

# SUBSPACE-BASED BLIND CHANNEL ESTIMATION IN CP SYSTEMS BY REPEATED USE OF REMODULATED RECEIVED BLOCKS

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## ABSTRACT

Blind channel estimation has been studied in various types of communication systems since it has a better bandwidth efficiency than training-based counterpart. In communication systems employed with cyclic prefixes (CP), subspace-based (SS) methods are among the most popular categories of blind channel estimation schemes. Existing SS methods, however, either require a large amount of received data or possess a high computational complexity. In this paper, a new algorithm for blind channel estimation in CP systems that require few received blocks with a reasonable complexity is proposed. The idea is based on combing advantages from two previously reported SS methods, namely, remodulation and repeated use of each received block. The combination of the two separate ideas turns out to be superior to each of them alone in many aspects. Simulation results not only confirm the capability of the proposed method to work properly with very few received blocks, but also show that it outperforms all previously reported methods. An extension of the proposed algorithm to MIMO case is promising.

**Index Terms**— Blind channel estimation; subspace-based; cyclic prefix; repetition index; remodulation

## 1. INTRODUCTION

Blind or semi-blind channel estimation has been studied in many modern communication systems due to its advantage to save bandwidth efficiency. Subspace-based (SS) methods belong to one of the most popular categories of blind channel estimation algorithms because they do not require additional constraints on transmitted signals such as finite-alphabet or constant modulus conditions and can be directly applied without much modification of transmitter structures. However, SS methods have some well-known drawbacks, such as requirement of knowledge of exact channel order, requirement of a large amount of received data, and a higher computational complexity than training-based channel estimators.

In recent years, research efforts of blind channel estimation methods have greatly switched to block transmission systems with guard intervals [1], such as orthogonal frequency division multiplexing (OFDM) systems [2, 3]. These studies have shown that many aforementioned drawbacks of SS methods, when applied in block transmission systems, are greatly improved or even resolved. For example, in block transmission systems employing zero-padding (ZP) [1] and those using cyclic prefixes (CP) [2], SS methods are no longer

sensitive to the problem of channel order overestimation as long as the channel order is upper bounded by the length of guard interval. In addition, while SS methods are usually expected to require a large amount of received data in order to obtain accurate second-order statistics, efforts in [3–5] have shown that the number of received blocks required for SS methods can be greatly reduced by using each received block repeatedly.

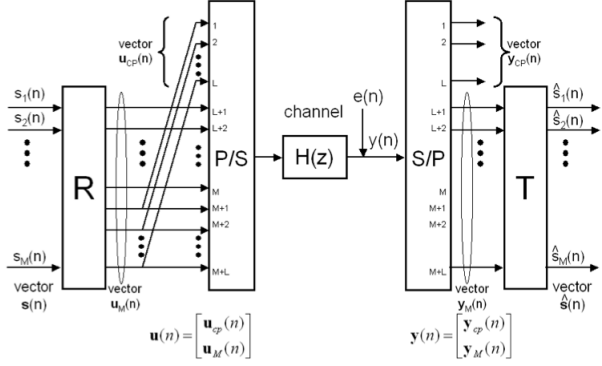
Among all blind channel estimation algorithms for redundant block transmission systems, the methods for ZP systems usually deal with a relatively simple matrix equations. However, these methods are not directly applicable to most of the currently popular systems such as CP-based OFDM systems. Existing methods for CP systems, on the contrary, usually have a much higher computational complexity and requires a larger amount of received data than their counterparts for ZP systems. Recently, a new subspace method for CP systems based on *remodulation* of received blocks is proposed [6] which yields to a matrix equation as simple as that in a ZP system. The method has a much better computational complexity than those in [2, 3] while having a rather satisfactory channel estimation performance. However, it still is not able to perform channel estimation when the number of received blocks is limited.

In this paper, we propose a method that combines ideas of remodulation and repetition of received blocks. Numerical results show that the proposed method not only is able to perform satisfactory channel estimation with a limited amount of received data, but also outperforms all existing methods with the same amount of received data. Furthermore, it possesses a fairly good computational complexity among all existing methods, just slightly greater than [6]. The proposed method is easily extended to the case of multiple-input-multiple-output (MIMO) scenario. Due to page length and for discussion simplicity, we focus only on single-input-single-output (SISO) systems in this presentation.

The rest of the paper is organized as follows. Section 2 gives the problem formulation and briefly reviews existing subspace-based methods for channel estimation in CP systems. Section 3 presents the proposed method and Section 4 contains the numerical results to compare the performances of all methods. Conclusions are given in Section 5.

### 1.1. Notations

Boldfaced lower case letters represent column vectors. Boldfaced upper case letters and calligraphic upper case letters



**Fig. 1.** System model for a SISO cyclic prefix block transmission system.

are reserved for matrices. Superscripts  $*$ ,  $T$ , and  $\dagger$  as in  $a^*$ ,  $\mathbf{A}^T$ , and  $\mathbf{A}^\dagger$  denote the conjugate, transpose, and transpose-conjugate operations, respectively. All the vectors and matrices in this paper are complex-valued. The matrix  $\mathbf{W}_M$  represents the  $M \times M$  normalized DFT matrix whose  $kl$ -th entry is  $e^{-j2\pi(k-1)(l-1)/M}/\sqrt{M}$ .  $\mathbf{I}_M$  is the  $M \times M$  identity matrix, and  $\mathbf{0}_{m \times n}$  is the  $m \times n$  zero matrix. For any  $m \times 1$  vector  $\mathbf{v} = [v_1 \ v_2 \ \dots \ v_m]^T$  and any positive integer  $n$ , we use  $\mathcal{T}_n(\mathbf{v})$  to denote the  $(m+n-1) \times n$  full-banded Toeplitz matrix

$$\mathcal{T}_n(\mathbf{v}) = \begin{bmatrix} v_1 & 0 & \dots & 0 \\ v_2 & v_1 & \ddots & \vdots \\ \vdots & v_2 & \ddots & 0 \\ v_m & \vdots & \ddots & v_1 \\ 0 & v_m & & v_2 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & v_m \end{bmatrix}. \quad (1)$$

## 2. SYSTEM MODELS AND PROBLEM FORMULATION

### 2.1. Problem Formulation

Figure 1 depicts a typical communication systems using cyclic prefix(CP). The source vector  $s(n)$  is first precoded by an  $M \times M$  invertible matrix  $\mathbf{R}$ , resulting in precoded data  $\mathbf{u}_M(n)$ . In particular, for OFDM or multi-carrier (MC) systems,  $\mathbf{R} = \mathbf{W}_M^\dagger$  is the normalized IDFT matrix; for single-carrier cyclic prefix (SC-CP) systems,  $\mathbf{R}$  is chosen as  $\mathbf{I}_M$ . A cyclic prefix of length  $L$ , taking from the last  $L$  elements of  $\mathbf{u}_M(n)$ , is defined as  $\mathbf{u}_{cp}(n) = [\mathbf{0}_{L \times (M-L)} \ \mathbf{I}_L] \mathbf{u}_M(n)$ . We assume  $L+1 < M$ . The cyclic prefix is appended to  $\mathbf{u}_M(n)$ , forming a vector  $\mathbf{u}(n) = [\mathbf{u}_{cp}(n)^T \ \mathbf{u}_M(n)^T]^T$  whose length is  $M+L$ . The vector  $\mathbf{u}(n)$ , after parallel-to-serial conversion, is sent over the channel  $H(z)$ , which is assumed to be an FIR channel with a maximum order  $L$ , i.e.,  $H(z) = \sum_{k=0}^L h_k z^{-k}$ . We define  $\mathbf{h}$  as the  $(L+1) \times 1$  column vector  $[h_0 \ h_1 \ \dots \ h_L]^T$ . The received symbols  $y(n)$  are corrupted by an additive white complex Gaus-

sian noise  $e(n)$  and are blocked into  $(M+L) \times 1$  vectors  $\mathbf{y}(n)$ . We assume perfect block synchronization between the transmitter and receiver. Also let  $\mathbf{e}(n)$  denote the blocked version of the noise  $e(n)$ . Denote  $\mathbf{y}_{cp}(n)$  as the first  $L$  entries and  $\mathbf{y}_M(n)$  as the last  $M$  entries of  $\mathbf{y}(n)$  so that  $\mathbf{y}(n) = [\mathbf{y}_{cp}(n)^T \ \mathbf{y}_M(n)^T]^T$ . It can be shown that

$$\mathbf{y}_M(n) = \mathbf{H}_{cir} \mathbf{u}_M(n) + \mathbf{e}_M(n) \quad (2)$$

where  $\mathbf{H}_{cir}$  is an  $M \times M$  circulant matrix whose first column is  $[h_0 \ \dots \ h_L \ 0 \ \dots \ 0]^T$  and  $\mathbf{e}_M(n) = [\mathbf{0} \ \mathbf{I}_M] \mathbf{e}(n)$ . The  $L \times 1$  vector  $\mathbf{y}_{cp}(n)$  contains interblock interference (IBI) and can be expressed as

$$\mathbf{y}_{cp}(n) = \mathbf{H}_l \mathbf{u}_{cp}(n) + \mathbf{H}_u \mathbf{u}_{cp}(n-1) + \mathbf{e}_{cp}(n) \quad (3)$$

where  $\mathbf{H}_l$  is an  $L \times L$  lower triangular matrix whose first column is  $[h_0 \ \dots \ h_{L-1}]^T$ ,  $\mathbf{H}_u$  is an  $L \times L$  upper triangular matrix whose first row is  $[h_L \ \dots \ h_1]^T$ , and  $\mathbf{e}_{cp}(n) = [\mathbf{I}_L \ \mathbf{0}] \mathbf{e}(n)$  is the noise component. We use the entire content of  $\mathbf{y}(n) = [\mathbf{y}_{cp}(n)^T \ \mathbf{y}_M(n)^T]^T$  in the formulation of the blind channel estimation problem.

The blind channel estimation problem in CP systems can be stated as follows. Given  $J$  received blocks  $\mathbf{y}(n)$ ,  $n = 0, 1, \dots, J-1$ , how do we estimate the channel coefficients  $h_0, h_1, \dots, h_L$  up to a scalar ambiguity?

### 2.2. Review of Existing Subspace-based Methods

One of the major difficulties for SS blind channel estimators in CP systems over ZP systems is the interblock interference (IBI) present in received CP  $\mathbf{y}_{cp}(n)$ . The  $n$ th received block,  $\mathbf{y}(n)$ , depends not only on the  $n$ th transmitted block  $\mathbf{u}_M(n)$ , but also on the CP part of the previous block  $\mathbf{u}_{cp}(n-1)$ . It is therefore not possible to use  $\mathbf{y}(n)$  alone in the subspace method since the signal space already occupies the whole observation space whose dimension is  $M+L$ . One needs to come up with either a method that has a larger observation space or one that has a smaller number of unknown parameters. In [2], a composite block  $\bar{\mathbf{y}}(n)$  composed of contents from two consecutive blocks  $\mathbf{y}(n-1)$  and  $\mathbf{y}(n)$  defined as

$$\bar{\mathbf{y}}(n) = [\mathbf{y}_M(n-1)^T \ \mathbf{y}_{cp}(n)^T \ \mathbf{y}_M(n)^T]^T \quad (4)$$

is used which has a length of  $2M+L$ . The observation space is  $2M+L$ , strictly larger than its signal space  $2M$ , therefore making subspace method possible. In [3], a generalization of the method in [2] was proposed by repeated use of each composite block. A parameter called *repetition index*  $Q$  is defined to indicate the number of columns each composite block can generate. It has an observation space whose dimension is  $2M+L+Q-1$ , strictly larger than the dimension of its signal space,  $2M+Q-1$ .

In [6], a different approach to formulate the signal-noise separation called *remodulation* is proposed. Instead of concatenating contents from two consecutive received blocks which results in a long vector, the remodulation method uses the *difference* of contents from two consecutive blocks, defined as

$$\mathbf{y}_{RM}(n) = \mathbf{y}(n) - \begin{bmatrix} \mathbf{y}_M(n-1) \\ \mathbf{y}_{cp}(n) \end{bmatrix}. \quad (5)$$

The remodulated received block can be shown to be written as

$$\mathbf{y}_{RM}(n) = \mathcal{T}_M(\mathbf{h})(\mathbf{u}'_M(n) - \mathbf{u}_M(n-1)) + \text{noise} \quad (6)$$

where  $\mathbf{u}'_M(n)$  is a permutation of the  $n$ th data block  $\mathbf{u}_M(n)$ . The remodulated block has a length  $M + L$ , equal to each cyclic-prefixed received block. Compared to the composite received block used in [2, 3], the remodulation method involves a smaller matrix (roughly half in size) and has a relatively small computation complexity. However, it still requires a large number of received blocks and has a limited applicability when the amount of received data is small.

### 3. PROPOSED METHOD

In this section, we propose our method for blind channel estimation in CP systems. The main idea is to repeatedly use each remodulation block mentioned in Eq. (5). In fact, from Eq. (6) we observe that the transfer function between the remodulated received block  $\mathbf{y}_{RM}(n)$  and the remodulated data block  $\mathbf{d}(n) \triangleq \mathbf{u}'_M(n) - \mathbf{u}_M(n-1)$  is a full-banded Toeplitz matrix composed of channel coefficients, which has an exactly same form to that appearing in a blind channel estimation problem for ZP systems [1]. We can therefore take advantage of what has been known in blind channel estimation for ZP systems. Starting from Eq. (6), The following equation can be verified (see, for example, [5]):

$$\mathcal{T}_Q(\mathbf{y}_{RM}(n)) = \mathcal{T}_{M+Q-1}(\mathbf{h}) \mathcal{T}_Q(\mathbf{d}(n)) + \text{noise}$$

where  $Q$  is any positive integer and  $\mathbf{d}(n)$  is the  $n$ th remodulated block. The parameter  $Q$  is called the repetition index as each remodulated block  $\mathbf{d}(n)$  is repeatedly used  $Q$  times. Suppose  $J$  received blocks  $\mathbf{y}(n), n = 0, 1, \dots, J-1$  are available at the receiver and we can generate  $J-1$  remodulated blocks  $\mathbf{y}_{RM}(n), n = 0, \dots, J-2$  as in Eq. (5). For each of these remodulated blocks  $\mathbf{y}_{RM}(n)$ , we first form a  $Q$ -column Toeplitz matrix  $\mathcal{T}_Q(\mathbf{y}_{RM}(n))$ . Then we concatenate all of these  $J-1$  Toeplitz matrices and construct the  $(M+L+Q-1) \times (J-1)Q$  matrix

$$\mathbf{Y}_{RM,Q} \triangleq [\mathcal{T}_Q(\mathbf{y}_{RM}(0)) \quad \cdots \quad \mathcal{T}_Q(\mathbf{y}_{RM}(J-2))]. \quad (7)$$

It is readily verified that [5]

$$\mathbf{Y}_{RM,Q} = \mathcal{T}_{M+Q-1}(\mathbf{h}) \mathbf{D}_Q + \text{noise}$$

where

$$\mathbf{D}_Q \triangleq [\mathcal{T}_Q(\mathbf{d}(0)) \quad \mathcal{T}_Q(\mathbf{d}(1)) \quad \cdots \quad \mathcal{T}_Q(\mathbf{d}(J-2))] \quad (8)$$

is a  $(M+Q-1) \times (J-1)Q$  matrix containing a repeated form of remodulated data blocks.

We first assume the noise is absent. The column space of  $\mathbf{Y}_{RM,Q}$  will be equal to that of  $\mathcal{T}_{M+Q-1}(\mathbf{h})$  as long as the matrix  $\mathbf{D}_Q$  has full row rank  $M+Q-1$ . The conditions on which  $\mathbf{D}_Q$  has full row rank are discussed later. In this case, the left null space of  $\mathbf{Y}_{RM,Q}$  is equal to the left null space of  $\mathcal{T}_{M+Q-1}(\mathbf{h})$  and therefore the channel coefficients  $\mathbf{h}$  can be blindly identified (with a scalar ambiguity) using only

$\mathbf{Y}_{RM,Q}$ . In presence of noise, the noise space of  $\mathbf{Y}_{RM,Q}$  can be found by choosing the left singular vectors of  $\mathbf{Y}_{RM,Q}$  corresponding to the smallest singular values. Specifically, suppose the singular value decomposition (SVD) of the  $\mathbf{Y}_{RM,Q}$  is expressed as

$$\mathbf{Y}_{RM,Q} = [\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \boldsymbol{\Sigma}_s & & \\ & \boldsymbol{\Sigma}_n & \\ & & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^\dagger \\ \mathbf{V}_n^\dagger \end{bmatrix}$$

in which the size of  $\boldsymbol{\Sigma}_s$  is  $(M+Q-1) \times (M+Q-1)$  and that of  $\boldsymbol{\Sigma}_n$  is  $L \times L$ . The matrix  $\boldsymbol{\Sigma}_n$  contains the smallest singular values and the corresponding left singular vectors are the columns of the matrix  $\mathbf{U}_n$ . When the noise is small, the columns of  $\mathbf{U}_n$  are approximately orthogonal to the columns of  $\mathcal{T}_{M+Q-1}(\mathbf{h})$ :

$$\|\mathbf{U}_n^\dagger \mathcal{T}_{M+Q-1}(\mathbf{h})\|_F^2 \approx 0.$$

Denote the  $(i, j)$ -entry of  $\mathbf{U}_n^\dagger$  as  $u_{ij}$ , we use contents of the  $k$ th column of  $\mathbf{U}_n$  and form the  $(M+Q-1) \times (L+1)$  Hankel matrix

$$\mathcal{U}_k \triangleq \begin{bmatrix} u_{k1} & u_{k2} & \cdots & u_{k,L+1} \\ u_{k2} & u_{k3} & \cdots & u_{k,L+2} \\ \vdots & \vdots & \ddots & \vdots \\ u_{k,M+Q-1} & u_{k2} & \cdots & u_{k,M+Q+L-1} \end{bmatrix} \quad (9)$$

for any  $k, 1 \leq k \leq L$ . Then we have  $\mathcal{U}_k \mathbf{h} \approx \mathbf{0}$ . Construct

$$\mathcal{U} = [\mathcal{U}_1^T \quad \mathcal{U}_2^T \quad \cdots \quad \mathcal{U}_L^T]^T. \quad (10)$$

Then the channel vector  $\mathbf{h}$  can be estimated up to a complex scalar ambiguity by choosing the vector  $\mathbf{h}$  which minimizes the norm of  $\mathcal{U}\mathbf{h}$ :

$$\hat{\mathbf{h}} = \arg \min_{\|\mathbf{h}\|=1} \|\mathcal{U}\mathbf{h}\|^2 = \arg \min_{\|\mathbf{h}\|=1} \mathbf{h}^\dagger (\mathcal{U}^\dagger \mathcal{U}) \mathbf{h}. \quad (11)$$

Recall that the above discussions are based on the assumption that the  $(M+Q-1) \times (J-1)Q$  matrix  $\mathbf{D}_Q$  in Eq. (8) has full row rank. This implies that the number of its columns must be greater than or equal to the number of its rows, i.e.,  $(J-1)Q \geq M+Q-1$ . Therefore, a necessary (but not sufficient) condition for the proposed algorithm to work appropriately is that

$$J \geq \frac{M-1}{Q} + 2.$$

This inequality essentially sets up a lower bound for the number of received blocks ( $J$ ) required for the proposed algorithm. Note that the receiver has the freedom to choose any positive integer as the repetition index  $Q$ . When a larger  $Q$  is chosen, the required number of received blocks is lower.

#### 3.1. Summary of Algorithm

The proposed algorithm can be summarized as follows.

- 1) Given block size  $M$ , CP length  $L$ , and the  $J$  available received blocks  $\mathbf{y}(n), n = 0, 1, \dots, J-1$ , choose the repetition index  $Q$  such that

$$Q \geq \frac{M-1}{J-2}.$$

Method	Feature	Number of received blocks	Complexity	Conditions when $M = 32$
Muquet <i>et al.</i> 2002 [2]	Composite blocks	$J \geq 2M + 1$	$\mathcal{O}((2M + L)^3)$	$J \geq 65$
Su and Vaidyanathan 2007 [3]	Composite blocks and repetition	$J \geq \frac{2M-1}{Q} + 2$	$\mathcal{O}((2M + L + Q - 1)^3)$	$J \geq 23$ when $Q = 3$
Gao <i>et al.</i> 2008 [6] (SISO case)	Remodulation	$J \geq M + 1$	$\mathcal{O}((M + L)^3)$	$J \geq 23$
Proposed method	Remodulation and repetition	$J \geq \frac{M-1}{Q} + 2$	$\mathcal{O}((M + L + Q - 1)^3)$	$J \geq 13$ when $Q = 3$

**Table 1.** Comparison of subspace-based blind channel estimation algorithms for CP systems.

- 2) Perform remodulation on  $\mathbf{y}(n)$  according to Eq. (5) and construct the  $(M + L + Q - 1) \times (J - 1)Q$  matrix  $\mathbf{Y}_{RM,Q}$  as defined in Eq. (7).
- 3) Perform singular value decomposition (SVD) on  $\mathbf{Y}_{RM,Q}$  so that

$$\mathbf{Y}_{RM,Q} = [\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \boldsymbol{\Sigma}_s & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_n \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^\dagger \\ \mathbf{V}_n^\dagger \end{bmatrix}$$

where the diagonal entries of  $\boldsymbol{\Sigma}_n$  are the  $L$  smallest singular values of  $\mathbf{Y}_{RM,Q}$ .

- 4) Use contents of  $\mathbf{U}_n$  and construct the  $(M + Q - 1)L \times (L + 1)$  matrix  $\mathcal{U}$  as in Eqs. (9)(10).
- 5) Let  $\hat{\mathbf{h}}$  be the eigenvector of  $\mathcal{U}\mathcal{U}^\dagger$  corresponding to the smallest eigenvalue. And  $\hat{\mathbf{h}}$  would be the estimated channel vector within a scalar ambiguity.

### 3.2. Complexity Analysis

Just like other SS methods, the main computational load of the proposed algorithm comes from the SVD operation on the  $\mathbf{Y}_{RM,Q}$  matrix. Since we only need to calculate left singular vectors of  $\mathbf{Y}_{RM,Q}$ , the complexity depends only on the number of rows of  $\mathbf{Y}_{RM,Q}$  and can be expressed as  $\mathcal{O}((M + Q - 1)^3)$ . When  $Q$  is chosen as a small integer, the complexity is just slightly greater than that in [6] but significantly smaller than other methods based on composite blocks [2, 3]. A full comparison of complexity of all algorithms is listed in Table 1. In addition, Table 1 also summarizes the lower bound of the numbers of received blocks of different methods. We observe that the proposed method is applicable with the smallest amount of received data among all existing SS blind channel estimation algorithms.

## 4. NUMERICAL RESULTS

In this section, we conduct Monte Carlo simulations to demonstrate the performance of the proposed method and compare it with those of previously reported methods. We assume perfect block synchronization in all simulations. The block size  $M$  is chosen as 32 and the length of cyclic prefix is  $L = 8$ . We test our methods in static channel environments. The Rayleigh fading channel of order  $L = 8$  is used. Source symbols are chosen from QPSK constellation and the precoder is chosen as  $\mathbf{R} = \mathbf{I}_M$  (i.e., in SC-CP systems).

Let  $N_{ch}$  be the number of statistically independent channel realizations generated for a simulation and  $N_S$  be the number of independent sets of data sources and noise generated for each channel realization. The normalized channel estimation mean square error, denoted as  $E_{ch}$ , is used as the figure of merit and is defined as

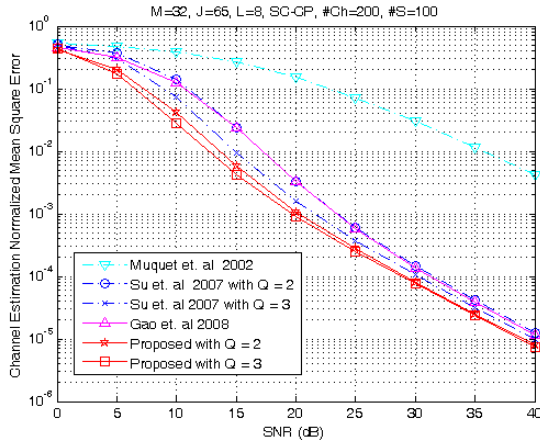
$$E_{ch} = \frac{1}{N_{ch}} \left[ \sum_{k=1}^{N_{ch}} \frac{1}{N_S} \sum_{l=1}^{N_S} \min_{c \in \mathbb{C}} \frac{\|c\hat{\mathbf{h}}_{k,l} - \mathbf{h}_k\|^2}{\|\mathbf{h}_k\|^2} \right]$$

where  $\mathbf{h}_k$  is the  $k$ th channel realization and  $\hat{\mathbf{h}}_{k,l}$  is the estimate of  $\mathbf{h}_k$  for the  $l$ th Monte Carlo trial (up to a complex scalar ambiguity). In all simulation plots, we use  $N_{ch} = 200$  and  $N_S = 100$ .

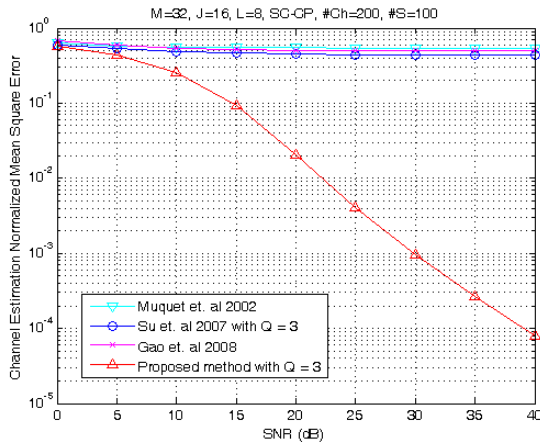
Figure 2 shows the simulation result when the number of received blocks is 65 (i.e., slightly more than twice of the data block size  $M$ ). This is the smallest number for the method in [2] to work properly (see remarks in Table 1). Both the methods in [3] and in [6] have a much better performance. Yet, the proposed method has an even improved performance compared to all previously reported methods in all SNR ranges. An approximately 2dB gain is observed if we compare the performance of proposed method with  $Q = 3$  to that of method in [3] with the same repetition index.

Figure 3 demonstrates the unique capability of the proposed method to work with a small amount of received data. Here the number of received blocks is set to  $J = 16$ , just half the block size. We observe that all previously reported methods do not work properly while the channel estimation error of the proposed method with  $Q = 3$  decreases as SNR increases. This is consistent with the conditions listed in the last column in Table 1.

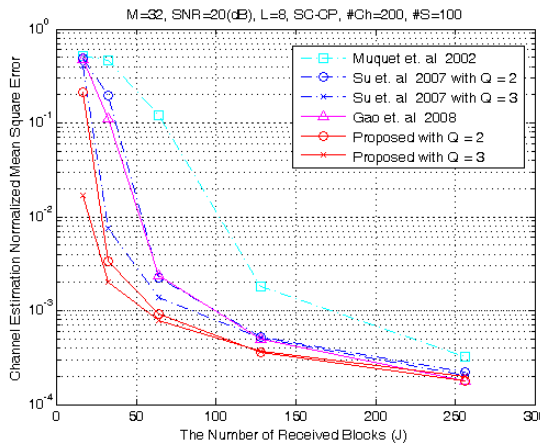
Finally, in Figure 4 we compare all above methods with different number of received blocks. The number of received blocks is ranging from  $J = 16$  to  $J = 256$  in this simulation plot and the SNR level is set to 20 dB. We still observe that the proposed method with  $Q = 2$  and  $Q = 3$  all outperforms all other methods, especially when  $J$  is small. To obtain a channel estimation error at the level of  $10^{-3}$ , the proposed method needs only around  $J = 64$  (twice the block size) received blocks while most of others need more than 100. When the amount of received data increases, the performance gap between the proposed method and previously reported methods gradually decreases. The advantage of the proposed method with a limited available amount of received data is clearly shown. In addition, compared to methods in [2, 3], the computational complexity is around only one fifth in this case.



**Fig. 2.** Normalized MSE of channel estimation for static channel with the QPSK constellation in SC-CP systems comparing with the repetition method.



**Fig. 3.** Normalized MSE of channel estimation for static channel with the QPSK constellation in SC-CP systems with a small number of received blocks.



**Fig. 4.** Normalized MSE of channel estimation for static channel with the QPSK constellation in SC-CP systems with different number of received blocks.

## 5. CONCLUSIONS

In this paper, a new algorithm for blind channel estimation in cyclic prefix (CP) systems is proposed based on repeated use of remodulated received blocks applied in subspace (SS) methods. An algorithm parameter called repetition index can be chosen as any positive integer. Compared to a previously reported method that also uses received blocks repeatedly, the proposed algorithm not only has a greatly reduced computational complexity, but also has an improved channel estimation performance when using the same repetition index. When it is compared with a previously reported method that uses remodulation only, the proposed method has a clear improvement on channel estimation performance at the expense of a slightly increased computational complexity. The proposed algorithm not only outperforms all previously reported methods, but also requires less amount of received data than all existing SS methods to yield the same channel estimation performance. Although the algorithm presented in this paper is based on single-input-single-output systems, the algorithm can be readily extended to the MIMO case.

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