in the future on more human subjects.

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