A BOLTZMANN MACHINE TO MODEL THE MULTIPATH ENVIRONMENT FOR PARTICLE FILTERING BASED GPS NAVIGATION

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ABSTRACT

In GPS navigation, an accuracy of about 10 m can be currently obtained, but this performance can be strongly degraded in a multipath environment. The multipath directly impact the distribution of the additive noise corrupting the distance measurements between the satellites and the GPS receiver. They are modeled either as variance jumps if there is a direct path between the satellites and the receiver or as mean-value jumps otherwise. The originality of our approach is to take into account the spatial dependencies between GPS measurements when modeling multipath occurrences. Indeed, if two signals from satellites have close directions of arrival, they are very likely to be simultaneously degraded by multipath. For that purpose, we suggest using a Boltzmann machine which provides a "natural" setting to define interactions between the GPS measurements. Then, as the proposed model is strongly non linear and non Gaussian, we jointly estimate the mobile location and perform the multipath detection/estimation by using particle filtering.

***Index Terms*—** Boltzmann machine, GPS navigation, multipath, particle filtering.

1. INTRODUCTION

Thanks to the Global Positioning System (GPS), every user can obtain his position anywhere on earth. For that purpose, the GPS receiver estimates the propagation delays of signals transmitted by a constellation of satellites of known locations. Then, range measurements are computed through multiplication by the velocity of light in vacuum i.e. $c = 3 \cdot 10^8\ m/s$. As both the receiver position and its clock offset with respect to the GPS reference time must be estimated, at least four satellite measurements are required to solve the navigation problem.

Today, an accuracy of about 10 m can be nominally achieved. However, the GPS performance can strongly deteriorate in urban environments due to the multipath phenomenon. It happens when different replicas of the satellite signal, incoming from reflections on nearby obstacles, reach the receiver. To mitigate multipath effects, several methods have been designed. See chapter 7 in [7]. Among them, antenna arrays can be used to reject the reflected signals. Alternative solutions consist in improving the correlation techniques implemented at the receiver to estimate the propagation delays of the satellite signals. Another class of approaches deals with multipath effects directly at the level of the navigation algorithm which estimates the position from the satellite ranging measurements. They have the advantage of avoiding any modification of the receiver architecture. In [4], the multipath are modeled by jumps of the variance of the additive measurement noise if there is a line of sight path between the satellite and the receiver. Otherwise, jumps of the mean value of the additive measurement noise are considered. In [4], Spangenberg *et al.* propose to detect and compensate for these jumps by using generalized likelihood ratio tests. More recently, in [8], the measurement noise is modeled by using non-parametric approaches based on Dirichlet Process Mixtures. Nevertheless, the satellite signals are assumed to be independent even if in reality two satellite signals whose directions of arrival are close to each other are very likely to be simultaneously affected by multipath.

This paper deals with line-of-sight multipath which are the more frequent in GPS navigation due to the satellite elevation angles. The detection of multipath is handled by introducing a random vector with as many components as GPS measurements. They can take two different values which indicate whether there is multipath or not. Our contribution is to model the dependencies between the above-mentioned binary random variables by using a Boltzmann machine (BM) [1]. Thus, the joint occurrence of multipath on close satellite signals can be favored. Indeed, BM admit a closed-form parametric expression so that the practitioner can easily define a dependence structure by tuning the parameters. A fully Bayesian hierarchical model is proposed to jointly address the multipath detection/estimation and the mobile location. Then, as the proposed model is strongly non linear and non Gaussian, particle filtering is used to perform this estimation. Note that an approximation of the so-called optimal simulation law is also derived [6].

The paper is organized as follows: in section 2, BM are presented. Section 3 details the Bayesian hierarchical model of the GPS navigation problem in the presence of multipath. Section 4 describes the proposed particle filter algorithm. Finally, the results obtained on simulated GPS data are discussed in section 5.

2. BOLTZMANN MACHINES

BM have been lately used in various applications including sparse representation modeling [1], neural-network-based pattern recognition [2], etc.
A BM is a Markov random field which consists of a set of binary random variables $\{\mathbf{c}(1), \cdots, \mathbf{c}(n)\}$ whose joint probability distribution takes the form:

$$Pr[\mathbf{c}|\mathbf{b}, \mathbf{W}] = \frac{1}{Z(\mathbf{b}, \mathbf{W})} \exp\left(\mathbf{b}^T\mathbf{c} + \frac{1}{2}\mathbf{c}^T\mathbf{W}\mathbf{c}\right) \quad (1)$$

where $\mathbf{c} = [\mathbf{c}(1), \cdots, \mathbf{c}(n)]^T$ takes its values in $\{-1, 1\}^n$, $\mathbf{W}$ is an n-by-n symmetric matrix and $\mathbf{b} = [\mathbf{b}(1), \cdots, \mathbf{b}(n)]^T$ is an n-component real-valued vector. To normalize the BM distribution, the so-called partition function $Z(\mathbf{b}, \mathbf{W})$ must be chosen as follows:

$$Z(\mathbf{b}, \mathbf{W}) = \sum_{\boldsymbol{\xi} \in \mathcal{S}} \exp\left(\mathbf{b}^T\boldsymbol{\xi} + \frac{1}{2}\boldsymbol{\xi}^T\mathbf{W}\boldsymbol{\xi}\right) \quad (2)$$

where $\mathcal{S}$ denotes the set of all possible states that can be taken by $\mathbf{c}$. In addition, by denoting w_{ij} the coefficient of the i^{th} row and j^{th} column of $\mathbf{W}$, one has:

$$\exp(\mathbf{b}^T\mathbf{c} + \frac{1}{2}\mathbf{c}^T\mathbf{W}\mathbf{c}) =$$
$$\exp\left(\mathbf{b}^T\mathbf{c} + \frac{1}{2}\mathbf{c}^T \begin{bmatrix} 0 & w_{12} & \dots & w_{1n} \\ w_{12} & \ddots & \ddots & w_{2n} \\ \vdots & \ddots & \ddots & \vdots \\ w_{1n} & w_{2n} & \dots & 0 \end{bmatrix} \mathbf{c} + \frac{1}{2}\sum_{i=1}^{n} w_{ii}\right)$$

Therefore, $\{w_{ii}\}_{i=1}^n$ can be set to 0 since they contribute to a constant in the function $\mathbf{c}^T\mathbf{W}\mathbf{c}$.

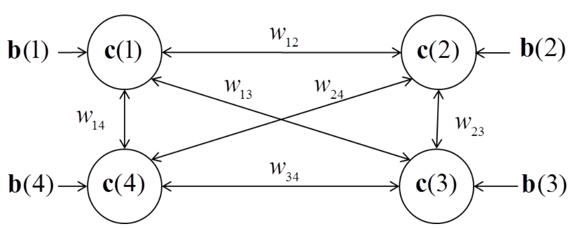

Fig. 1. representation of the BM as an undirected graph for $n = 4$

The BM allows the practitioner to define in a convenient manner a structure of dependence between the binary random variables $\{\mathbf{c}(k)\}_{k=1}^n$ by means of $\mathbf{W}$. To better understand the role played by $\mathbf{W}$ and $\mathbf{b}$, the BM distribution can be represented by using an undirected graph as shown in Fig. 1.
1) Thus, a non-zero entry w_{ij} in the matrix $\mathbf{W}$ results in an edge connecting nodes i and j with a specific weight. It characterizes the interaction between the random variables $\mathbf{c}(i)$ and $\mathbf{c}(j)$. For an excitatory interaction defined by $w_{ij} > 0$, both $\mathbf{c}(i)$ and $\mathbf{c}(j)$ tend to take the same value whereas they tend to have opposite values for an inhibitory interaction characterized by $w_{ij} < 0$.
2) In addition, the so-called bias $\mathbf{b}(k)$ is associated with the node k and defines the marginal behavior of the random variable $\mathbf{c}(k)$. Setting $\mathbf{W} = \mathbf{0}$ in the BM distribution (1) leads to $Pr[\mathbf{c}|\mathbf{b}, \mathbf{W} = \mathbf{0}] = \frac{1}{Z(\mathbf{b},\mathbf{W})} \prod_{k=1}^n \exp(\mathbf{b}(k)\mathbf{c}(k))$.
This corresponds to statistically independent $\{\mathbf{c}(k)\}_{k=1}^n$. In this case, it can be easily shown that $Pr[\mathbf{c}(k) = -1] = \exp(-2\mathbf{b}(k))Pr[\mathbf{c}(k) = 1]$ for $k = 1, \cdots, n$. Since $Pr[\mathbf{c}(k) = -1] + Pr[\mathbf{c}(k) = 1] = 1$, one has:

$$Pr[\mathbf{c}(k) = 1] = \frac{1}{1 + \exp(-2\mathbf{b}(k))} \quad (3)$$

Note that when $\mathbf{W}$ is nonzero, (3) no longer holds. However, the intuition remains true that a positive value for $\mathbf{b}(k)$ favors a value equal to 1 for $\mathbf{c}(k)$ whereas a negative value for $\mathbf{b}(k)$ favors a value equal to -1 for $\mathbf{c}(k)$.

3. BAYESIAN MODELING OF THE PROBLEM

Here, the problem is to decide whether the GPS measurements are affected or not by multipath and, if so, to take into account this unwanted phenomenon when estimating the dynamics of the mobile equipped by the GPS receiver. In the following, the dynamics of the mobile is described by a set of variables contained in the state vector $\mathbf{x}_t$ of size n_x. 3 of its components include the position coordinates of the mobile $\mathbf{p}_t = [x_t, y_t, z_t]^T$ in a reference coordinate system, here the ECEF, but the others depend on the considered motion model.

3.1. Observation equation

The estimation of the state vector $\mathbf{x}_t$ is based on the measurement vector $\mathbf{Z}_t$ storing the GPS measurements from n_t satellites at time instant t. n_t is likely to vary with time due to the relative geometry of the receiver and the satellites. The k^{th} component of the column vector $\mathbf{Z}_t$, with $k = 1, \cdots, n_t$, can be expressed as follows:

$$\mathbf{Z}_t(k) = \|\mathbf{p}_t - \mathbf{p}_t^k\| + b_t + \sqrt{\boldsymbol{\phi}_t(k)}\mathbf{v}_t(k) \quad (4)$$

where the vector $\mathbf{p}_t^k = [x_t^k, y_t^k, z_t^k]^T$ contains the 3 position coordinates of the satellite k and $\|.\|$ is the Euclidian norm. b_t denotes the GPS receiver clock offset with respect to the GPS reference time and $\mathbf{v}_t(k)$ is a zero-mean white Gaussian random process with unit variance. Finally, $\boldsymbol{\phi}_t(k)$ denotes the variance of the additive measurement noise. Its value differs if multipath occur or not. When the measurement from the k^{th} satellite is not affected by multipath, $\boldsymbol{\phi}_t(k)$ is conservatively set to σ_0^2, with σ_0 the nominal measurement noise standard deviation equal to 8 m. On the contrary, if multipath degrade the measurement, $\boldsymbol{\phi}_t(k) \neq \sigma_0^2$. Its value becomes unknown and hence must be estimated.
Therefore, multipath detection has also to be addressed and can be handled by introducing a discrete-valued n_t-component

vector $\mathbf{c}_t$. If the k^{th} measurement is affected by multipath at time t, $\mathbf{c}_t(k) = 1$. Otherwise, it is equal to -1.
As we aim at estimating these parameters in a Bayesian framework, the prior distribution of $\mathbf{c}_t$ has to be defined. A simple choice of prior probabilities, considering independent $\mathbf{c}_t(k)$, could be $Pr[\mathbf{c}_t(k) = -1] = 0.5$ and $Pr[\mathbf{c}_t(k) = 1] = 0.5$. Nevertheless, the latter choice would not be realistic since the characteristics of the environment vary in time. Furthermore, as the satellite measurements are not independent of each other, we decide to adopt a joint distribution for the vector $\mathbf{c}_t$ that allows the practitioner to define a dependence structure between its components. This distribution is chosen by using (1) and is denoted as $Pr[\mathbf{c}_t|\mathbf{b}_t, \mathbf{W}_t]$ with $\mathbf{b}_t$ the n_t-component real-valued bias vector at time t and $\mathbf{W}_t$ the n_t-by-n_t interaction matrix at time t. The vector $\mathbf{b}_t$ represents the marginal behavior of the random variables $\mathbf{c}_t(k)$ and needs also to be estimated since it varies according to the environment. In this work, we propose to set the matrix $\mathbf{W}_t$ in order to define the desired dependence structure between the components of $\mathbf{c}_t$.

Therefore, our purpose in the following is to recursively estimate from the sets of measurements the $(n_x + 3n_t)$-component extended state vector $\mathbf{X}_t = [\mathbf{x}_t^T, \boldsymbol{\phi}_t^T, \mathbf{c}_t^T, \mathbf{b}_t^T]^T$, containing the variables to be estimated. Before expressing the transition distribution of $\mathbf{X}_t$, note that it can be factored by using Bayes' rule and by taking into account the independencies between the random variables as follows:

$$p(\mathbf{X}_t|\mathbf{X}_{t-1}) = p(\boldsymbol{\phi}_t|\boldsymbol{\phi}_{t-1}, \mathbf{c}_t, \mathbf{c}_{t-1}) Pr[\mathbf{c}_t|\mathbf{b}_t, \mathbf{W}_t] \times p(\mathbf{b}_t|\mathbf{b}_{t-1}) p(\mathbf{x}_t|\mathbf{x}_{t-1}) \quad (5)$$

3.2. Prior distributions of the state vector components

A) The state vector $\mathbf{x}_t = \left[\mathbf{p}_t^T, \dot{\mathbf{p}}_t^T, b_t, d_t\right]^T$ is assumed to satisfy a second-order model, with $\dot{\mathbf{p}}_t = [\dot{x}_t, \dot{y}_t, \dot{z}_t]^T$ a vector containing the 3 velocity coordinates of the mobile and d_t the receiver clock drift with respect to the GPS reference time. The evolution of $\mathbf{x}_t$ is described classically by the transition law:

$$p(\mathbf{x}_t|\mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_t; \mathbf{F}\mathbf{x}_{t-1}, \mathbf{Q}) \quad (6)$$

where $\mathcal{N}(\mathbf{x}_t; \mathbf{F}\mathbf{x}_{t-1}, \mathbf{Q})$ denotes the multivariate Gaussian law with argument $\mathbf{x}_t$, mean vector $\mathbf{F}\mathbf{x}_{t-1}$ and covariance matrix $\mathbf{Q}$, $\mathbf{F}$ and $\mathbf{Q}$ being block diagonal matrices. These latter are for instance detailed in [3].

B) We propose to model the vector $\mathbf{c}_t$ as a BM. Thus, its distribution is:

$$Pr[\mathbf{c}_t|\mathbf{b}_t, \mathbf{W}_t] = \frac{1}{Z(\mathbf{b}_t, \mathbf{W}_t)} \exp\left(\mathbf{b}_t^T \mathbf{c}_t + \frac{1}{2}\mathbf{c}_t^T \mathbf{W}_t \mathbf{c}_t\right) \quad (7)$$

Let us recall that $\mathbf{W}_t$ is user-chosen whereas $\mathbf{b}_t$ is estimated, hence its evolution must be modeled.
How to select $\mathbf{W}_t$:
Two satellite signals having close directions of arrival have a high probability of being simultaneously affected by multipath. Thus, the components of $\mathbf{c}_t$ should not be independent. Therefore $\mathbf{W}_t \neq 0$. To favor the simultaneous occurrence of multipath in measurements associated to two satellites, we suggest defining the entries of the matrix $\mathbf{W}_t$ as follows:
The entry w_{ij} is used to represent the angular proximity of satellites i and j. A possible measure of the latter is the director cosine of the angle $\theta_{i,j}$ between the directions of the two satellites with respect to the receiver (See Fig. 2):

$$w_{ij} = \cos\theta_{i,j} = \frac{< \mathbf{p}_t^i - \mathbf{p}_t, \mathbf{p}_t^j - \mathbf{p}_t >}{\|\mathbf{p}_t^i - \mathbf{p}_t\| \cdot \|\mathbf{p}_t^j - \mathbf{p}_t\|} \quad (8)$$

where $\mathbf{p}_t^i$, $\mathbf{p}_t^j$ contains respectively the 3 position coordinates of satellites i, j and $< \cdot, \cdot >$ denotes the dot product operator.

Note that when the dot product in (8) becomes negative, the angle $\theta_{i,j}$ between the directions of the satellites exceeds $\pi/2$. In this case, we can reasonably consider them as independent and we suggest setting: $w_{ij} = 0$.

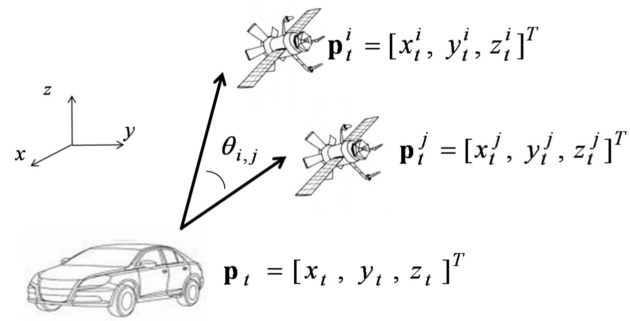

Fig. 2. deviation between the angular directions of two satellites in the constellation

How to model $\mathbf{b}_t$:
The way the real-valued biases $\mathbf{b}_t(k)$ evolve in time can be modeled by a normal distribution as follows:

$$p(\mathbf{b}_t(k)|\mathbf{b}_{t-1}(k)) = \mathcal{N}(\mathbf{b}_t(k); \boldsymbol{\mu}_b(k), \boldsymbol{\sigma}_b^2(k)) \quad (9)$$

Here, the parameters $\boldsymbol{\mu}_b(k)$ and $\boldsymbol{\sigma}_b^2(k)$ must be tuned to introduce a temporal dependency favoring the following phenomenon: when operating in a multipath-free environment at time instant t, the probability that no multipath occur for future time instants must be high. Likewise, when operating in a multipath environment, the probability that the multipath are still present for future time instants must also be high. Then, a possible choice of parameters consists of $\boldsymbol{\mu}_b(k) = \mathbf{b}_{t-1}(k)$ and $\boldsymbol{\sigma}_b^2(k) = \sigma_b^2$ with $\mathbf{b}_{t-1}(k)$ the k^{th} component of the bias vector at the previous time instant $t-1$ and σ_b^2 the variance of $\mathbf{b}_t(k)$ that must be chosen sufficiently small to favor the desired temporal correlation on $\mathbf{b}_t(k)$.
The joint probability density function (PDF) of $\mathbf{b}_t$ is then defined as follows:

$$p(\mathbf{b}_t|\mathbf{b}_{t-1}) = \prod_{k=1}^{n_t} p(\mathbf{b}_t(k)|\mathbf{b}_{t-1}(k)) = \mathcal{N}(\mathbf{b}_t; \boldsymbol{\mu}_b, \mathbf{R}_b) \quad (10)$$

where $\mathbf{R}_b = \sigma_b^2 \mathbf{I}_{n_t}$ and $\mathbf{I}_{n_t}$ is the identity matrix of size n_t-by-n_t.
C) We propose to define the prior distribution of the vector $\boldsymbol{\phi}_t$ by considering different scenarios as follows:
C.1) when multipath appear on the k^{th} satellite at time instant t, i.e. $\mathbf{c}_t(k) = 1$ and $\mathbf{c}_{t-1}(k) = -1$, the prior distribution of $\boldsymbol{\phi}_t(k)$ must favor values of the variances higher than σ_0^2 for the observation noise. Classically, truncated Gaussian laws and log-Normal laws have been used to define the prior PDF of noise variances. Here, we suggest using an Inverse Gamma ($\mathcal{IG}$) [5] distribution to model $\boldsymbol{\phi}_t(k)$. As this will be explained in section 4.2, the latter makes it possible to compute analytically the simulation law of the particle filter. Therefore, one has:

$$p(\boldsymbol{\phi}_t(k)|\boldsymbol{\phi}_{t-1}(k), \mathbf{c}_t(k), \mathbf{c}_{t-1}(k)) = \mathcal{IG}(\boldsymbol{\phi}_t(k); \boldsymbol{\varepsilon}) \quad (11)$$

where $\mathcal{IG}(\boldsymbol{\phi}_t(k); \boldsymbol{\varepsilon})$ is the $\mathcal{IG}$ PDF with argument $\boldsymbol{\phi}_t(k)$ and parameters $\boldsymbol{\varepsilon} = [\alpha, \beta]^T$. It can be expressed as follows:

$$\mathcal{IG}(\boldsymbol{\phi}_t(k); \boldsymbol{\varepsilon}) = \frac{\beta^\alpha}{\Gamma(\alpha)} \boldsymbol{\phi}_t(k)^{-\alpha-1} \exp\left(-\frac{\beta}{\alpha}\right)$$

where $\frac{\beta^\alpha}{\Gamma(\alpha)}$ is a normalizing constant.
The parameters are tuned so that the prior is not informative (See Fig. 3, dashed curve).
C.2) when the k^{th} satellite is affected by multipath at time instant t that were already detected at time instant $t-1$, i.e. $\mathbf{c}_t(k) = 1$ and $\mathbf{c}_{t-1}(k) = 1$, the prior distribution of $\boldsymbol{\phi}_t(k)$ must favor variances approximately equal to the corresponding previous values. This hence means that there is still a multipath. In this case, we assign to $\boldsymbol{\phi}_t(k)$ an $\mathcal{IG}$ prior with its parameters tuned so that the latter is peaked around the previous value of the observation noise variance, namely $\boldsymbol{\phi}_{t-1}(k)$ (See Fig. 3, plain curve).

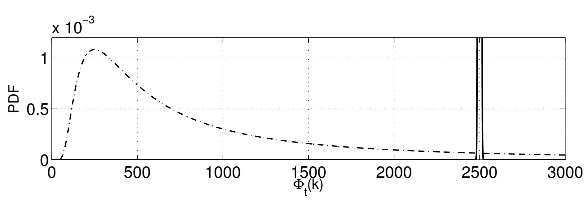

Fig. 3. inverse-gamma PDF in a non-informative case (dashed curve) and a peaked case (plain curve)

4. ESTIMATION WITH PARTICLE FILTERING

4.1. Presentation of particle filtering

The aim is to estimate the *a posteriori* distribution $p(\mathbf{X}_{1:t}|\mathbf{Z}_{1:t})$, where $\mathbf{X}_{1:t} = \mathbf{X}_1, \cdots, \mathbf{X}_t$ and $\mathbf{Z}_{1:t} = \mathbf{Z}_1, \cdots, \mathbf{Z}_t$, and more particularly the marginal density $p(\mathbf{X}_t|\mathbf{Z}_{1:t})$. In general, this latter cannot be determined analytically.
To estimate the extended state vector $\mathbf{X}_t$, we choose a particle filter (PF). This class of algorithms allows not only non linear and/or non Gaussian models to be dealt with, but also state vectors comprising both continuous and discrete-valued variables. PF, also known as sequential Monte Carlo methods, provide a discrete approximation of this distribution:

$$\widehat{P}_N(\mathbf{X}_t) = \sum_{i=1}^{N} w_t^{(i)} \delta(\mathbf{X}_t - \mathbf{X}_t^{(i)}). \quad (12)$$

The N support points are called particles and they are propagated sequentially according to a proposal law $q(\mathbf{X}_t|\mathbf{X}_{t-1}, \mathbf{Z}_t)$. They are assigned weights $w_t^{(i)}$ to correct for the discrepancy between the target law and the proposal law. Note that the weights are updated sequentially as follows:

$$w_t^{(i)} = w_{t-1}^{(i)} \frac{p(\mathbf{X}_t^{(i)}|\mathbf{X}_{t-1}^{(i)}) p(\mathbf{Z}_t|\mathbf{X}_t^{(i)})}{q(\mathbf{X}_t^{(i)}|\mathbf{X}_{t-1}^{(i)}, \mathbf{Z}_t)}. \quad (13)$$

To prevent degeneracy, the particles are usually resampled on a regular basis according to (12). If the formalism presented in this section appears to be classic, do note that the major difficulty lies in computing the simulation law which is an approximation of the so-called optimal proposal distribution $p(\mathbf{X}_t|\mathbf{X}_{t-1}, \mathbf{Z}_t)$ [6] that takes into account the measurement at the current time instant. In this way, the particles are generated in regions of the state space corresponding to a high likelihood. It is of the utmost importance due to uncertainty on the value of $\boldsymbol{\phi}_t(k)$ after a variance jump.

4.2. Simulation laws design

Using Bayes' rule and taking advantage of the independencies between random variables we can write the following factorization for the optimal simulation law:

$$\begin{aligned} p(\mathbf{X}_t|\mathbf{X}_{t-1}, \mathbf{Z}_t) &= p(\boldsymbol{\phi}_t|\mathbf{x}_t, \boldsymbol{\phi}_{t-1}, \mathbf{c}_t, \mathbf{c}_{t-1}, \mathbf{Z}_t) \\ &\times p(\mathbf{b}_t|\mathbf{c}_t, \mathbf{b}_{t-1}) p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{c}_t, \mathbf{c}_{t-1}, \boldsymbol{\phi}_{t-1}, \mathbf{Z}_t) \\ &\times p(\mathbf{c}_t|\mathbf{x}_{t-1}, \boldsymbol{\phi}_{t-1}, \mathbf{c}_{t-1}, \mathbf{b}_{t-1}, \mathbf{Z}_t) \end{aligned} \quad (14)$$

Its computation requires substantial developments which are not detailed in this paper for lack of space. However, the expression of an approximation of this simulation law is given in this section. More precisely, this latter is based on a linearization of the observation equation and takes advantage of the conjugation properties of the selected prior densities.
A) The simulation law for $\mathbf{c}_t$ can be expressed as follows:

$$\begin{aligned} &p(\mathbf{c}_t|\mathbf{x}_{t-1}, \boldsymbol{\phi}_{t-1}, \mathbf{c}_{t-1}, \mathbf{b}_{t-1}, \mathbf{Z}_t) \\ &= \exp\left(\frac{1}{2}\mathbf{c}_t^T(\mathbf{R}_b + \mathbf{W}_t)\mathbf{c}_t - \mathbf{b}_{t-1}^T\mathbf{c}_t\right) \\ &\times \prod_{k=1}^{n_t} \left[\delta_{-1}(\mathbf{c}_t(k)) a_{-1,t}(k) + \delta_1(\mathbf{c}_t(k)) a_{1,t}(k)\right] \end{aligned} \quad (15)$$

with δ_a the Dirac impulse function centered around a and $a_{1,t}(k)$ and $a_{-1,t}(k)$ two terms incoming from the computations that directly depends on $\mathbf{Z}_t$.
B) The simulation law for $\mathbf{x}_t$ is the following:

$$p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{c}_t, \mathbf{c}_{t-1}, \boldsymbol{\phi}_{t-1}, \mathbf{Z}_t) = \mathcal{N}(\mathbf{x}_t; \boldsymbol{\mu}_x, \boldsymbol{\Sigma}_x) \quad (16)$$

Table 1. percentage of time the measurement states are correctly estimated

satellite	1	2	3	4	5	6
without BM (%)	65.3	70.2	86.2	84.6	86.1	84.2
with BM (%)	69.7	73.1	86.6	85.3	85.2	85

where the parameters $\boldsymbol{\mu}_x$ and $\boldsymbol{\Sigma}_x$ take different expressions depending on $\mathbf{c}_t$.
C) The simulation law for $\mathbf{b}_t$ is Gaussian:

$$p(\mathbf{b}_t|\mathbf{c}_t, \mathbf{b}_{t-1}) = \mathcal{N}(\mathbf{b}_t; \boldsymbol{\mu}_b^{'}, \boldsymbol{\Sigma}_b^{'}) \quad (17)$$

where $\boldsymbol{\Sigma}_b^{'} = \mathbf{R}_b = \sigma_b^2 \mathbf{I}_{n_t}$ and $\boldsymbol{\mu}_b^{'} = \mathbf{R}_b \mathbf{c}_t + \mathbf{b}_{t-1}$.
D) Finally, the simulation law for $\boldsymbol{\phi}_t$ is as follows:

$$p(\boldsymbol{\phi}_t|\mathbf{x}_t, \boldsymbol{\phi}_{t-1}, \mathbf{c}_t, \mathbf{c}_{t-1}, \mathbf{Z}_t) = \prod_{k=1}^{n_t} \mathcal{IG}(\boldsymbol{\phi}_t(k); \boldsymbol{\varepsilon}(k)) \quad (18)$$

where $\boldsymbol{\varepsilon}(k)$ is a vector whose components depend on whether the GPS measurements at time instant t are affected by multipath or not.

5. SIMULATION RESULTS

Our algorithm is tested on simulated GPS data corresponding to a nearly constant velocity trajectory of 200 s in an urban environment. The receiver has 6 satellites in view during the whole trajectory. The satellite measurements are generated by a routine of our own where GPS almanac data are used to compute the satellite positions. To simulate the multipath appearance, a white Gaussian noise is added on the measurements from the two angularly closest couples of satellites. This happens for the couple (1,4) (respectively (2,6)) in the temporal intervals [10,80]s, [100,140]s and [160,190]s (respectively [20,90]s, [110,150]s and [170,200]s). The standard-deviation of the multipath error is chosen equal to 20 m for satellites 1 and 2, 80 m for satellite 4 and 40 m for satellite 6.
To show the relevance of taking into account the spatial dependencies, we run our algorithm with the entries of $\mathbf{W}_t$ either set as in (8) or set to zero. In both cases, $N = 1000$ particles are used. Multipath are considered detected when the *a posteriori* probability $Pr[\mathbf{c}_t(k) = 1|\mathbf{Z}_{1:t}]$ becomes greater than 0.5. Fig. 4 shows, for 3 distinct satellites, this probability averaged on 50 realizations of simulated measurement noise. We observe that the presence of multipath is correctly detected and table 1 shows that using the BM improves the percentage of good detections.
Fig. 5 shows the evolution of the square root of the horizontal estimation mean-square error (RMSE) associated with both tested algorithms. It should be noted that the BM helps detecting multipath of small amplitudes, hence the impact on the positioning error is not significant.

6. CONCLUSION

Our main motivation was to study the relevance of the BM to define dependencies between GPS satellite measurements for

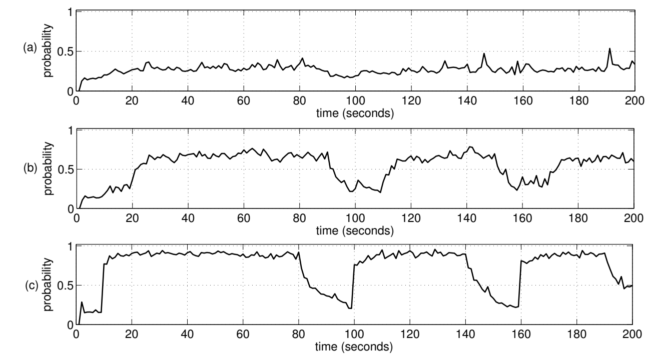

Fig. 4. Probability $Pr[\mathbf{c}_t(k) = 1|\mathbf{Z}_{1:t}]$ for (a) satellite 3, (b) satellite 2 and (c) satellite 4 for the algorithm with BM.

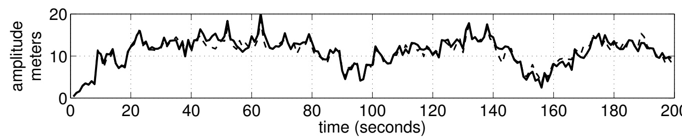

Fig. 5. Evolution of the RMSE for the algorithm with $\mathbf{W}_t = 0$ (dashed curve) and with $\mathbf{W}_t \neq 0$ (plain curve)

positioning in an urban environment. A fully Bayesian hierarchical model was proposed. The latter automatically adapts to the varying multipath environment with few parameters to be tuned by the practitioner. Furthermore, we are currently working on the development of a procedure to adjust on-line the entries of the interaction matrix from the measurements gathered by the GPS receiver.

7. REFERENCES

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